

APOLLO

Calculus

SL

Mathematics: applications and interpretation

First assessment 2021

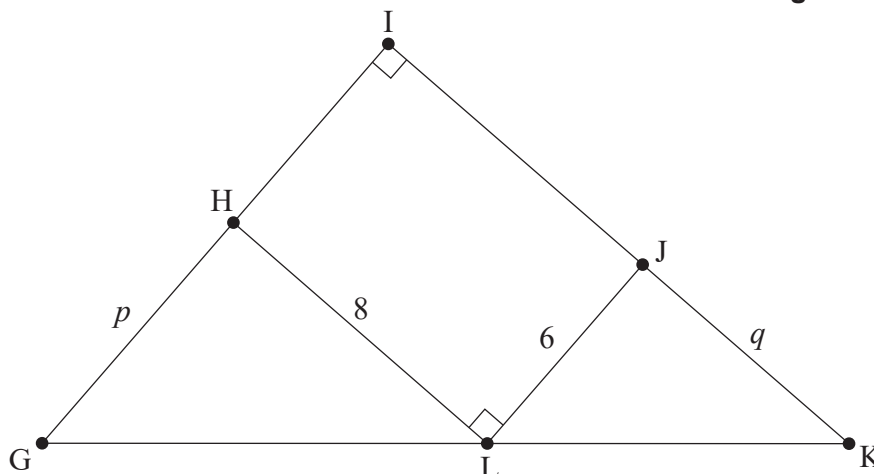
54 questions | questions

12. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is $A \text{ cm}^2$.

(a) (i) Find A in terms of p and q .

(ii) Show that $A = \frac{192}{q} + 3q + 48$.

[4]

(b) Find $\frac{dA}{dq}$.

[2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q .

(ii) Hence, or otherwise, find this value of q .

[2]

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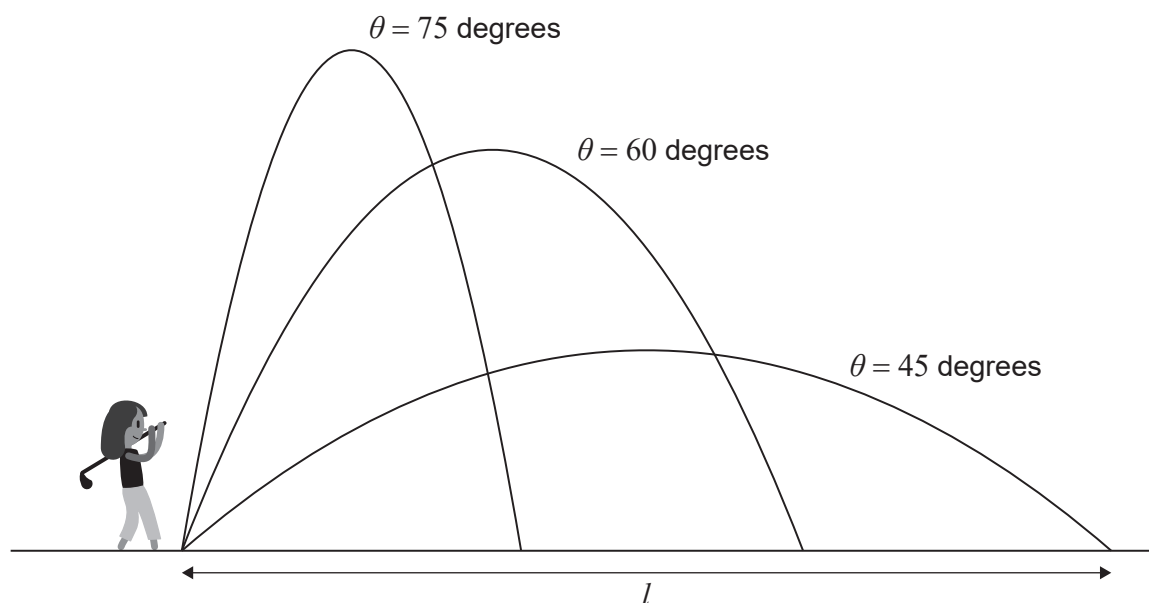
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13. [Maximum mark: 8]

Sieun hits golf balls into the air. Each time she hits a ball she records θ , the angle at which the ball is launched into the air, and l , the horizontal distance, in metres, which the ball travels from the point of contact to the first time it lands. The diagram below represents this information.



Sieun analyses her results and concludes:

$$\frac{dl}{d\theta} = -0.2\theta + 9, \quad 35^\circ \leq \theta \leq 75^\circ.$$

- (a) Determine whether the graph of l against θ is increasing or decreasing at $\theta = 50^\circ$. [3]

Sieun observes that when the angle is 40° , the ball will travel a horizontal distance of 205.5 m.

- (b) Find an expression for the function $l(\theta)$. [5]

(This question continues on the following page)



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13. [Maximum mark: 7]

A company produces and sells electric cars. The company’s profit, P , in thousands of dollars, changes based on the number of cars, x , they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{dP}{dx} = -1.6x + 48, \quad x \geq 0.$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

- (a) Find an expression for P in terms of x . [5]

The company regularly increases the number of cars it produces.

- (b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer. [2]

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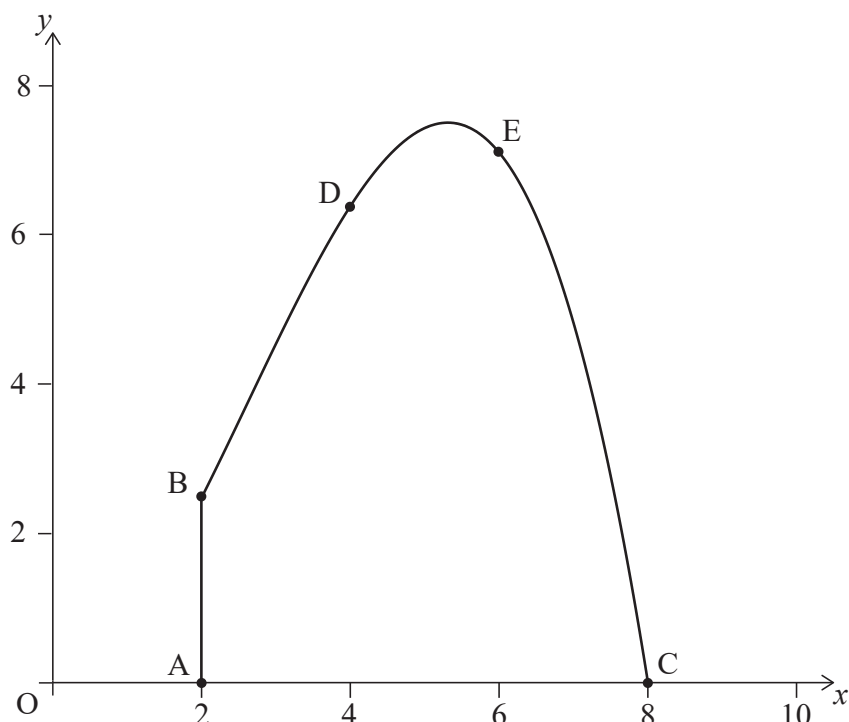
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5. [Maximum mark: 13]

The cross-sectional view of a tunnel is shown on the axes below. The line $[AB]$ represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y = -0.1x^3 + 0.8x^2$, $2 \leq x \leq 8$, relative to an origin O .



Point A has coordinates $(2, 0)$, point B has coordinates $(2, 2.4)$, and point C has coordinates $(8, 0)$.

- (a) (i) Find $\frac{dy}{dx}$.
- (ii) Hence find the maximum height of the tunnel. [6]
- When $x = 4$ the height of the tunnel is 6.4 m and when $x = 6$ the height of the tunnel is 7.2 m. These points are shown as D and E on the diagram, respectively.
- (b) Use the trapezoidal rule, with three intervals, to estimate the cross-sectional area of the tunnel. [3]
- (c) (i) Write down the integral which can be used to find the cross-sectional area of the tunnel.
- (ii) Hence find the cross-sectional area of the tunnel. [4]

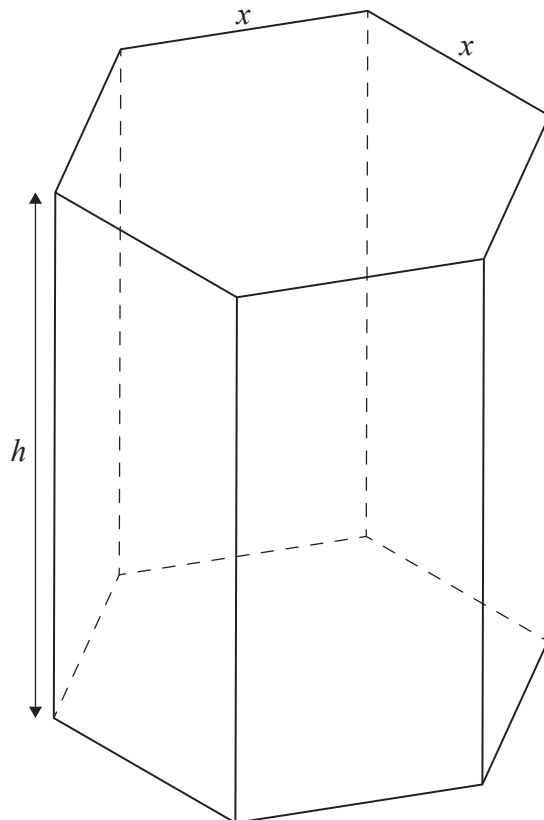
References:

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5. [Maximum mark: 16]

A hollow chocolate box is manufactured in the form of a right prism with a regular hexagonal base. The height of the prism is h cm, and the top and base of the prism have sides of length x cm.

diagram not to scale



- Given that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, show that the area of the base of the box is equal to $\frac{3\sqrt{3}x^2}{2}$. [2]
- Given that the total external surface area of the box is 1200 cm^2 , show that the volume of the box may be expressed as $V = 300\sqrt{3}x - \frac{9}{4}x^3$. [5]
- Sketch the graph of $V = 300\sqrt{3}x - \frac{9}{4}x^3$, for $0 \leq x \leq 16$. [2]
- Find an expression for $\frac{dV}{dx}$. [2]
- Find the value of x which maximizes the volume of the box. [2]
- Hence, or otherwise, find the maximum possible volume of the box. [2]

The box will contain spherical chocolates. The production manager assumes that they can calculate the exact number of chocolates in each box by dividing the volume of the box by the volume of a single chocolate and then rounding down to the nearest integer.

- Explain why the production manager is incorrect. [1]

References:

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6. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t , the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t -axis.

The rate, R , is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

- (a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours.
- [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

- (b) Find the percentage error of the estimate found in part (a).
- [2]

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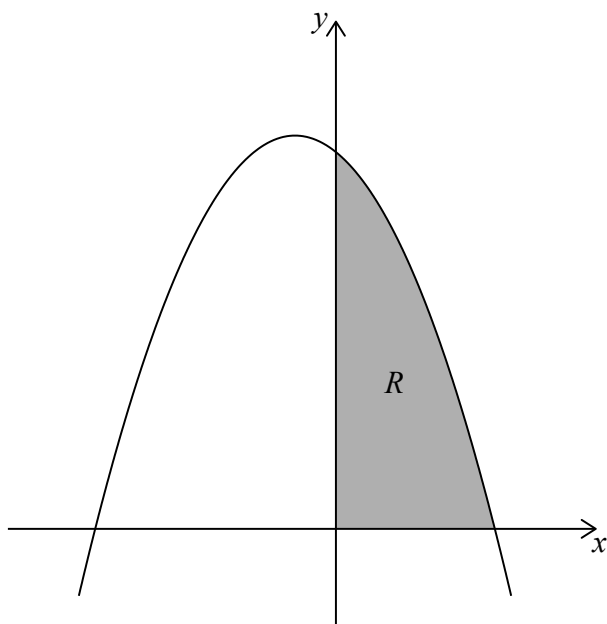
The surface area of an open box with a volume of 32 cm^3 and a square base with sides of length $x \text{ cm}$ is given by $S(x) = x^2 + \frac{128}{x}$ where $x > 0$.

- (a) Find $S'(x)$. [3]
- (b) (i) Solve $S'(x) = 0$.
- (ii) Interpret your answer to (b)(i) in context. [3]

[illegible]

10. [Maximum mark: 5]

The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x -axis, y -axis and the graph of f .



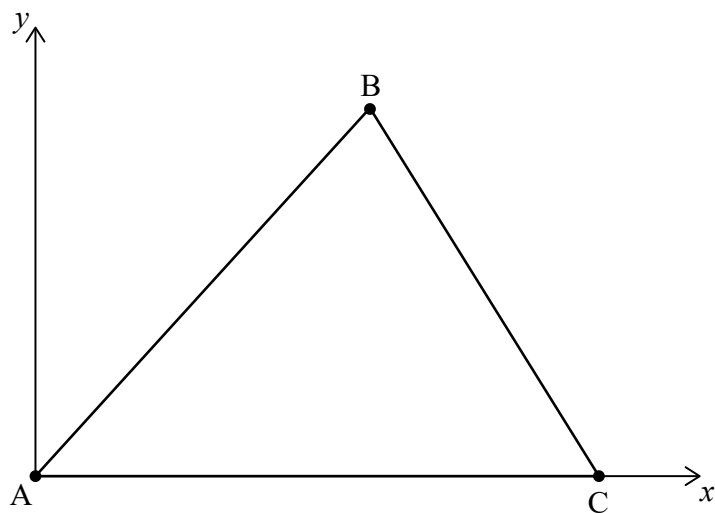
(a) Write down an integral for the area of region R .

[2]

(b) Find the area of region R .

[1]

The three points $A(0, 0)$, $B(3, 10)$ and $C(a, 0)$ define the vertices of a triangle.



(c) Find the value of a , the x -coordinate of C , such that the area of the triangle is equal to the area of region R .

[2]

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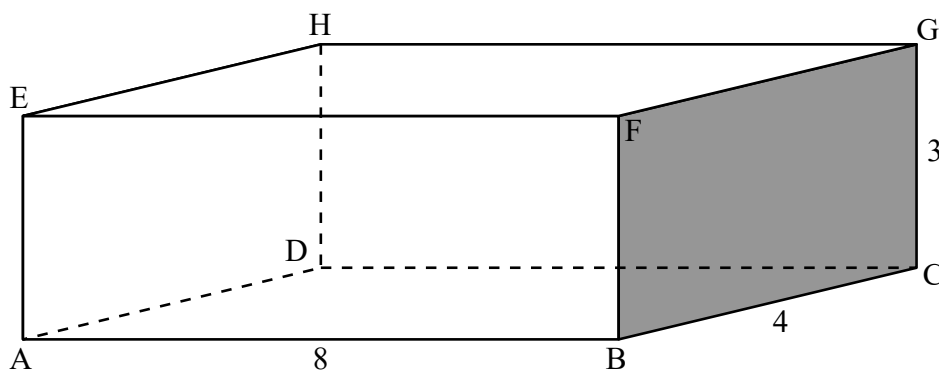
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4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length 8 cm, width 4 cm and height 3 cm. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm^2 . [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{dP}{dx} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of 20 000 boxes is \$1700.

(d) Find $P(x)$. [5]

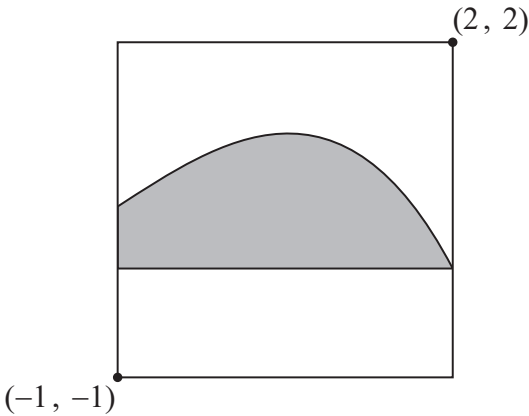
(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

Turn over

6. [Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1, -1)$ and the top right corner has coordinates $(2, 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x -axis. Distances are measured in metres.

- (a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	-1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \frac{-x^3 - 3x^2 + 4x + 12}{10}$ to draw the curve.

- (b) Find the exact area of the shaded region in the painting. [2]
- (c) Find the area of the unshaded region in the painting. [2]

(This question continues on the following page)



(Question 6 continued)

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Turn over

9. [Maximum mark: 7]

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3, x \neq 0$.

- (a) Find $f'(x)$.

[3]
- (b) Find the equation of the normal to the curve $y = f(x)$ at $(1, 2)$ in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$.

[4]

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12. [Maximum mark: 6]

A company’s profit per year was found to be changing at a rate of

$$\frac{dP}{dt} = 3t^2 - 8t$$

where P is the company’s profit in thousands of dollars and t is the time since the company was founded, measured in years.

- (a) Determine whether the profit is increasing or decreasing when $t = 2$. [2]

One year after the company was founded, the profit was 4 thousand dollars.

- (b) Find an expression for $P(t)$, when $t \geq 0$. [4]



Turn over

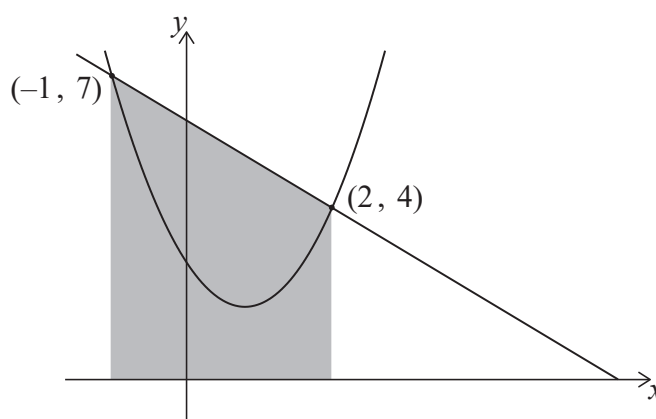
6. [Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x -axis has been shaded.

diagram not to scale

Diagram 1



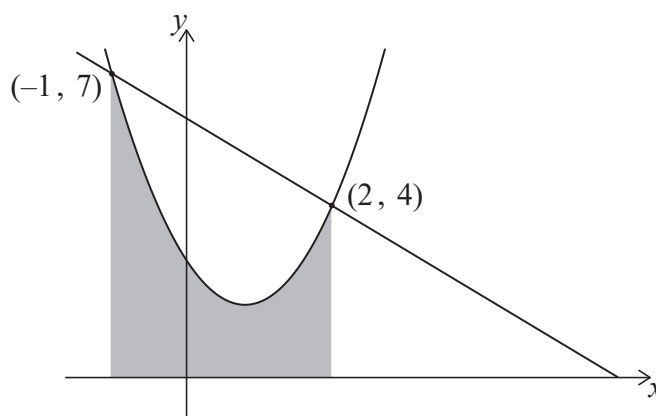
- (a) Calculate the area of the shaded region in **diagram 1**.

[2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x -axis has been shaded.

diagram not to scale

Diagram 2



- (b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

- (ii) Calculate the area of this region.

[3]

- (c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$.

[2]

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11. [Maximum mark: 7]

Consider the function $f(x) = x^2 - \frac{3}{x}$, $x \neq 0$.

(a) Find $f'(x)$. [2]

Line L is a tangent to $f(x)$ at the point $(1, -2)$.

(b) Use your answer to part (a) to find the gradient of L . [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]



Turn over

5. [Maximum mark: 15]

A cafe makes x litres of coffee each morning. The cafe's profit each morning, C , measured in dollars, is modelled by the following equation

$$C = \frac{x}{10} \left(k^2 - \frac{3}{100} x^2 \right)$$

where k is a positive constant.

(a) Find an expression for $\frac{dC}{dx}$ in terms of k and x . [3]

(b) Hence find the maximum value of C in terms of k . Give your answer in the form pk^3 , where p is a constant. [4]

The cafe's manager knows that the cafe makes a profit of \$426 when 20 litres of coffee are made in a morning.

(c) (i) Find the value of k .
(ii) Use the model to find how much coffee the cafe should make each morning to maximize its profit. [3]

(d) Sketch the graph of C against x , labelling the maximum point and the x -intercepts with their coordinates. [3]

The manager of the cafe wishes to serve as many customers as possible.

(e) Determine the maximum amount of coffee the cafe can make that will not result in a loss of money for the morning. [2]

References:

4. Aline Escobar., n.d. Cowboy. [image online] Available at: <https://thenounproject.com/search/?q=cowboy&i=1080130>
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<https://creativecommons.org/licenses/by-sa/3.0/deed.en> [Accessed 13/05/2021]. Source adapted.

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13. [Maximum mark: 8]

Giles charges a customer per hour to hire his boat. It is known that

$$\frac{dP}{dt} = 20 - \frac{980}{t^2}, \quad 0 < t \leq 12$$

where P is the cost per hour, in Norwegian krone (NOK), that the customer is charged and t is the time, in hours, spent on the boat.

The cost per hour has a local minimum when the boat is hired for h hours.

- (a) Find the value of h . [2]

Sandra hired Giles' boat for 5 hours and was charged NOK 328 per hour. Yvonne hires Giles' boat for 7 hours.

- (b) Show that the cost per hour for Yvonne is NOK 312. [6]



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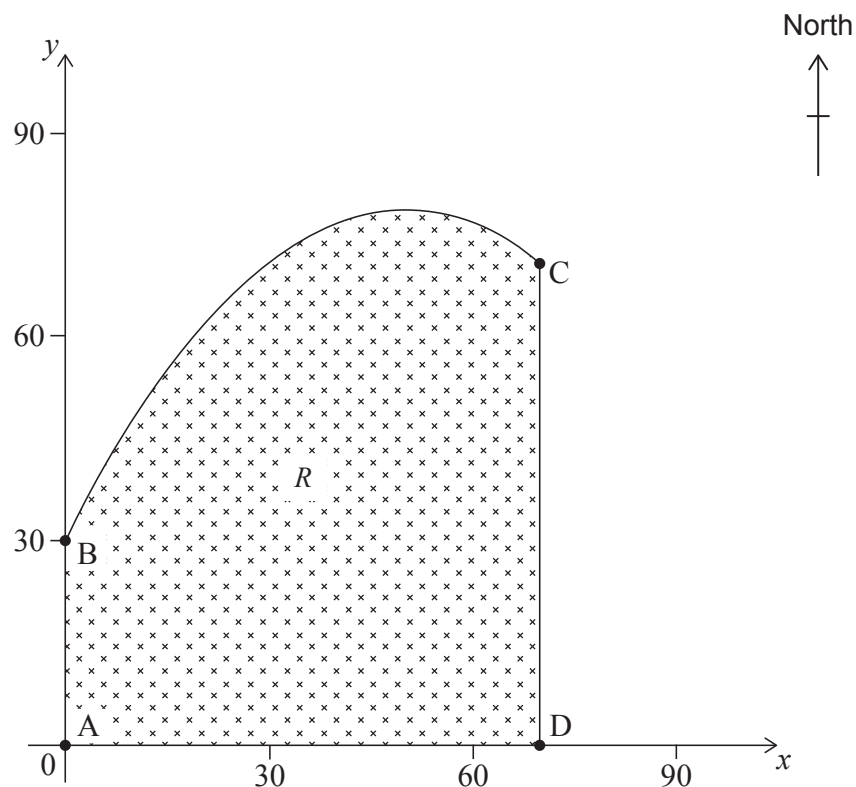
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5. [Maximum mark: 17]

Linda owns a field, represented by the shaded region R . The plan view of the field is shown in the following diagram, where both axes represent distance and are measured in metres.



The segments $[AB]$, $[CD]$ and $[AD]$ respectively represent the western, eastern and southern boundaries of the field. The function, $f(x)$, models the northern boundary of the field between points B and C and is given by

$$f(x) = \frac{-x^2}{50} + 2x + 30, \text{ for } 0 \leq x \leq 70.$$

(a) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the point on the field that is furthest north.

[5]

Point A has coordinates $(0, 0)$, point B has coordinates $(0, 30)$, point C has coordinates $(70, 72)$ and point D has coordinates $(70, 0)$.

(b) (i) Write down the integral which can be used to find the area of the shaded region R .

(ii) Find the area of Linda's field.

[4]

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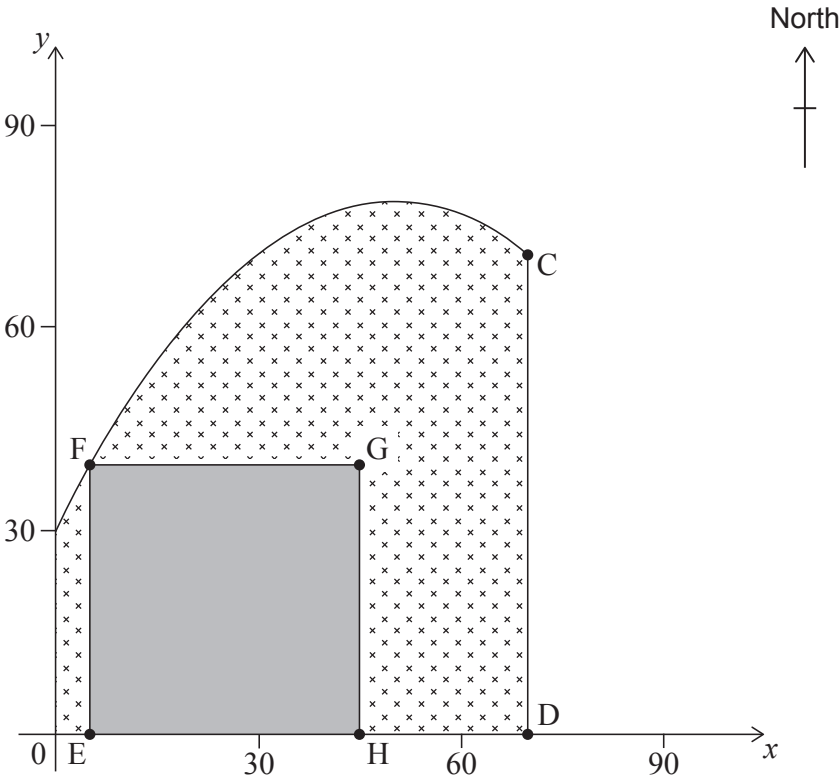
(Question 5 continued)

Linda used the trapezoidal rule with ten intervals to estimate the area. This calculation underestimated the area by 11.4m^2 .

- (c) (i) Calculate the percentage error in Linda’s estimate.
- (ii) Suggest how Linda might be able to reduce the error whilst still using the trapezoidal rule.

[3]

Linda would like to construct a building on her field. The **square** foundation of the building, EFGH, will be located such that [EH] is on the southern boundary and point F is on the northern boundary of the property. A possible location of the foundation of the building is shown in the following diagram.



The area of the square foundation will be largest when [GH] lies on [CD].

- (d) (i) Find the x -coordinate of point E for the largest area of the square foundation of building EFGH.
- (ii) Find the largest area of the foundation.

[5]

References:

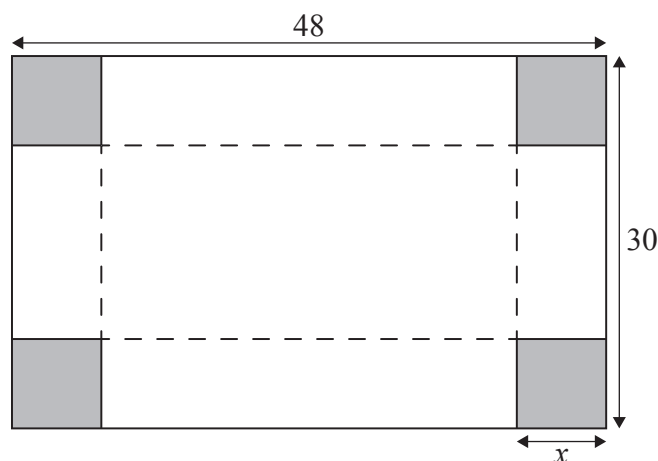
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7. [Maximum mark: 6]

A rectangular box, with an open top, is to be constructed from a piece of cardboard that measures 48 cm by 30 cm.

Squares of equal size will be cut from the corners of the cardboard, as indicated by the shading in the diagram. The sides will then be folded along the dotted lines to form the box.

diagram not to scale



The volume of the box, in cubic centimetres, can be modelled by the function $V(x) = (48 - 2x)(30 - 2x)(x)$, for $0 < x < k$, where x is the length of the sides of the squares removed in centimetres.

(a) Write down the maximum possible value of k in this context. [1]

(b) Find the value of x that maximizes the volume of the box. [2]

A second piece of 48 cm by 30 cm cardboard is damaged and a strip 2 cm wide must be removed from all four sides. A box will then be constructed in a similar manner from the remaining cardboard.

(c) Calculate the maximum possible volume of the box made from the second piece of cardboard. [3]

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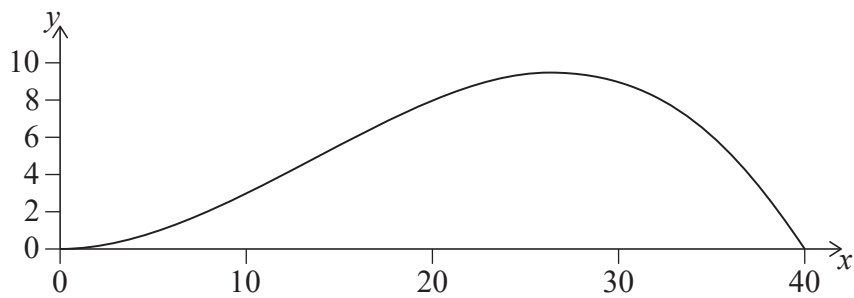
(Question 7 continued)

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9. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

- (a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model.
- [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3$, $0 \leq x \leq 40$.

- (b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places.
- [4]
- (c) Find the percentage error in the area found using the trapezoidal rule.
- [2]

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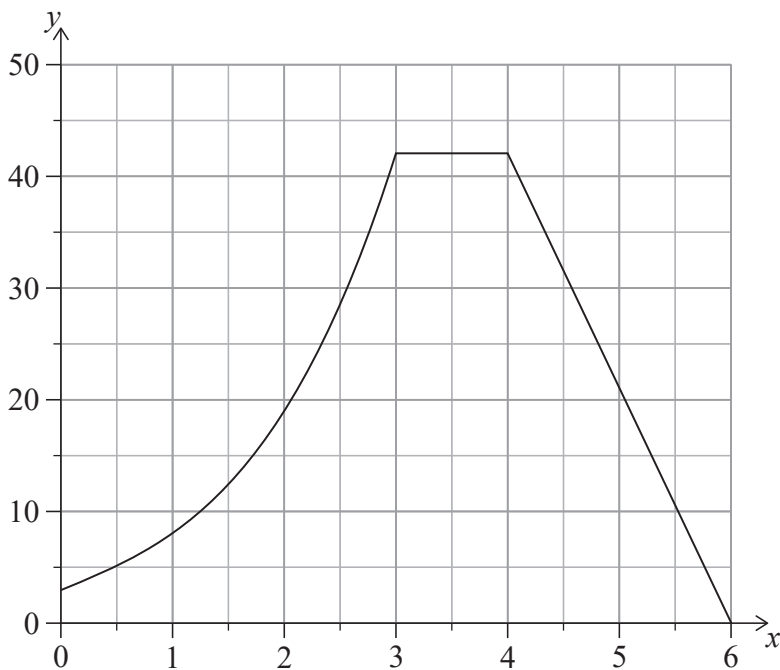
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Turn over

13. [Maximum mark: 9]

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

- (a) Calculate an estimate for the area in the interval $0 \leq x \leq 3$ by using the trapezoidal rule with three equal intervals. [2]

It is known that $f'(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

- (b) Find an expression for $f(x)$, in the domain $0 < x < 3$. [4]
- (c) **Hence** find the actual area of the **entire** cross-section. [3]

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[illegible]

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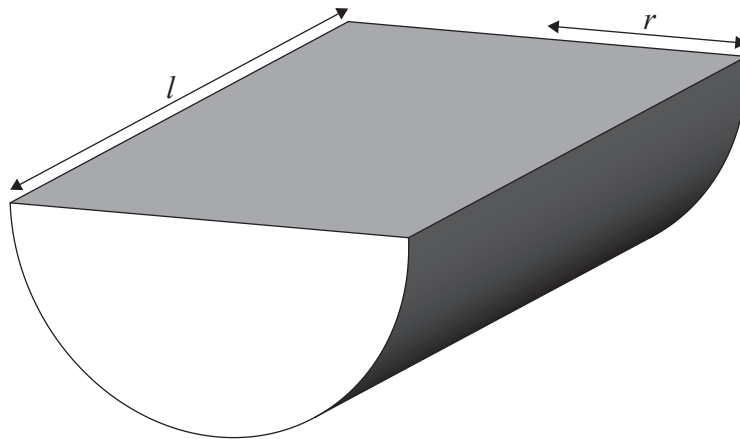
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5. [Maximum mark: 17]

A large closed container, in the shape of a half cylinder with a rectangular lid, is to be constructed with a volume of 0.8 m^3 . The container has a length of l metres and a radius of r metres.

diagram not to scale



- (a) Find an exact expression for l in terms of r and π .

[2]

The container will be constructed using two different materials. The material for both the curved surface and the rectangular lid of the container costs \$4.40 per square metre. The material for the semicircular ends of the container costs \$ p per square metre.

The cost, C , of the materials to construct the container can be written in terms of r and p (where $p > 0$ and $r > 0$).

- (b) Show that $C = 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$.

[4]

- (c) Find $\frac{dC}{dr}$.

[3]

The cost of materials to construct the container is minimized when the radius of the container, r , is 0.7 m.

- (d) Find the value of p .

[3]

In total, 350 containers will be constructed at this minimum cost.

- (e) Calculate the cost of materials, to the nearest dollar, to construct all 350 containers.

[3]

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(Question 5 continued)

The materials for constructing the containers can be purchased at a discount according to the information in the table.

Cost of materials (\$ C) before discount	Discount applied to entire order
$1000 \leq C < 2500$	1 %
$2500 \leq C < 5000$	4 %
$5000 \leq C < 10\,000$	8 %
$C \geq 10\,000$	10 %

- (f) Determine the cost of materials for 350 containers after the discount is applied. [2]

References:

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5. [Maximum mark: 16]

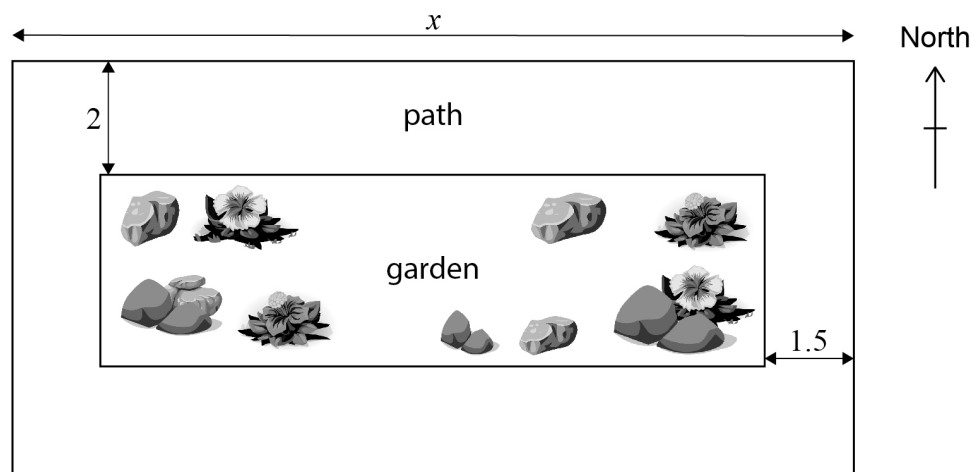
A particular park consists of a rectangular garden, of area $A \text{ m}^2$, and a concrete path surrounding it. The park has a total area of 1200 m^2 .

The width of the path at the north and south side of the park is 2 m.

The width of the path at the west and east side of the park is 1.5 m.

The length of the park (along the north and south sides) is x metres, $3 < x < 300$.

diagram not to scale



- (a) (i) Write down the length of the garden in terms of x . [5]
- (ii) Find an expression for the width of the garden in terms of x . [4]
- (iii) Hence show that $A = 1212 - 4x - \frac{3600}{x}$. [5]
- (b) Find the possible dimensions of the park if the area of the garden is 800 m^2 . [4]
- (c) Find an expression for $\frac{dA}{dx}$. [3]
- (d) Use your answer from part (c) to find the value of x that will maximize the area of the garden. [2]
- (e) Find the maximum possible area of the garden. [2]

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3. [Maximum mark: 6]

Consider the graph of the following function:

$$f(x) = \frac{16}{x} + \frac{x^2}{8}, \text{ for } x \neq 0.$$

- (a) Write down the equation of the vertical asymptote of $f(x)$. [1]
- (b) Find $f'(x)$. [3]
- (c) Write down the interval in which $f(x)$ is increasing. [2]



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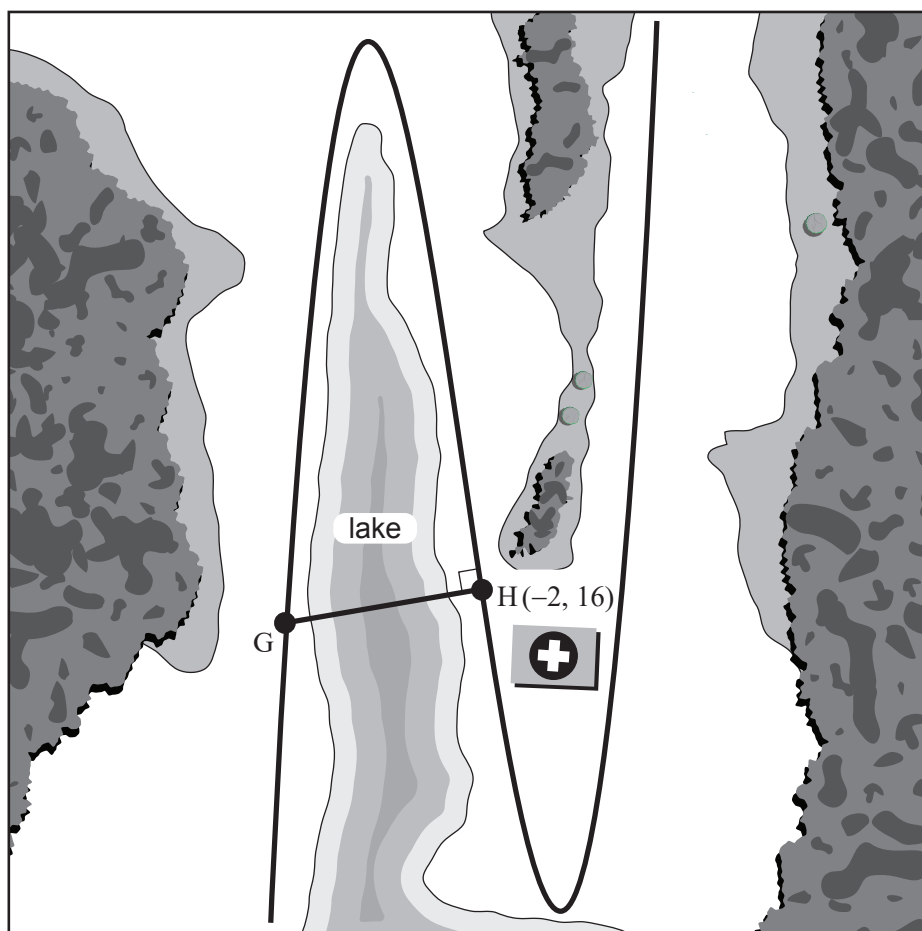
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x + 2)^3 + 3x^2 - 2x$. All distances in the map are in kilometres.

The local hospital is located at point H, which has coordinates $(-2, 16)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point H and will connect the hospital to point G.

diagram not to scale



- Using your graphic display calculator, find the value of $r'(-2)$. [2]
- Find the equation of the line normal to $r(x)$ at point H, which can be used to model the new bridge. [2]
- Hence, determine the length of the new bridge. [3]

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[illegible]

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3. [Maximum mark: 6]

Consider the graph of the following function:

$$g(x) = \frac{8}{x} + \frac{x^2}{2}, \text{ for } x \neq 0.$$

- (a) Write down the equation of the vertical asymptote of $g(x)$. [1]
- (b) Find $g'(x)$. [3]
- (c) Write down the interval in which $g(x)$ is increasing. [2]



Turn over

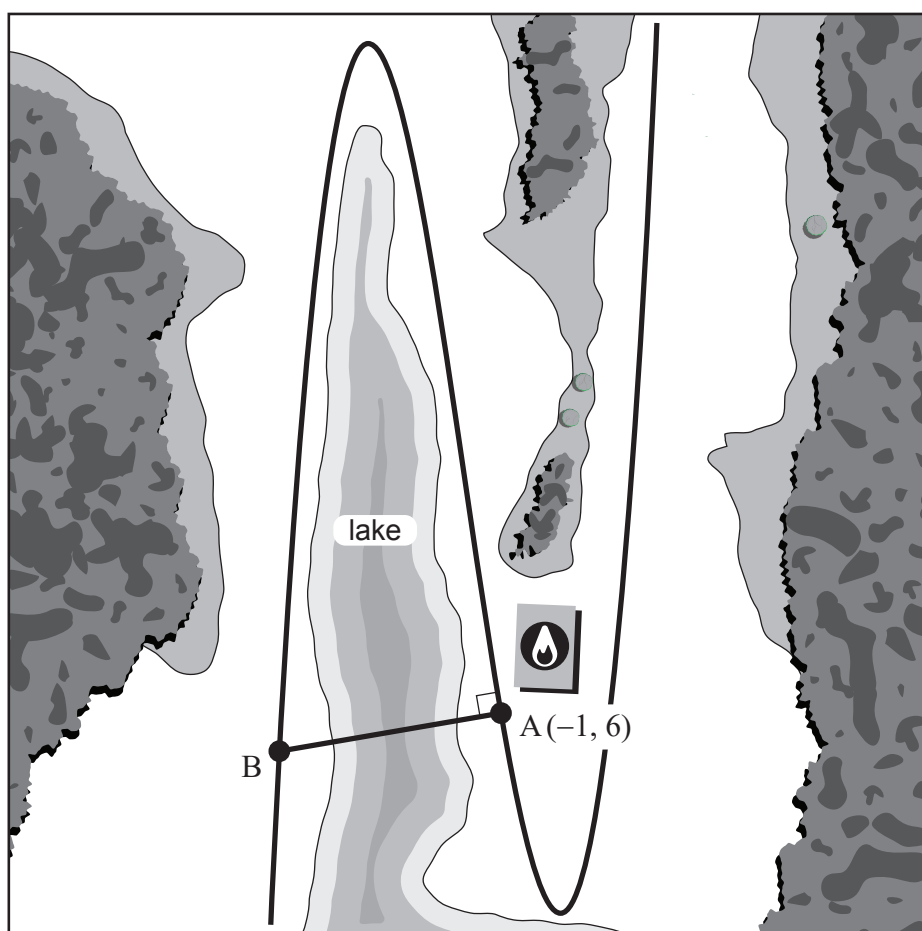
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x + 1)^3 + 2x^2 - 4x$. All distances in the map are in kilometres.

The local fire station is located at point A, which has coordinates $(-1, 6)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point A and will connect the fire station to point B.

diagram not to scale



- Using your graphic display calculator, find the value of $r'(-1)$. [2]
- Find the equation of the line normal to $r(x)$ at point A, which can be used to model the new bridge. [2]
- Hence, determine the length of the new bridge. [3]

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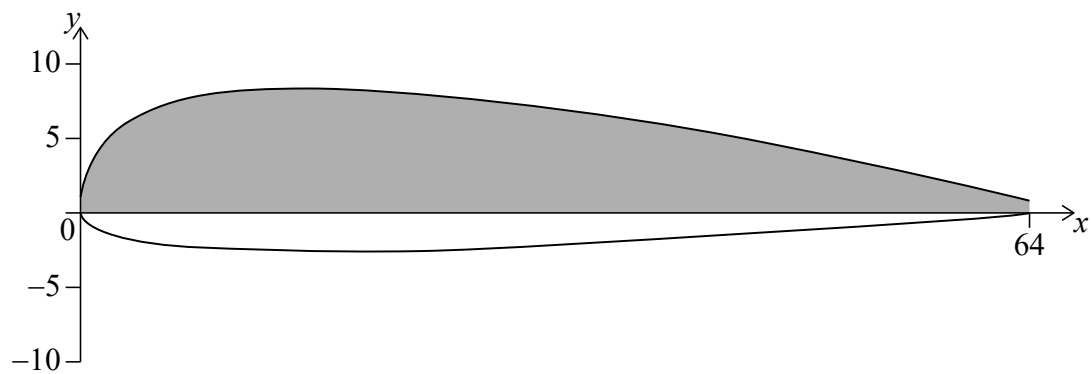


[illegible]

Apollo | Original source crop | 43

1. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x -axis and the curve with equation

$$y = 4\sqrt{x} - \frac{x}{2} + 1, \text{ for } 0 \leq x \leq 64$$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

- (a) Find the values of a , b and c , shown in the table. [3]

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

- (b) Find Jan’s estimate of the shaded area of the cross-section. [3]
- (c) (i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section.
- (ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]
- (d) Calculate the percentage error of Jan’s estimate in part (b). [2]

Turn over

5. [Maximum mark: 19]

Mosaic Industries is an international company that sells cell phone cases. Their market research predicts that the average number of cases they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each case.

A salesperson suggests that Mosaic Industries should sell the cases for 16 EUR each.

- (a) Find the average number of cases that this model predicts Mosaic Industries will sell at this price. [2]
- (b) Calculate Mosaic Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function $R(x)$ that can be used to predict Mosaic Industries' average monthly income, before expenses, at any selling price, x . [1]

Mosaic Industries has 10 000 EUR of fixed monthly operational costs. Additionally, Mosaic Industries must pay their phone case supplier 10 EUR for each case.

- (d) Calculate Mosaic Industries' average monthly profit if they sell each case at a price of 16 EUR. [3]
- (e) Show that the average monthly profit for any selling price, x , can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]
- (f) (i) Find $P'(x)$.
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Mosaic Industries negotiates a new deal with their phone case supplier. Under the new deal, the supplier agrees to discount the cost of each case based on the number of cases purchased by Mosaic Industries. The cost charged by the supplier for each case can be found using the function

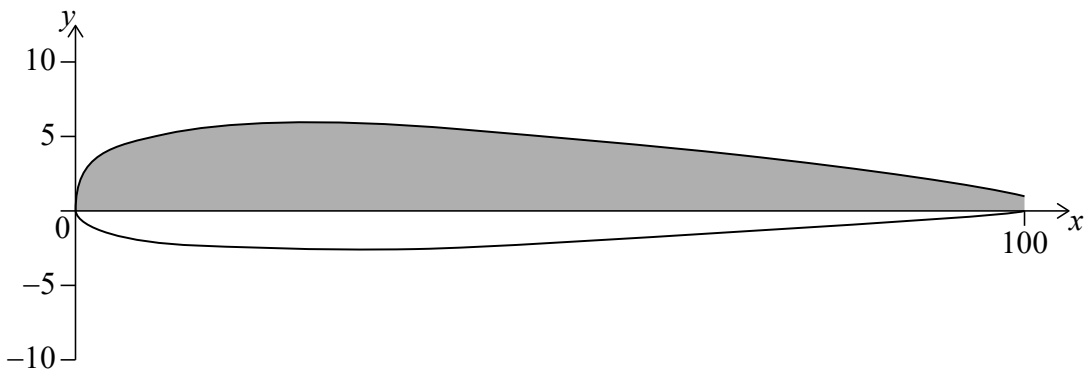
$$C(n) = 10 - 0.0001n$$

where n represents the number of cases sold by Mosaic Industries.

- (g) Find the function that can be used to find Mosaic Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per case, that Mosaic Industries should choose in order to maximize their average monthly profit under the new deal. [2]

1. [Maximum mark: 12]

Gabriel is investigating the shape of model airplane wings. A cross-section of one of the wings is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x -axis and the curve with equation

$$y = 2\sqrt{x} - \frac{x}{5} + 1, \text{ for } 0 \leq x \leq 100$$

where x is the distance, in cm, from the front of the wing and y is the height, in cm, above the horizontal axis through the wing, as shown in the diagram.

- (a) Find the values of a , b and c , shown in the table. [3]

x (cm)	0	25	50	75	100
y (cm)	1	a	b	c	1

Gabriel uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the wing.

- (b) Find Gabriel's estimate of the shaded area of the cross-section. [3]
- (c) (i) Write down the integral that Gabriel can use to find the exact area of the shaded part of the cross-section.
- (ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]
- (d) Calculate the percentage error of Gabriel's estimate in part (b). [2]

Turn over

5. [Maximum mark: 19]

Howell Industries is an international company that sells travel coffee mugs. Their market research predicts that the average number of mugs they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each mug.

A salesperson suggests that Howell Industries should sell the mugs for 16 EUR each.

- (a) Find the average number of mugs that this model predicts Howell Industries will sell at this price. [2]
- (b) Calculate Howell Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function $R(x)$ that can be used to predict Howell Industries' average monthly income, before expenses, at any selling price, x . [1]

Howell Industries has 10 000 EUR of fixed monthly operational costs. Additionally, Howell Industries must pay their mug supplier 10 EUR for each coffee mug.

- (d) Calculate Howell Industries' average monthly profit if they sell each mug at a price of 16 EUR. [3]
- (e) Show that the average monthly profit for any selling price, x , can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]
- (f) (i) Find $P'(x)$.
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Howell Industries negotiates a new deal with their mug supplier. Under the new deal, the supplier agrees to discount the cost of each mug based on the number of mugs purchased by Howell Industries. The cost charged by the supplier for each mug can be found using the function

$$C(n) = 10 - 0.0001n$$

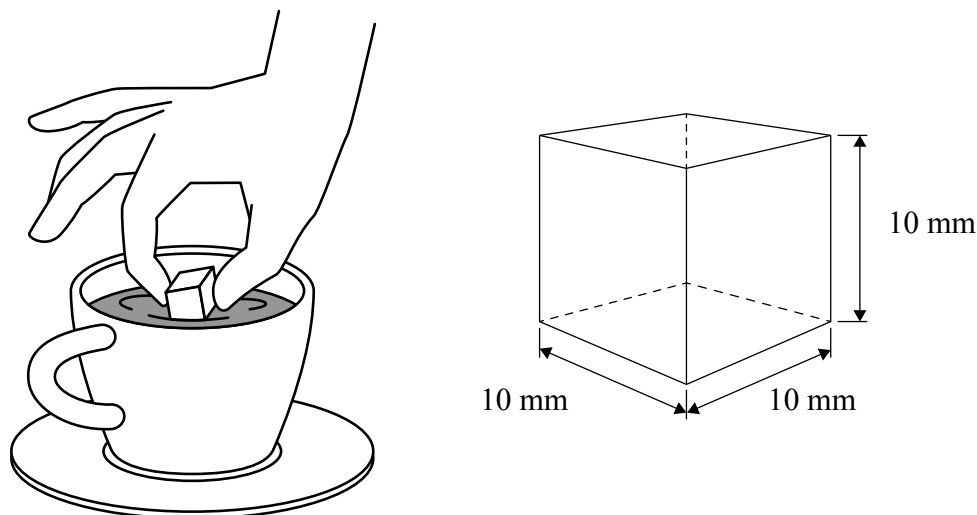
where n represents the number of mugs sold by Howell Industries.

- (g) Find the function that can be used to find Howell Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per mug, that Howell Industries should choose in order to maximize their average monthly profit under the new deal. [2]

9. [Maximum mark: 8]

Kyungyoon investigates the rate at which a cubical block of sugar dissolves in hot coffee. Initially, the cube has side lengths of 10 mm. This information is illustrated in the following diagrams.

diagram not to scale



Kyungyoon predicts that, as the block of sugar dissolves, each side length will decrease at a constant rate of 0.2 mm per second.

- (a) According to this model, find
- (i) the length of one side of a block of sugar, 20 seconds after it is placed in hot coffee.
 - (ii) the volume of a block of sugar, 20 seconds after it is placed in hot coffee. [3]

Let the function $V(t)$ represent the volume of the block of sugar, mm^3 , t seconds after it is placed in hot coffee. $V(t)$ is given by

$$V(t) = 1000 - 60t + 1.2t^2 - 0.008t^3, \text{ for } 0 \leq t \leq 50.$$

- (b) Find $V'(t)$. [2]
- (c) Find the rate of change of the volume of the block of sugar at $t = 20$. [2]
- (d) State one reason why the side length of the cube may not always decrease at a constant rate. [1]

(This question continues on the following page)



(Question 9 continued)

[illegible]

Turn over

10. [Maximum mark: 7]

The gradient of the **normal** to the curve $y = ax^2 + bx - 10$ at the point T(2, 4) is $-\frac{1}{3}$.

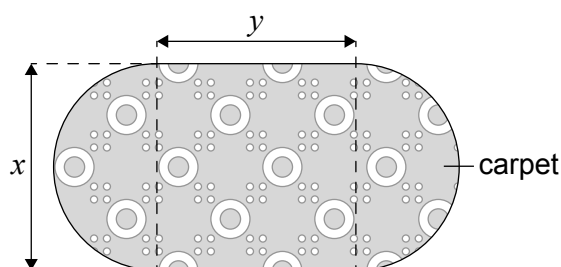
Calculate the value of a and the value of b .

[illegible]

12. [Maximum mark: 8]

A company is designing a new carpet. The intended design of the carpet is in the shape of a rectangle with a semi-circle at each end.

The width of the rectangle is y metres and the diameter of each semi-circle is x metres, with $x > 0$ and $y \geq 0$.



The company has decided that the perimeter of the carpet will be 20 metres and would like to maximize its area.

- (a) Find an expression for the perimeter in terms of x and y . [1]
- (b) Show that the area, $A \text{ m}^2$, of the carpet can be expressed as $A = 10x - \frac{\pi x^2}{4}$. [3]
- (c) Find $\frac{dA}{dx}$. [2]
- (d) Hence find the **exact** value of x for which the area is a maximum. [2]

(This question continues on the following page)



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will not be marked.



3. [Maximum mark: 13]

A shop uses the following model to estimate n , the number of smoothies sold per day, in terms of x , the price of a single smoothie in pesos.

$$n = \frac{40\,000}{x^2}$$

The maximum number of smoothies the shop can make in a day is 400.

- (a) Find the maximum price they could charge per smoothie for the shop to sell 400 in one day. [2]

- (b) On a day when the shop sells smoothies at 50 pesos each, use the model to find
- (i) the number of smoothies sold.
 - (ii) the total income from the smoothies sold. [2]

The cost of making each smoothie is 20 pesos. The profit per day (P) is the total income from the sale of smoothies that day minus the cost of making them.

- (c) (i) Show that, according to the model, $P = \frac{40\,000}{x} - \frac{800\,000}{x^2}$.
- (ii) Find $\frac{dP}{dx}$.
- (iii) Find the value of x for which $\frac{dP}{dx} = 0$.
- (iv) Find the number of smoothies sold when the profit is maximized. [9]

Turn over

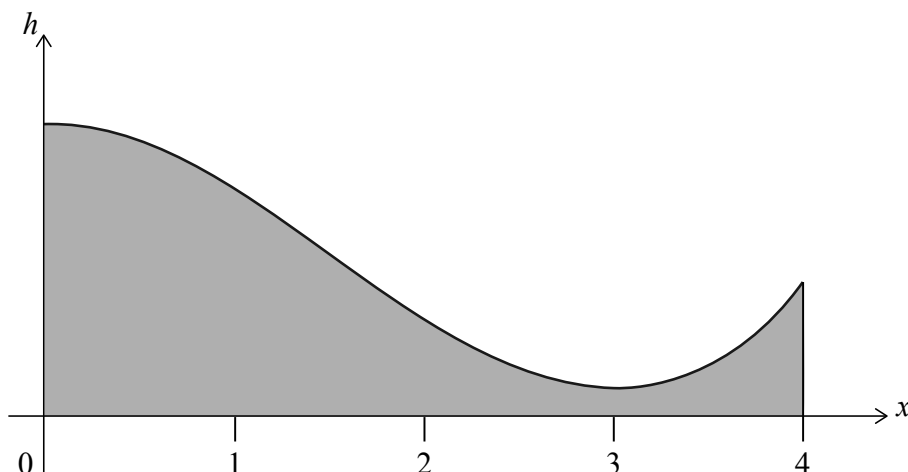
5. [Maximum mark: 21]

Andrew designs a fibreglass water slide for a water park.

Let x be the horizontal distance, in metres, from the start of the water slide.

Let h be the height, in metres, of the water slide above the horizontal ground.

The following diagram shows a cross section of the water slide. The water slide will be supported by a concrete structure, represented by the shaded region in the diagram.



It is known that $h'(x) = 0.6x^2 - 1.8x$.

- (a) Find the gradient of the water slide when $x = 2$. [2]

At the exit point of the water slide, $x = 4$ and $h = 1.4$.

- (b) Find an expression for $h(x)$. [4]

- (c) (i) Write down an integral that can be used to find the area of the shaded region.

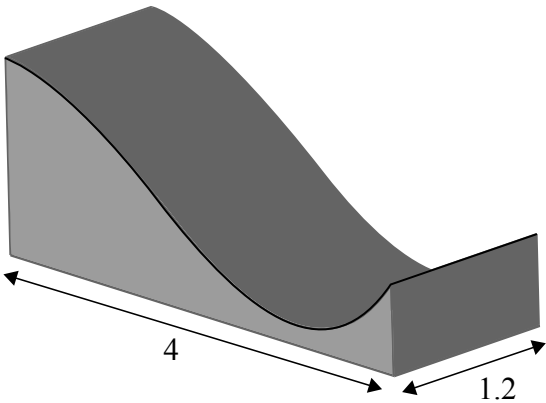
- (ii) Find the cross-sectional area of the concrete structure. [4]

(This question continues on the following page)

(Question 5 continued)

The water slide and concrete structure have a uniform cross section and a width of 1.2 metres, as shown in the following diagram.

diagram not to scale



- (d) Find the volume of the concrete structure. [1]

To comply with safety regulations, Andrew updates this design. The start of the design will stay the same. Then, over the domain $2 \leq x \leq 4$, the updated design must satisfy the following conditions.

$$h(x) = ax^2 + bx + c \qquad h(4) = 1 \qquad h'(2) = -1.2 \qquad h'(3) = 0$$

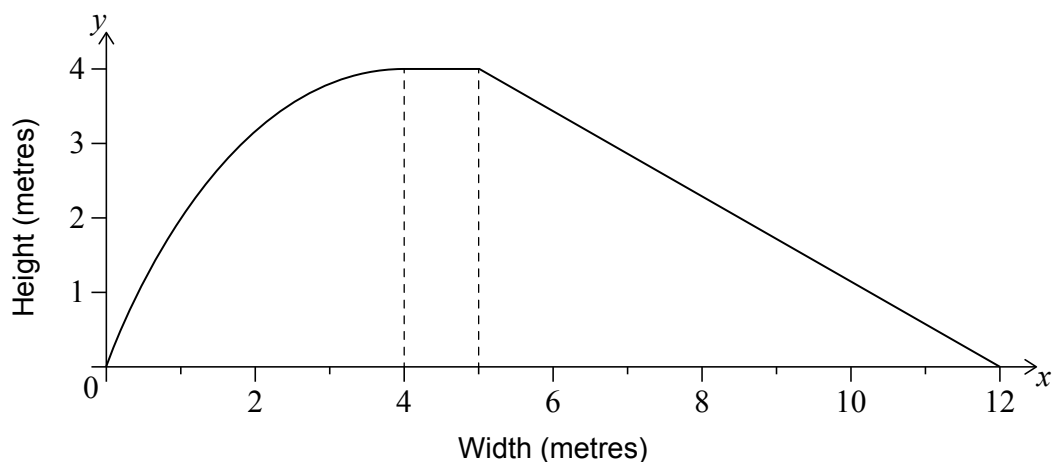
- (e) Write down **three** equations, in terms of a , b and c , using the given conditions. [6]
- (f) Hence find the values of a , b and c . [2]

Andrew claims the updated design will still provide a smooth path for a person using the slide.

- (g) Explain why Andrew's claim is correct at the point where $x = 2$. [2]

3. [Maximum mark: 15]

The following diagram shows a model of the side view of a water slide. All lengths are measured in metres.



The curved edge of the slide is modelled by

$$f(x) = -\frac{1}{4}x^2 + 2x \text{ for } 0 \leq x \leq 4.$$

The remainder of the slide is modelled by

$$g(x) = \begin{cases} 4, & \text{for } 4 \leq x \leq 5 \\ \frac{48}{7} - \frac{4x}{7}, & \text{for } 5 \leq x \leq 12 \end{cases}$$

- (a) Use the trapezoidal rule with an interval width of 1 to calculate the approximate area under the model of the slide in the interval $0 \leq x \leq 4$. [5]
- (b) Find $\int \left(-\frac{1}{4}x^2 + 2x \right) dx$. [3]
- (c) Calculate the exact area under the entire model of the slide, for $0 \leq x \leq 12$. [4]
- (d) Find the percentage error in the **total** area under the entire model of the slide when using the approximate value from part (a). [3]

Turn over

12. [Maximum mark: 7]

Consider the curve $y = 5x^2 - \frac{3}{x^3}$.

- (a) Find $\frac{dy}{dx}$. [3]
- (b) Write down the gradient of the curve at $x = 1$. [1]
- (c) Hence, find the equation of the normal to the curve at $x = 1$. [3]



Turn over

12. [Maximum mark: 7]

Consider the curve $y = 4x^3 - \frac{2}{x^2}$.

- (a) Find $\frac{dy}{dx}$. [3]
- (b) Write down the gradient of the curve at $x = 1$. [1]
- (c) Hence, find the equation of the normal to the curve at $x = 1$. [3]

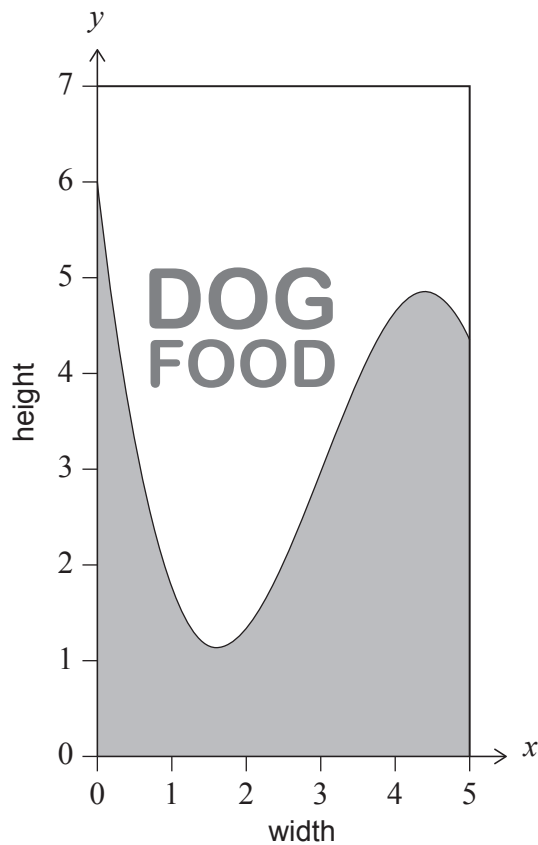


Turn over

4. [Maximum mark: 20]

A type of dog food is sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is 5 cm wide and 7 cm high.

The design on the label is a curve, as shown on the following axes, where one unit represents 1 cm for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	6	1.7	1.3	3.0	4.7	4.3

- (a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve.

[3]

The curve used in the label design can be modelled by:

$$y = -\frac{x^3}{3} + 3x^2 - 7x + 6, \text{ for } 0 \leq x \leq 5.$$

- (b) (i) Write down the integral that represents the area of the shaded region.
- (ii) Hence, find the area of the shaded region.

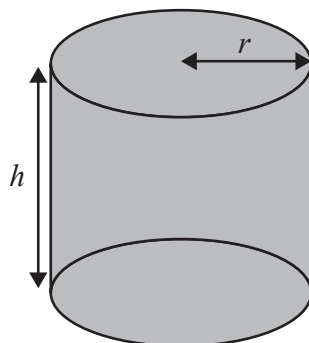
[4]

(This question continues on the following page)

(Question 4 continued)

The dog food is sold in closed cylindrical containers, with radius r and height h .

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of 400 cm^3 .

- (c) Write down an equation, in terms of r and h , that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A , of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

- (d) Find the value of k . [4]

- (e) (i) Find $\frac{dA}{dr}$.
 (ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

The containers are made so that the surface area is minimized. The 5 cm by 7 cm rectangular label is to be glued to the curved surface of the container.

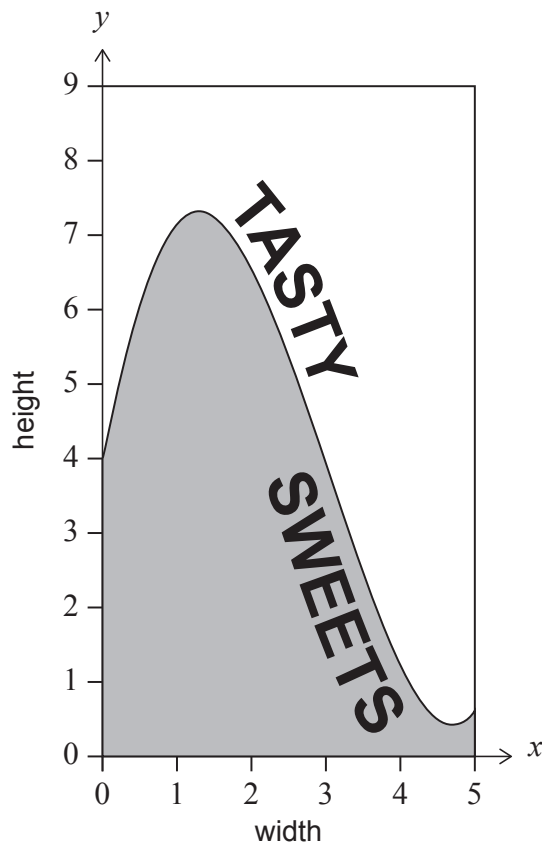
- (f) Show that the label will fit on the container. [3]

Turn over

4. [Maximum mark: 20]

Sweets are sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is 5 cm wide and 9 cm high.

The design on the label is a curve, as shown on the following axes, where one unit represents 1 cm for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	4	7.3	6.7	4.0	1.3	0.7

- (a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve.

[3]

The curve used in the label design can be modelled by:

$$y = \frac{x^3}{3} - 3x^2 + 6x + 4, \text{ for } 0 \leq x \leq 5.$$

- (b) (i) Write down the integral that represents the area of the shaded region.
- (ii) Hence, find the area of the shaded region.

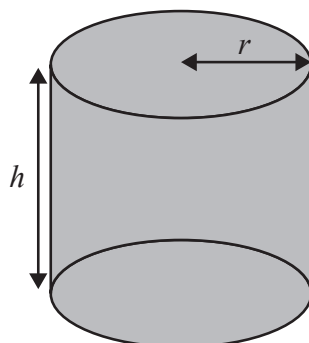
[4]

(This question continues on the following page)

(Question 4 continued)

The sweets are sold in closed cylindrical containers, with radius r and height h .

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of 600 cm^3 .

- (c) Write down an equation, in terms of r and h , that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A , of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

- (d) Find the value of k . [4]

- (e) (i) Find $\frac{dA}{dr}$.
 (ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

The containers are made so that the surface area is minimized. The 5 cm by 9 cm rectangular label is to be glued to the curved surface of the container.

- (f) Show that the label will fit on the container. [3]

Turn over

Consider a function of the form $f(x) = 2x^{-2} + bx^{-1} + c$, where b and c are constants. The graph of $y = f(x)$ has a gradient of 0.208 at $x = 5$.

- (a) Determine the value of b . [5]
- (b) Show that $f(x)$ is increasing at $x = 3.5$ [2]

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Turn over

Consider the function $f(x) = x^3 - 9x^2 + 23x - 15$ for $0 \leq x \leq 5$.

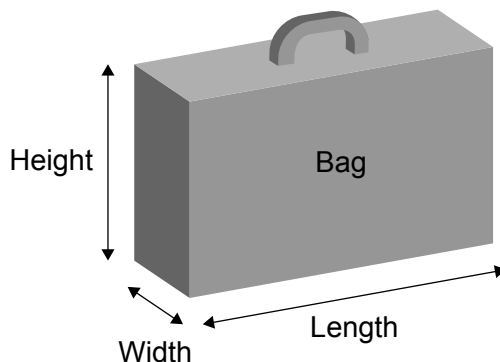
(a) Find the value of a . [2]

(b) Use your graphic display calculator to find the area enclosed by the curve $y = f(x)$ and the x -axis, when $1 \leq x \leq a$. [2]

12. [Maximum mark: 9]

An airline restricts the size of bags by stating that the sum of a bag's width, length, and height (ignoring the handle) must not exceed a fixed value, M .

diagram not to scale



$$\text{Width} + \text{Length} + \text{Height} \leq M$$

A company is designing a cuboid-shaped bag so that the sum of the bag's width, length, and height **equals** M . Their design will have a length equal to three times its width.

As the width of the design varies, its volume changes according to the equation

$$\frac{dV}{dw} = 690w - 36w^2,$$

where w is the width in centimetres and V is the volume in cubic centimetres.

- (a) Use your graphic display calculator to find the value of w that will produce the maximum volume. [2]
- (b) Show that the maximum volume of the bag to three significant figures is $42\,200\text{ cm}^3$. [4]
- (c) Hence find the value of M . [3]

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(Question 12 continued)

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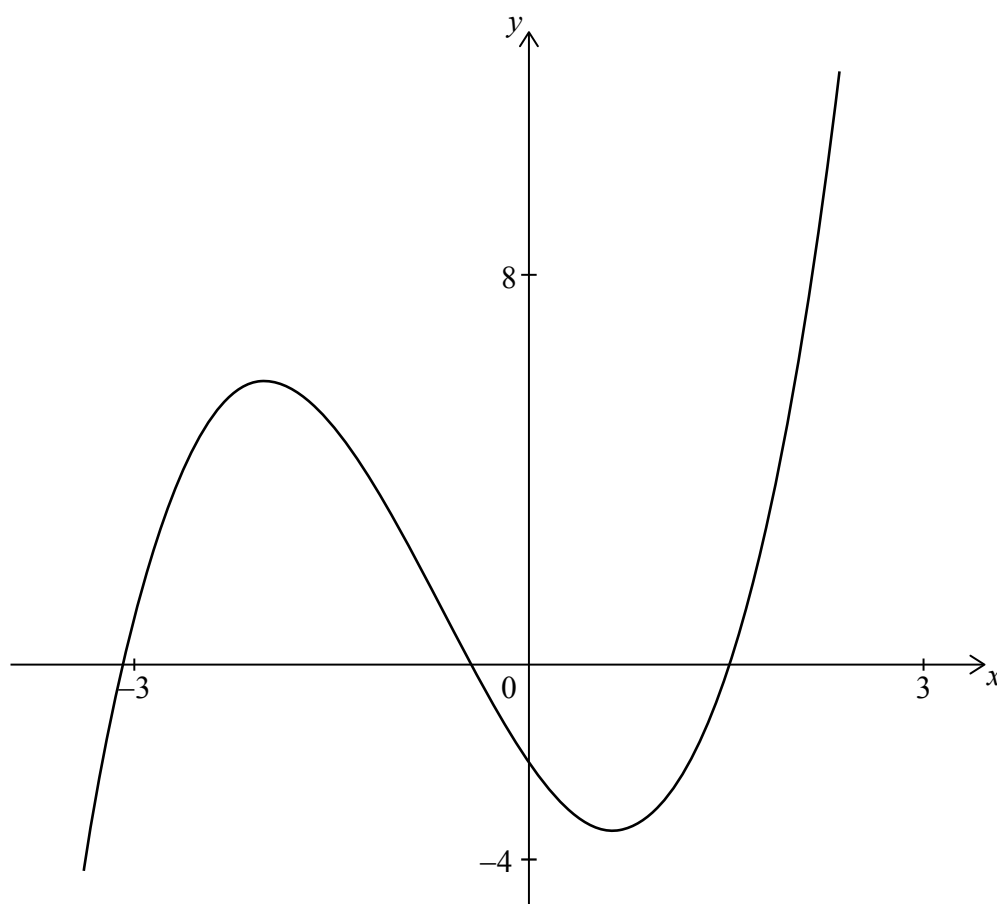
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Turn over

4. [Maximum mark: 5]

Consider the graph of the cubic function $f(x) = x^3 + 2x^2 - 4x - 2$. Part of the graph of $y = f(x)$ is shown in the following diagram.



(a) Write down the x -coordinate of

(i) the local maximum.

(ii) the local minimum.

[2]

(b) Hence, write down the interval where the function is decreasing.

[1]

The tangent to the curve at $(1, -3)$ is parallel to the straight line $y = 3x + 5$.

(c) Write down

(i) the gradient of the tangent.

(ii) the equation of the tangent.

[2]

(This question continues on the following page)



(Question 4 continued)

[illegible]

Turn over

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9. [Maximum mark: 7]

Pascale owns a company that produces and sells curry powder. The rate of change of the company’s profit, P , in Mauritian rupees (MUR) from producing x kilograms (kg) of curry powder is modelled by

$$\frac{dP}{dx} = -10x + 460, \, x \geq 0.$$

She makes a profit of 3300 MUR when producing 10 kg of curry powder.

- (a) Find an expression for the company’s profit, P , in terms of x . [5]

Pascale decides to increase the production of curry powder from 25 kg to 50 kg.

- (b) Find the increase in profit. [2]



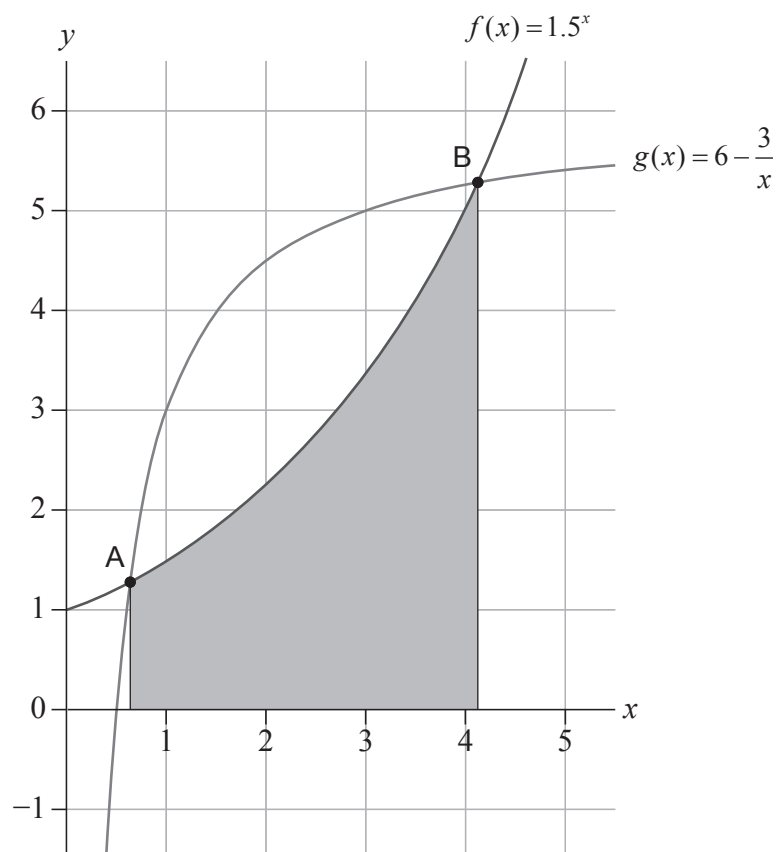
Turn over

2. [Maximum mark: 12]

The diagram shows part of the graphs of the functions

$$f(x) = 1.5^x \quad x \geq 0$$

$$g(x) = 6 - \frac{3}{x} \quad x > 0.$$



(a) Solve $f(x) = g(x)$. [3]

(b) (i) Write down the integral that represents the area of the shaded region.

(ii) Calculate the area of this shaded region.

(iii) Hence, or otherwise, calculate the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$. [6]

The tangent to the graph of $y = f(x)$ is parallel to the tangent to the graph of $y = g(x)$ at $x = k$.

(c) Find the value of k . [3]

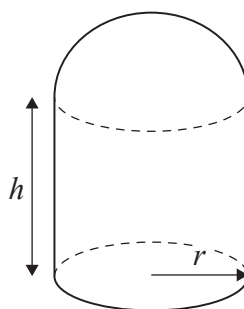
5. [Maximum mark: 16]

Ju Shen designs a plastic cover, in the shape of a cylinder combined with a hemisphere on top, as shown in the diagram.

The plastic used to make the cover forms the curved surface of both the hemisphere and the cylinder; there is no bottom to the cover, however it stands on a flat horizontal surface.

Let the height of the cylinder be h cm and the internal radius of its base be r cm.

diagram not to scale



- (a) Find an expression for the total internal surface area, $A \text{ cm}^2$, of the cover in terms of r and h . [2]
- (b) Show that the total volume, $V \text{ cm}^3$, enclosed by the cover and the horizontal surface is given by the expression

$$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}. \quad [2]$$

The total volume enclosed by the cover is $10\,000 \text{ cm}^3$.

- (c) Hence show that $h = \frac{30\,000 - 2\pi r^3}{3\pi r^2}$. [2]

Ju Shen uses the total internal surface area to model the amount of plastic used to construct the cover.

- (d) Show that A is given by the expression

$$A = \frac{2\pi r^2}{3} + \frac{20\,000}{r}. \quad [2]$$

(This question continues on the following page)

(Question 5 continued)

Ju Shen wants to use the minimum amount of plastic in the construction of the cover.

- (e) Find an expression for $\frac{dA}{dr}$. [3]
 - (f) Find the value of r and the value of h that minimizes the use of plastic. [4]
 - (g) By interpreting your answer to part (f), suggest the best shape for Ju Shen's plastic cover. [1]
-

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References:

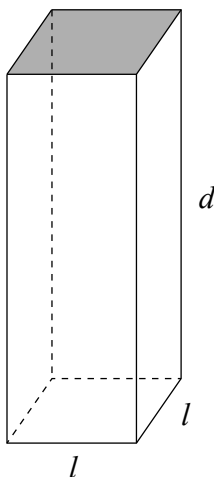
1. naisupakit, 2016. *Triangle Pillow tradition native Thai style pillow*. [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/triangle-pillow-tradition-native-thai-style-pillow-royalty-free-image/623127206> [Accessed 9 April 2024]. Source adapted.
4. bortonia, 2020. *Antique Grandfather Clock Icon on Transparent Background*. [image online] Available at: <https://www.gettyimages.co.uk/detail/illustration/antique-grandfather-clock-icon-on-royalty-free-illustration/1284116855> [Accessed 9 April 2024]. Source adapted.

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3. [Maximum mark: 20]

Kailash manufactures drink containers in the shape of a cuboid. The container has a square top and a square base of length, l cm. Its height, d cm, is three times the length of the base.

diagram not to scale



- (a) Write down an expression for d in terms of l .

[1]

The container can hold 375 cm^3 of drink.

- (b) Find the value of l and d .

[3]

- (c) Calculate the total external surface area of the container.

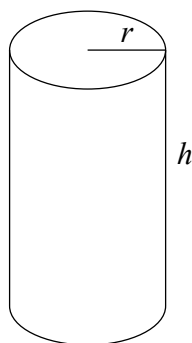
[3]

(This question continues on the following page)

(Question 3 continued)

To reduce environmental impact, Kailash is trying to minimize the amount of material needed for the production of the 375 cm^3 container.

He is willing to change the shape to a cylinder with radius $r \text{ cm}$, and height $h \text{ cm}$, as shown below.



The cylindrical container of drink must also hold 375 cm^3 .

- (d) Find an expression for the height, h , of the container in terms of r . [2]

Let the total external surface area be $A \text{ cm}^2$.

- (e) Show that $A = 2\pi r^2 + \frac{750}{r}$. [2]

- (f) Find $\frac{dA}{dr}$. [3]

- (g) Hence or otherwise

- (i) find the value of r that will minimize A .
 (ii) find the minimum value of A needed for the cylinder. [3]

To produce the containers, additional material is required:

- 10% additional surface area for the cuboid
- 25% additional surface area for the cylinder.

Kailash will choose the container that requires the least total amount of material.

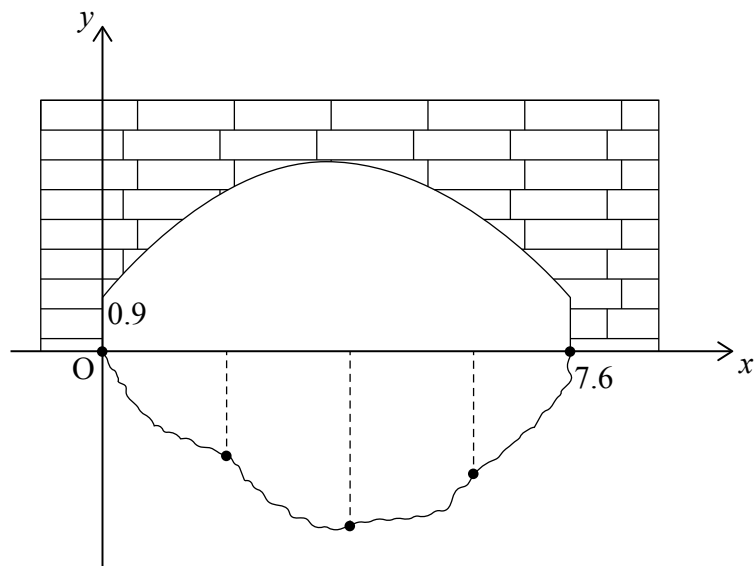
- (h) Determine which container Kailash should choose. Justify your answer. [3]

Turn over

5. [Maximum mark: 12]

The diagram shows the cross-section of a bridge and a river. A coordinate system has been added with the origin, O , at the point where the bridge meets the water on one side. All units are in metres.

diagram not to scale



A researcher wants to calculate the volume of water that flows under the bridge. To do this he takes measurements of the depth every 1.9 m from O . The depths are shown in the following table.

Horizontal distance from O in metres	0	1.9	3.8	5.7	7.6
Vertical depth of water in metres	0	1.68	2.81	2.32	0

- (a) Use the trapezoidal rule to find the cross-sectional area of the river as it passes under the bridge. [3]

The water flows under the bridge at a rate of 0.3 m s^{-1} .

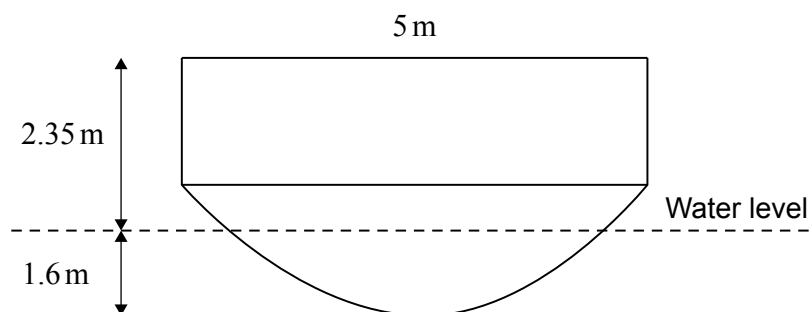
- (b) Find the volume of water that passes under the bridge each second. [2]

(This question continues on the following page)

(Question 5 continued)

A boat is travelling along the river. The cross-section of the boat and the water level is shown in the following diagram.

The top of the boat is parallel to the water level and has a width of 5 m. The height of the boat is 2.35 m above the water level and the lowest part of the boat is 1.6 m below the water level.



The boat is travelling down the centre of the river.

- (c) Find the vertical distance between the lowest part of the boat and the bottom of the river as it passes under the bridge. [1]

The curved arch of the bridge can be modelled by the equation

$$y = -0.15x^2 + 1.14x + 0.9, \quad 0 \leq x \leq 7.6.$$

- (d) Find the maximum height of the curved arch above the water level. [2]
- (e) Determine whether the top of the boat will be able to pass under the bridge. [4]

11. [Maximum mark: 8]

Consider the function $B(x) = 20 + \frac{32}{x} + 2x$, where $x \geq 1$, $x \in \mathbb{R}$.

- (a) (i) Find $B'(x)$.
- (ii) Use your answer to part (a)(i) to find the minimum value of $B(x)$. You may assume that $B(x)$ has no local maximum point over the given domain. [6]

Kim goes to a restaurant that offers a large family banquet. The time, T , in minutes, to prepare the family banquet depends on the number of chefs, n .

To predict the value of T , Kim uses the model $T(n) = 20 + \frac{32}{n} + 2n$, where $n \in \mathbb{Z}^+$.

The restaurant informs Kim that the time taken to prepare the family banquet is less than 40 minutes. There are k chefs preparing the family banquet.

- (b) Find the possible values of k . [2]



Turn over

9. [Maximum mark: 6]

Consider the function $f(x) = x^3 - 2x^2 + 5x - 7$.

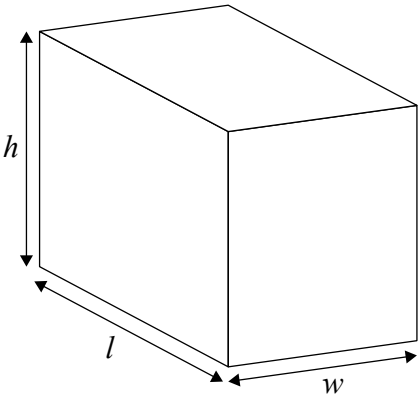
- Find $f(1)$. [2]
- Find $f'(x)$. [2]
- Find the equation of the line that is tangent to the curve $y = f(x)$ at $x = 1$. [2]

[illegible]

13. [Maximum mark: 5]

Soraya is designing a gift box in the shape of a rectangular prism, as shown in the diagram.

diagram not to scale



The length, l , of the box is 1.5 times the width, w , of the box. This can be expressed as $l = 1.5w$. The height, h , of the box is also labelled on the diagram.

The box has a volume of $10\,000\text{ cm}^3$. Soraya wants to minimize the surface area of the box. Find the width of the box.

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(This question continues on the following page)



(Question 13 continued)

[illegible]

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Answers written on this page will not be
marked.



2. [Maximum mark: 16]

The height of a plant, h cm, depends on the time, t days, after it is planted.

For the first 15 days, the rate of change of the height of the plant is modelled as

$$\frac{dh}{dt} = 1.2 - 0.1t, \text{ where } 0 \leq t \leq 15.$$

- (a) (i) Find the value of $\frac{dh}{dt}$ at $t = 3$.

- (ii) Interpret the meaning of your answer to part (a)(i) in context. [2]

- (b) Use $\frac{dh}{dt}$ to find the value of t when the plant reaches its maximum height. [2]

Two days after it is planted, the height of the plant is 6.5 cm.

- (c) Find an expression for h in terms of t , for $0 \leq t \leq 15$. [5]

- (d) Hence, find the maximum height of the plant. [2]

A second plant was planted at the same time as the first plant. Its height, H cm, is modelled by $H = 2(1.2)^t$, where $0 \leq t \leq 15$.

- (e) Write down the initial height of the second plant. [1]

Each day, the height of the second plant increases by $p\%$.

- (f) Find the value of p . [2]

- (g) Find the value of t when the two plants have the same height. [2]

5. [Maximum mark: 18]

Zheng models the height of a section of a rollercoaster.

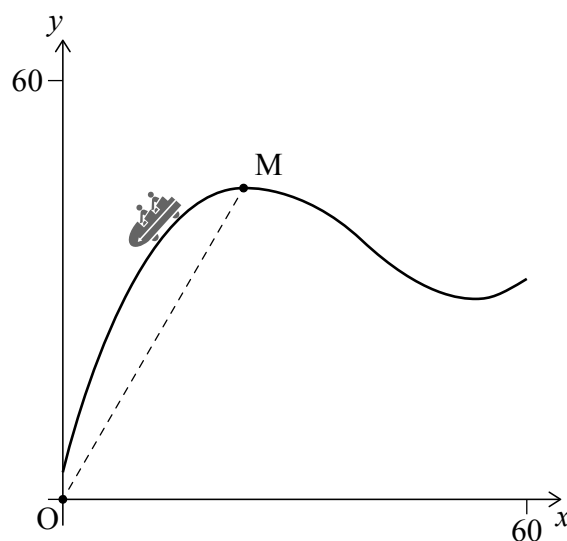
She uses the function

$$f(x) = 0.001x^3 - 0.12x^2 + 4.05x + 3, \text{ for } 0 \leq x \leq 60,$$

where x is the horizontal distance from the left-hand end of the section and $f(x)$ is the vertical height. Distances are measured in metres.

Zheng's model is shown on the following graph of $y = f(x)$.

diagram not to scale



Point O has coordinates $(0, 0)$.

- (a) (i) Write down the definite integral for the exact area between the curve and the x -axis.
- (ii) Calculate this area. [4]

A light beam from O passes through the local maximum at point M.

- (b) (i) Find the coordinates of M.
- (ii) Find the angle of elevation from O to M. [4]

(This question continues on the following page)

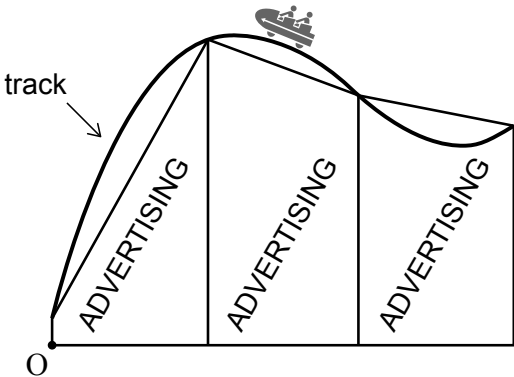
(Question 5 continued)

Zheng wants to place advertising boards in front of this section of the rollercoaster.
At four positions along the rollercoaster track, she records the horizontal distance from O and the vertical height of the track.

Her values are shown in the table and represented on the graph.

x (horizontal distance)	0	20	40	60
y (vertical distance)	3	44	37	30

diagram not to scale



- (c) Find the total area of the advertising boards. [3]

The cost of space on the advertising boards per square metre varies according to the amount purchased.

The **rate of change** in the cost, in dollars (\$), for space on the advertising boards can be modelled by the function

$$C'(a) = -0.00292a + 4.71, \text{ for } a > 60,$$

where a is the total square metres purchased.

A company, E-Byte, wishes to advertise its new gaming device. Zheng tells E-Byte it will cost \$1750 for 500 m^2 of space on the advertising boards.

- (d) Find the cost function $C(a)$. [5]

E-Byte instead wants to purchase the entire space of all three boards.

- (e) Using the cost function from part (d), estimate how much this would cost E-Byte. [2]