APOLLO

# Mathematics: analysis and approaches SL Answers

First assessment 2021
Whole questions arranged by syllabus topic.
Original question and scheme crops preserve source diagrams.

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# Number and algebra

47 questions

**2.** (a) $3\times10^4$ OR $30000\,(\text{km})$ (accept $3\bullet10^4$) ***A1***

***[1 mark]***

(b) $\frac{4}{3}\pi\left(3\times10^4\right)^3$ OR $\frac{4}{3}\pi\left(30000\right)^3$ ***(A1)***

$=\frac{4}{3}\pi\times27\times10^{12}\left(=\pi\left(36\times10^{12}\right)\right)$ OR $=\frac{4}{3}\pi\times27000000000000$ ***(A1)***

$=\pi\left(3.6\times10^{13}\right)$ (km$^3$) OR $a=3.6, k=13$ ***A1***

***[3 marks]***

***Total [4 marks]***

**3.** **METHOD 1 (finding $u_1$ first, from $S_8$)**

$4(u_1+8)=8$ ***(A1)***

$u_1=-6$ ***A1***

$u_1+7d=8$ OR $4(2u_1+7d)=8$ (may be seen with their value of $u_1$) ***(A1)***

attempt to substitute their $u_1$ ***(M1)***

$d=2$ ***A1***

**METHOD 2 (solving simultaneously)**

$u_1+7d=8$ ***(A1)***

$4(u_1+8)=8$ OR $4(2u_1+7d)=8$ OR $u_1=-3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1=-6, d=2$ ***A1A1***

***[5 marks]***

**2.** attempt to subtract squares of integers ***(M1)***

$(n+1)^2 - n^2$

**EITHER**

correct order of subtraction and correct expansion of $(n+1)^2$, seen anywhere ***A1A1***

$= n^2 + 2n + 1 - n^2 (= 2n+1)$

**OR**

correct order of subtraction and correct factorization of difference of squares ***A1A1***

$= (n+1-n)(n+1+n)(= 2n+1)$

**THEN**

$= n + n + 1$ =RHS ***A1***

| **Note:** Do not award final ***A1*** unless all previous working is correct. |
|---|

which is the sum of $n$ and $n+1$ ***AG***

| **Note:** If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$ . |
|---|

***[4 marks]***

**4. EITHER**

attempt to use the binomial expansion of $(x+k)^7$ ***(M1)***

${}^7C_0x^7k^0 + {}^7C_1x^6k^1 + {}^7C_2x^5k^2 + \ldots$ (or ${}^7C_0k^7x^0 + {}^7C_1k^5x^1 + {}^7C_2k^5x^2 + \ldots$)

identifying the correct term ${}^7C_2x^5k^2$ (or ${}^7C_5k^2x^5$) ***(A1)***

**OR**

attempt to use the general term ${}^7C_rx^rk^{7-r}$ (or ${}^7C_rk^rx^{7-r}$) ***(M1)***

$r=2$ (or $r=5$) ***(A1)***

**THEN**

${}^7C_2=21$ (or ${}^7C_5=21$) (seen anywhere) ***(A1)***

$21x^5k^2=63x^5$ $(21k^2=63,\ k^2=3)$ ***A1***

$k=\pm\sqrt{3}$ ***A1***

**Note:** If working shown, award ***M1A1A1A1A0*** for $k=\sqrt{3}$.

***[5 marks]***

**6. METHOD 1**

product of a binomial coefficient, a power of 3 (and a power of $x^2$) seen ***(M1)***

evidence of correct term chosen ***(A1)***

$^{n+1}C_2 \times 3^{n+1-2} \times \left(x^2\right)^2 \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4\right)$ OR $n - r = 1$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

**EITHER**

$^{n+1}C_2 \times 3^{n-1} = 20412$ ***(A1)***

**OR**

$^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$ ***(A1)***

**THEN**

$n = 7$ ***A1***

**METHOD 2**

$$3^{n+1}\left(1+\frac{x^2}{3}\right)^{n+1}$$

product of a binomial coefficient, and a power of $\frac{x^2}{3}$ OR $\frac{1}{3}$ seen ***(M1)***

evidence of correct term chosen ***(A1)***

$$3^{n+1} \times \frac{n(n+1)}{2!} \times \left(\frac{x^2}{3}\right)^2 \left(= \frac{3^{n-1}}{2} n(n+1)x^4\right)$$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

$$3^{n-1} \times \frac{n(n+1)}{2} = 20412$$ ***(A1)***

$n = 7$ ***A1***

***Total [5 marks]***

## Section B

**7.** (a) (i) **EITHER**

$9000 \times \left(1 + \frac{7}{100}\right)^5$ ***(A1)***

$12622.965...$ ***(A1)***

**OR**

$n = 5$
$\text{I}\% = 7$
$\text{PV} = \mp 9000$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(A1)***
$\pm 12622.965...$ ***(A1)***

**THEN**

$(\$)\, 12600$ ***A1***

(ii) **EITHER**

$9000\left(1 + \frac{7}{100}\right)^x = 20000$ ***(A1)***

**OR**

$\text{I}\% = 7$
$\text{PV} = \mp 9000$
$\text{FV} = \pm 20000$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(A1)***

**THEN**

$= 12\,(\text{years})$ ***A1***

***[5 marks]***

(b) **METHOD 1**

attempt to substitute into compound interest formula (condone absence of compounding periods) ***(M1)***

$$9000\left(1+\frac{r}{100\times 12}\right)^{12\times 10}=20000$$

$8.01170...$ ***(A1)***

$r=8.02\,(\%)$ ***A1***

**METHOD 2**

$n=10$
$\text{PV}=\pm 9000$
$\text{FV}=\mp 20000$
$\text{P/Y}=1$
$\text{C/Y}=12$
$r=8.01170...$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for ( $r=$ ) $8.01170\ldots$

$r=8.02\,(\%)$ ***A1***

(c) (i) recognising geometric series (seen anywhere) ***(M1)***

$$r=\frac{4500}{9000}\quad\left(=\frac{1}{2}\right)$$ ***(A1)***

**EITHER**

considering $S_\infty$ ***(M1)***

$$\frac{9000}{1-0.5}\left(=18000\right)$$ ***A1***

correct reasoning that $18000<20000$ ***R1***

**Note**: Accept $S_\infty < 20000$ only if $S_\infty$ has been calculated.

**OR**

considering $S_n$ for a large value of $n$, $n \geq 80$ ***(M1)***

**Note:** Award ***M1*** only if the candidate gives a valid reason for choosing a value of $n$, where $50 \leq n < 80$.

correct value of $S_n$ for their $n$ ***A1***

valid reason why Chris will not reach the target, which involves their choice of $n$, their value of $S_n$ and Chris' age OR using two large values of $n$ to recognize asymptotic behaviour of $S_n$ as $n \to \infty$. ***R1***

**Note:** Do not award the ***R*** mark without the preceding ***A*** mark.

**THEN**

Therefore, Chris will never reach the target. ***AG***

(ii) recognising geometric sum ***M1***

$$\frac{u_1\left(1-0.5^5\right)}{0.5} = 20000$$ ***(A1)***

$10322.58...$

$(\$)\ 10323$ ***A1***

***[8 marks]***

***Total [16 marks]***

**3.** (a) attempt to use $u_1 + (n-1)d = 0$ ***(M1)***

$60 - 2.5(k-1) = 0$

$k = 25$ ***A1***

***[2 marks]***

(b) **METHOD 1**

attempting to express $S_n$ in terms of $n$ ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

**METHOD 2**

**EITHER**

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60 + 0),\ S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ ***(A1)***

**OR**

attempting to calculate $S_{24}$ ***(M1)***

$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ ***(A1)***

**THEN**

$= 750$ ***A1***

***[3 marks]***

***Total [5 marks]***

**8.** (a) $f\left(\frac{2}{3}\right)=4$ OR $a^{\frac{2}{3}}=4$ ***(M1)***

$a=4^{\frac{3}{2}}$ OR $a=\left(2^2\right)^{\frac{3}{2}}$ OR $a^2=64$ OR $\sqrt[3]{a}=2$ ***A1***

$a=8$ ***AG***

***[2 marks]***

(b) $f^{-1}(x)=\log_8 x$ ***A1***

**Note**: Accept $f^{-1}(x)=\log_a x$.

Accept any equivalent expression for $f^{-1}$ e.g. $f^{-1}(x)=\frac{\ln x}{\ln 8}$.

***[1 mark]***

(c) correct substitution ***(A1)***

$\log_8\sqrt{32}$ OR $8^x=32^{\frac{1}{2}}$

correct working involving log/index law ***(A1)***

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2=\frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8=3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x}=2^{\frac{5}{2}}$

$f^{-1}\left(\sqrt{32}\right)=\frac{5}{6}$ ***A1***

***[3 marks]***

*continued…*

*Question 8 continued.*

(d) (i) **METHOD 1**

equating a pair of differences ***(M1)***

$u_2 - u_1 = u_4 - u_3 (= u_3 - u_2)$

$\log_8 p - \log_8 27 = \log_8 125 - \log_8 q$

$\log_8 125 - \log_8 q = \log_8 q - \log_8 p$

$\log_8\left(\frac{p}{27}\right) = \log_8\left(\frac{125}{q}\right)$, $\log_8\left(\frac{125}{q}\right) = \log_8\left(\frac{q}{p}\right)$ ***A1A1***

$\frac{p}{27} = \frac{125}{q}$ and $\frac{125}{q} = \frac{q}{p}$ ***A1***

$27, p, q$ and $125$ are in geometric sequence ***AG***

**Note:** If candidate assumes the sequence is geometric, award no marks for part (i). If $r = \frac{5}{3}$ has been found, this will be awarded marks in part (ii).

**METHOD 2**

expressing a pair of consecutive terms, in terms of $d$ ***(M1)***

$p = 8^d \times 27$ and $q = 8^{2d} \times 27$ OR $q = 8^{2d} \times 27$ and $125 = 8^{3d} \times 27$

two correct pairs of consecutive terms, in terms of $d$ ***A1***

$\frac{8^d \times 27}{27} = \frac{8^{2d} \times 27}{8^d \times 27} = \frac{8^{3d} \times 27}{8^{2d} \times 27}$ (must include 3 ratios) ***A1***

all simplify to $8^d$ ***A1***

$27, p, q$ and $125$ are in geometric sequence ***AG***

*continued…*

*Question 8 continued.*

(ii) **METHOD 1 (geometric, finding *r*)**

$u_4 = u_1 r^3$ OR $125 = 27(r)^3$ ***(M1)***

$r = \frac{5}{3}$ (seen anywhere) ***A1***

$p = 27r$ OR $\frac{125}{q} = \frac{5}{3}$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

**METHOD 2 (arithmetic)**

$u_4 = u_1 + 3d$ OR $\log_8 125 = \log_8 27 + 3d$ ***(M1)***

$d = \log_8\left(\frac{5}{3}\right)$ (seen anywhere) ***A1***

$\log_8 p = \log_8 27 + \log_8\left(\frac{5}{3}\right)$ OR $\log_8 q = \log_8 27 + 2\log_8\left(\frac{5}{3}\right)$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

**METHOD 3 (geometric using proportion)**

recognizing proportion ***(M1)***

$pq = 125 \times 27$ OR $q^2 = 125p$ OR $p^2 = 27q$

two correct proportion equations ***A1***

attempt to eliminate either $p$ or $q$ ***(M1)***

$q^2 = 125 \times \frac{125 \times 27}{q}$ OR $p^2 = 27 \times \frac{125 \times 27}{p}$

$p = 45,\ q = 75$ ***A1A1***

***[9 marks]***
***Total [15 marks]***

**Special note**: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of $f$ or the gradient of $f'$ to earn the ***R*** mark.

**6.** (a) $u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24} \left( = \frac{7}{12} = 0.583333\ldots \right)$ ***A1***

***[2 marks]***

(b) $r = \frac{7}{8} \left( = 0.875 \right)$ ***(A1)***

substituting their values for $u_1$ and $r$ into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3} \left( = 4.66666\ldots \right)$ ***A1***

***[3 marks]***

(c) attempt to substitute their values into the inequality or formula for $S_n$ ***(M1)***

$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3} \left( \frac{7}{8} \right)^r < 0.001$ OR $S_n = \frac{\frac{7}{12} \left( 1 - \left( \frac{7}{8} \right)^n \right)}{\left( 1 - \frac{7}{8} \right)}$

attempt to solve their inequality using a table, graph or logarithms (must be exponential) ***(M1)***

| |
|---|
| **Note:** Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$. |

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036...$ OR $S_\infty - S_{64} = 0.000906...$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683...$ OR $S_\infty - S_{64} - 0.001 = -0.0000931777...$

least value is $n = 64$ ***A1***

***[4 marks]***

***Total [9 marks]***

## Section B

**3.** (a) attempting to expand the LHS ***(M1)***

$\text{LHS} = \left(4n^2 - 4n + 1\right) + \left(4n^2 + 4n + 1\right)$ ***A1***

$= 8n^2 + 2 \left(= \text{RHS}\right)$ ***AG***

***[2 marks]***

(b) ***METHOD 1***

recognition that $2n-1$ and $2n+1$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$8n^2 + 2 = 2\left(4n^2 + 1\right)$ ***A1***

valid reason *eg* divisible by $2$ ($2$ is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

***[3 marks]***

***METHOD 2***

recognition, *eg* that $n$ and $n+2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$n^2 + (n+2)^2 = 2\left(n^2 + 2n + 2\right)$ ***A1***

valid reason *eg* divisible by $2$ ($2$ is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

***[3 marks]***

***Total [5 marks]***

**3.** (a) $\left(1+\frac{5.5}{4\times100}\right)$ ***(M1)(A1)***

$1.056$ ***A1***

***[3 marks]***

*continued…*



*Question 3 continued*

(b) **EITHER**

$2P = P\times\left(1+\frac{5.5}{100\times4}\right)^{4n}$ **OR** $2P = P\times\left(\text{their }(a)\right)^{m}$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

**OR**

$\text{PV} = \pm1$
$\text{FV} = \mp2$
$\text{I\%} = 5.5$
$\text{P/Y} = 4$
$\text{C/Y} = 4$
$n = 50.756\ldots$ ***(M1)(A1)***

**OR**

$\text{PV} = \pm1$
$\text{FV} = \mp2$
$\text{I\%} = 100\,(\text{their }(a)-1)$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(M1)(A1)***

**THEN**

$\Rightarrow$ $12.7$ years
Laurie will have double the amount she invested during $2032$ ***A1***

***[3 marks]***

***Total [6 marks]***

**8.** (a) (i) **EITHER**

attempt to use a ratio from consecutive terms ***M1***

$$\frac{p\ln x}{\ln x}=\frac{\frac{1}{3}\ln x}{p\ln x} \quad \text{OR} \quad \frac{1}{3}\ln x=(\ln x)r^2 \quad \text{OR} \quad p\ln x=\ln x\left(\frac{1}{3p}\right)$$

**Note:** Candidates may use $\ln x^1+\ln x^p+\ln x^{\frac{1}{3}}+...$ and consider the powers of $x$ in geometric sequence.

Award ***M1*** for $\frac{p}{1}=\frac{\frac{1}{3}}{p}$.

**OR**

$r=p$ and $r^2=\frac{1}{3}$ ***M1***

**THEN**

$p^2=\frac{1}{3}$ OR $r=\pm\frac{1}{\sqrt{3}}$ ***A1***

$p=\pm\frac{1}{\sqrt{3}}$ ***AG***

**Note:** Award ***M0A0*** for $r^2=\frac{1}{3}$ or $p^2=\frac{1}{3}$ with no other working seen.

(ii) $\frac{\ln x}{1-\frac{1}{\sqrt{3}}}\ \left(=3+\sqrt{3}\right)$ ***(A1)***

$\ln x=3-\frac{3}{\sqrt{3}}+\sqrt{3}-\frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x=3-\sqrt{3}+\sqrt{3}-1\ \left(\Rightarrow \ln x=2\right)$ ***A1***

$x=e^2$ ***A1***

***[5 marks]***

*continued…*

*Question 8 continued*

(b) (i) **METHOD 1**

attempt to find a difference from consecutive terms or from $u_2$ ***M1***

correct equation ***A1***

$p\ln x - \ln x = \frac{1}{3}\ln x - p\ln x$ OR $\frac{1}{3}\ln x = \ln x + 2\left(p\ln x - \ln x\right)$

**Note:** Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + ...$ and consider the powers of $x$ in arithmetic sequence.
Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$.

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

**METHOD 2**

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p\ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1***

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

*continued…*

*Question 8 continued*

**METHOD 3**

attempt to find difference using $u_3$ ***M1***

$\frac{1}{3}\ln x = \ln x + 2d \quad \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$

$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right)$ OR $p\ln x - \ln x = -\frac{1}{3}\ln x$ ***A1***

$p\ln x = \frac{2}{3}\ln x$ ***A1***

$p = \frac{2}{3}$ ***AG***

(ii) $d = -\frac{1}{3}\ln x$ ***A1***

*continued…*

*Question 8 continued*

(iii) **METHOD 1**

$S_n = \frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right]$

attempt to substitute into $S_n$ and equate to $-3\ln x$ ***(M1)***

$\frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right] = -3\ln x$

correct working with $S_n$ (seen anywhere) ***(A1)***

$\frac{n}{2}\left[2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right]$ OR $n\ln x - \frac{n(n-1)}{6}\ln x$ OR $\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$

correct equation without $\ln x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ or equivalent

**Note:** Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to $\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$n^2 - 7n - 18 = 0$

attempt to solve their quadratic ***(M1)***

$(n-9)(n+2) = 0$

$n = 9$ ***A1***

*continued…*

*Question 8 continued*

**METHOD 2**

listing the first 7 terms of the sequence ***(A1)***

$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + ...$

recognizing first 7 terms sum to 0 ***M1***

8th term is $-\frac{4}{3}\ln x$ ***(A1)***

9th term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of 8th and 9th terms $= -3\ln x$ ***(A1)***

$n = 9$ ***A1***

***[10 marks]***

***Total [15 marks]***

**2.** (a) $u_1 = 12$ ***A1***

***[1 mark]***

(b) $15 - 3n = -33$ ***(A1)***

$n = 16$ ***A1***

***[2 marks]***

(c) valid approach to find $d$ ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is $-3$ OR attempts to solve $-33 = 12 + 15d$

$d = -3$ ***A1***

***[2 marks]***

***Total [5 marks]***

**3.** (a) $(n-1)+n+(n+1)$ ***(A1)***

$=3n$ ***A1***

which is always divisible by $3$ ***AG***

***[2 marks]***

(b) $(n-1)^2+n^2+(n+1)^2$ $(= n^2-2n+1+n^2+n^2+2n+1)$ ***A1***

attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept $n^2-1$ or $n^2+1$) ***(M1)***

$=3n^2+2$ ***A1***

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct expression divided by 3 ***R1***

$3n^2$ is divisible by $3$ and so $3n^2+2$ is never divisible by $3$

OR the first term is divisible by 3, the second is not

OR $3\left(n^2+\frac{2}{3}\right)$ OR $\frac{3n^2+2}{3}=n^2+\frac{2}{3}$

hence the sum of the squares is never divisible by $3$ ***AG***

***[4 marks]***

***Total [6 marks]***

**6.** (a) **EITHER**

recognises the required term (or coefficient) in the expansion ***(M1)***

$bx^5 = {}^7C_2\, x^5 1^2$ OR $b = {}^7C_2$ OR ${}^7C_5$

$b = \frac{7!}{2!5!}\ \left(= \frac{7!}{2!(7-2)!}\right)$

correct working ***A1***

$\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$ OR $\frac{7\times6}{2!}$ OR $\frac{42}{2}$

**OR**

lists terms from row $7$ of Pascal's triangle ***(M1)***

$1, 7, 21, \ldots$ ***A1***

**THEN**

$b = 21$ ***AG***

***[2 marks]***

(b) $a = 7$ ***(A1)***

correct equation ***A1***

$21x^5 = \frac{ax^6 + 35x^4}{2}$ OR $21x^5 = \frac{7x^6 + 35x^4}{2}$

correct quadratic equation ***A1***

$7x^2 - 42x + 35 = 0$ OR $x^2 - 6x + 5 = 0$ (or equivalent)

valid attempt to solve **their** quadratic ***(M1)***

$(x-1)(x-5) = 0$ OR $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$

$x = 1,\ x = 5$ ***A1***

**Note:** Award final ***A0*** for obtaining $x = 0,\ x = 1, x = 5$.

***[5 marks]***

***Total [7 marks]***

## Section B

**1.**

**Note:** The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

(a) **EITHER**

| | | |
|---|---|---|
| $N = 10$ | OR | $N = 20$ |
| $I\% = 2.74$ | | $I\% = 2.74$ |
| $PV = (\mp)1700$ | | $PV = (\mp)1700$ |
| $P/Y = 1$ | | $P/Y = 2$ |
| $C/Y = 2$ | | $C/Y = 2$ |

***(M1)(A1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. Accept a positive or negative value for $PV$.

**OR**

$$1700\left(1+\frac{0.0274}{2}\right)^{2\times 10}$$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for substitution into compound interest formula.
Award ***(A1)*** for correct substitution.

**THEN**

$\$2231.71$ ***A1***

***[3 marks]***

*continued…*

*Question 1 continued*

(b) **EITHER**

$N = 10$

$PV = \mp 1700$

$FV = \pm 2231.71...$

$P/Y = 1$

$C/Y = 1$ ***(M1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen.

**OR**

$1700\left(1+\frac{r}{100}\right)^{10} = 2231.71...$ ***(M1)***

**THEN**

$r = 2.75876\ldots$

$r = 2.76$ ***A1***

**Note:** Ignore omission of opposite signs for $PV$ and $FV$ if $r = 2.76$ is obtained.

***[2 marks]***

(c) $\$531.71$ ***A1***

***[1 mark]***

***Total [6 marks]***

**4.** (a)

**Note:** Award a maximum of ***M1A0A0*** if the candidate manipulates both sides of the equation ( such as moving terms from one side to the other ).

**METHOD 1 (working with LHS)**

attempting to expand $\left(a^2-1\right)^2$ (do not accept $a^4+1$ or $a^4-1$) ***(M1)***

$\text{LHS} = a^2 + \frac{a^4-2a^2+1}{4}$ or $\frac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$= \frac{a^4+2a^2+1}{4}$ ***A1***

$= \left(\frac{a^2+1}{2}\right)^2 \ (= \text{RHS})$ ***AG***

**Note:** Do not award the final ***A1*** if further working contradicts the AG.

**METHOD 2 (working with RHS)**

attempting to expand $\left(a^2+1\right)^2$ ***(M1)***

$\text{RHS} = \frac{a^4+2a^2+1}{4}$

$= \frac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$= a^2 + \frac{a^4-2a^2+1}{4}$ ***A1***

$= a^2 + \left(\frac{a^2-1}{2}\right)^2$ (=LHS) ***AG***

**Note:** Do not award the final ***A1*** if further working contradicts the ***AG***.

***[3 marks]***

*continued…*

*Question 4 continued*

(b) recognise base and height as $a$ and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) ***(M1)***

correct substitution into triangle area formula ***A1***

$\text{Area} = \frac{a}{2}\left(\frac{a^2-1}{2}\right)$ (or equivalent) $\left(= \frac{a\left(a^2-1\right)}{4} = \frac{a^3-a}{4}\right)$

***[2 marks]***

***Total [5 marks]***

**4.** $86.4 = 50r^3$ ***(A1)***

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere ***(A1)***

$\frac{50\left(1.2^n - 1\right)}{0.2} > 33500$ OR $250\left(1.2^n - 1\right) = 33500$ ***(A1)***

attempt to solve their geometric $S_n$ inequality or equation ***(M1)***

sketch OR $n > 26.9045, n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ ***A1***

***Total [5 marks]***

**6.**

**Note:** Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^9C_r + (ax)^{9-r} + (1)^r$ .

valid approach for expansion (must be the product of a binomial coefficient with $n = 9$ and a power of $ax$) ***(M1)***

${}^9C_r(ax)^{9-r}(1)^r$ OR ${}^9C_{9-r}(ax)^r(1)^{9-r}$ OR ${}^9C_0(ax)^0(1)^9 + {}^9C_1(ax)^1(1)^8 + ...$

recognizing that the term in $x^6$ is needed ***(M1)***

$\frac{\text{Term in } x^6}{21x^2} = kx^4$ OR $r = 6$ OR $r = 3$ OR $9 - r = 6$

correct term or coefficient in binomial expansion (seen anywhere) ***(A1)***

${}^9C_6(ax)^6(1)^3$ OR ${}^9C_3\,a^6x^6$ OR $84\left(a^6x^6\right)(1)$ OR $84a^6$

**EITHER**

correct term in $x^4$ or coefficient (may be seen in equation) ***(A1)***

$\frac{{}^9C_6}{21}a^6x^4$ OR $4a^6x^4$ OR $4a^6$

Set their term in $x^4$ or coefficient of $x^4$ equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of *x*) ***(M1)***

$\frac{{}^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4$ OR $4a^6 = \frac{8}{7}a^5$

*continued…*

*Question 6 continued*

**OR**

correct term in $x^6$ or coefficient of $x^6$ (may be seen in equation) ***(A1)***

$84a^6x^6$ OR $84a^6$

set their term in $x^6$ or coefficient of $x^6$ equal to $24a^5x^6$ or $24a^5$ (do not accept other powers of *x*) ***(M1)***

$84a^6x^6 = 24a^5x^6$ OR $84a = 24$

**THEN**

$a = \frac{2}{7} \approx 0.286\left(0.285714...\right)$ ***A1***

**Note:** Award ***A0*** for the final mark for $a = \frac{2}{7}$ and $a = 0$.

***Total [6 marks]***

## Section B

**8.** (a) (i) recognition that $n = 5$ ***(M1)***

$S_5 = 45$ ***A1***

(ii) **METHOD 1**

recognition that $S_5 + u_6 = S_6$ ***(M1)***

$u_6 = 15$ ***A1***

**METHOD 2**

recognition that $60 = \frac{6}{2}(S_1 + u_6)$ ***(M1)***

$60 = 3(5 + u_6)$

$u_6 = 15$ ***A1***

**METHOD 3**

substituting their $u_1$ and $d$ values into $u_1 + (n-1)d$ ***(M1)***

$u_6 = 15$ ***A1***

***[4 marks]***

(b) recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their $u_6$ into $S_6$ ***(M1)***

OR equations for $S_5$ and $S_6$ in terms of $u_1$ and $d$

$1 + 4$ OR $60 = \frac{6}{2}(u_1 + 15)$

$u_1 = 5$ ***A1***

***[2 marks]***

*continued…*

*Question 8 continued*

(c) **EITHER**

valid attempt to find $d$ (may be seen in (a) or (b)) ***(M1)***

$d = 2$ ***(A1)***

**OR**

valid attempt to find $S_n - S_{n-1}$ ***(M1)***

$n^2 + 4n - \left(n^2 - 2n + 1 + 4n - 4\right)$ ***(A1)***

**OR**

equating $n^2 + 4n = \frac{n}{2}\left(5 + u_n\right)$ ***(M1)***

$2n + 8 = 5 + u_n$ (or equivalent) ***(A1)***

**THEN**

$u_n = 5 + 2(n-1)$ OR $u_n = 2n + 3$ ***A1***

***[3 marks]***

(d) recognition that $v_2 r^2 = v_4$ OR $(v_3)^2 = v_2 \times v_4$ ***(M1)***

$r^2 = 3$ OR $v_3 = (\pm)5\sqrt{3}$ ***(A1)***

$r = \pm\sqrt{3}$ ***A1***

**Note:** If no working shown, award ***M1A1A0*** for $\sqrt{3}$.

***[3 marks]***

(e) recognition that $r$ is negative ***(M1)***

$v_5 = -15\sqrt{3} \quad \left(= -\frac{45}{\sqrt{3}}\right)$ ***A1***

***[2 marks]***

***Total [14 marks]***

**2.** (a) $9\%$ (accept $0.09$) ***A1***

***[1 mark]***

(b) $t = 5$ (seen anywhere) ***(A1)***

$24961.28...$

$25000$ (dollars) ***A1***

***[2 marks]***

*continued…*

*Question 2 continued*

(c) **EITHER**

$n = 5$

$\mathrm{I}\% = 3$

$\mathrm{PV} = (\mp)15000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 1$ ***(A1)***

**Note:** Award ***(A1)*** for use of a financial app in their technology with all entries correct.

$(\Rightarrow \mathrm{FV} = (\pm)17389.11...)$

**OR**

$15000\left(1+\frac{3}{100}\right)^5 (=17389.11...)$ ***(A1)***

**THEN**

subtracting their value from their answer to part (b) ***(M1)***

$7572.17\ldots$

$7570$ (dollars) ***A1***

***[3 marks]***

***Total [6 marks]***

**6.** product of a binomial coefficient, a power of $ax^3$ and a power of $b$ seen ***(M1)***

evidence of correct term chosen

for $n=8:\ r=2$ (or $r=6$) OR for $n=10:\ r=2$ (or $r=8$) ***(A1)***

correct equations (may include powers of $x$) ***A1A1***

${}^8C_2a^2b^6=448\ \left(28a^2b^6=448\Rightarrow a^2b^6=16\right)$, ${}^{10}C_2a^2b^8=2880\ \left(45a^2b^8=2880\Rightarrow a^2b^8=64\right)$

attempt to solve their system in $a$ and $b$ algebraically or graphically ***(M1)***

$b=2\ ;\ a=\frac{1}{2}$ ***A1A1***

**Note:** Award a maximum of ***(M1)(A1)A1A1(M1)A1A0*** for $b=\pm 2$ and/or $a=\pm\frac{1}{2}$.

***[7 marks]***

## Section B

**5.** (a) attempt to use the binomial expansion of $(x+h)^8$ ***(M1)***

$^8C_0x^8h^0 + {}^8C_1x^7h^1 + {}^8C_2x^6h^2 + \ldots$

(i) $a = 8h$ (accept $^8C_1\,h$) ***A1***

(ii) $b = 28h^2$ (accept $^8C_2\,h^2$) ***A1***

(iii) $d = 70h^4$ (accept $^8C_4\,h^4$) ***A1***

***[4 marks]***

(b) recognition that there is a common ratio between their terms ***(M1)***

$8h \times r = 28h^2$ **OR** $28h^2 \times r = 70h^4$ **OR** $8h \times r^2 = 70h^4$

correct equation in terms of $h$ ***(A1)***

$\frac{28h^2}{8h} = \frac{70h^4}{28h^2}$ (or equivalent)

$h = 1.4$ ***A1***

***[3 marks]***

***Total [7 marks]***

**8.**

**Note:** The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

(a) (i) $4200\times 36$ ***(A1)***

$=151200$

$=(\$)\,151000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$\dfrac{1500\left(1-1.04^{36}\right)}{1-1.04}$ ***(A1)***

$=116397.4707...$

$=(\$)116397.47$ ***A1***

***[5 marks]***

(b) Sorin's future value after $n$ years = $160000\left(1+\dfrac{5}{100\times 12}\right)^{12n}$ ***A1***

***[1 mark]***

*continued…*

*Question 8 continued*

(c) (i) Sorin's total $= 160000\left(1+\frac{5}{100\times 12}\right)^{6} \left(=164041.89...\right)$ (***A1***)

Daniela's total $= \frac{1500\left(1-1.04^{6}\right)}{1-1.04}\left(=9949.46...\right)$ (***A1***)

total value = ($)$173991.36$ ***A1***

(ii)

**EITHER** (finding number of months, $m$)

$160000\left(1+\frac{5}{100\times 12}\right)^{m}+\frac{1500\left(1-1.04^{m}\right)}{1-1.04}\left(\geq 257000\right)$ ***(A1)***

$m \geq 28.4412...$ OR $\left(m=28 \Rightarrow\right)\ 254707$ AND $\left(m=29 \Rightarrow\right)\ 259954$ ***(A1)***

**Note:** Condone use of an equation or strict inequality.

**OR** (finding number of years, $n$)

$160000\left(1+\frac{5}{100\times 12}\right)^{12\times n}+\frac{1500\left(1-1.04^{12\times n}\right)}{1-1.04}\left(\geq 257000\right)$ ***(A1)***

$n \geq 2.37010...$ (years) ***(A1)***

**Note:** Condone use of an equation or strict inequality.

**THEN**

$m=29$ (months) ***A1***

***[6 marks]***

*continued…*

*Question 8 continued*

(d) **EITHER**

| | | |
|---|---|---|
| $N = 24$ | OR | $N = 6$ |
| $PV = \mp 30000$ | | $PV = \mp 30000$ |
| $PMT = 0$ | | $PMT = 0$ |
| $FV = \pm 41000$ | | $FV = \pm 41000$ |
| $P/Y = 4$ | | $P/Y = 1$ |
| $C/Y = 4$ | | $C/Y = 4$ |

***(M1)(A1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

**OR**

$$30000\left(1+\frac{r}{100\times 4}\right)^{6\times 4} = 41000$$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

**THEN**

$5.24027\ldots$

$(r =) 5.24\ (\%)$ ***A1***

***[3 marks]***

***Total [15 marks]***

**8.**

**Note:** The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

(a) (i) $5500 \times 36$ ***(A1)***

$= (\$)\, 198000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$\frac{2000\left(1-1.06^{36}\right)}{1-1.06}$ ***(A1)***

$= 238241.7333...$

$= (\$)238241.73$ ***A1***

***[5 marks]***

(b) Sorin's future value after $n$ years = $120000\left(1+\frac{4}{100\times 12}\right)^{12n}$ ***A1***

***[1 mark]***

*continued…*

*Question 8 continued*

(c) (i) Sorin's total $= 120000\left(1+\frac{4}{100\times 12}\right)^{6}\ (=122420.09)$ ***(A1)***

Daniela's total $= \frac{2000\left(1-1.06^{6}\right)}{1-1.06}\ (=13950.64)$ ***(A1)***

total value = ($)136370.73 ***A1***

(ii) **EITHER** (finding number of months, $m$)

$120000\left(1+\frac{4}{100\times 12}\right)^{m}+\frac{2000\left(1-1.06^{m}\right)}{1-1.06}\ (\geq 250000)$ ***(A1)***

$m \geq 26.0905...$ OR $(m=26\Rightarrow)\ 249157...$ AND $(m=27\Rightarrow)\ 258692...$ ***(A1)***

**Note:** Condone use of an equation or strict inequality.

**OR** (finding number of years, $n$)

$120000\left(1+\frac{4}{100\times 12}\right)^{12\times n}+\frac{2000\left(1-1.06^{12\times n}\right)}{1-1.06}\ (\geq 250000)$ ***(A1)***

$n \geq 2.17421...$ (years) ***(A1)***

**Note:** Condone use of an equation or strict inequality.

**THEN**

$m=27$ (months) ***A1***

***[6 marks]***

*continued…*

*Question 8 continued*

(d) **EITHER**

| | | |
|---|---|---|
| $N = 24$ | OR | $N = 6$ |
| $PV = \mp 40000$ | | $PV = \mp 40000$ |
| $PMT = 0$ | | $PMT = 0$ |
| $FV = \pm 53000$ | | $FV = \pm 53000$ |
| $P/Y = 4$ | | $P/Y = 1$ |
| $C/Y = 4$ | | $C/Y = 4$ |

***(M1)(A1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

**OR**

$$40000\left(1+\frac{r}{100\times 4}\right)^{6\times 4} = 53000$$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

**THEN**

$4.71781\ldots$

$(r =)\ 4.72\ (\%)$ ***A1***

***[3 marks]***

***Total [15 marks]***

**1.** (a) valid method to find the common difference ***(M1)***

$d = \frac{22-10}{2}$ OR $10+2d=22$ OR $u_1+d=10,\ u_1+3d=22$ OR $u_3=16$

$d=6$ ***A1***

***[2 marks]***

(b) $u_1 = 10-6\left(=4\right)$ ***(A1)***

$u_n = 4+6\left(n-1\right)$ OR $u_n = 6n-2$ ***A1***

***[2 marks]***

***Total [4 marks]***

**3.** (a) $\log_{10} 1 - \log_{10} a$ OR $\log_{10} a^{-1} = -\log_{10} a$ OR $\log_{10} 10^{-\frac{1}{3}}$ OR $10^x = \frac{1}{10^{\frac{1}{3}}}$ ***(A1)***

$= -\frac{1}{3}$ ***A1***

***[2 marks]***

(b) $\frac{\log_{10} a}{\log_{10} 1000}$ OR $\frac{1}{3}\log_{1000} 10$ OR $\log_{1000} \sqrt[3]{1000^{\frac{1}{3}}}$ OR $10^{\frac{1}{3}} = 1000^x \left(= \left(10^3\right)^x\right)$ ***(A1)***

$\frac{\log_{10} a}{3}$ OR $\frac{1}{3}\log_{1000} 1000^{\frac{1}{3}}$ OR $\log_{1000} 1000^{\frac{1}{9}}$ OR $3x = \frac{1}{3}$ ***(A1)***

$= \frac{1}{9}$ ***A1***

***[3 marks]***

***Total [5 marks]***

**6.** (a) $S_n = \frac{10^n - 1}{9}$ ***A1***

$(a = 10, b = 9)$

***[1 mark]***

(b) **METHOD 1**

$S_1 + S_2 + S_3 + \ldots + S_n$

$= \frac{10-1}{9} + \frac{10^2-1}{9} + \ldots + \frac{10^n - 1}{9}$ ***(A1)***

$= \frac{10-1+10^2-1+10^3-1+\ldots+10^n-1}{9}$ OR $\frac{9\left(10-1+10^2-1+10^3-1+\ldots+10^n-1\right)}{81}$

attempt to use geometric series formula on powers of 10, and collect -1's together ***M1***

$10+10^2+10^3+\ldots+10^n = \frac{10\left(10^n-1\right)}{10-1}$ and $-1-1-1\ldots = -n$ ***A1***

$= \frac{\frac{10\left(10^n-1\right)}{10-1} - n}{9}$ OR $\frac{9\left(\frac{10\left(10^n-1\right)}{10-1}\right) - 9n}{81}$ ***A1***

**Note:** Award ***A1*** for any correct intermediate expression.

$= \frac{10\left(10^n-1\right)-9n}{81}$ ***AG***

*continued…*

*Question 6 continued*

**METHOD 2**

attempt to create sum using sigma notation with $S_n$ ***M1***

$\sum_{i=1}^{n} \frac{10^i - 1}{9} \quad \left( = \frac{1}{9}\left( \sum_{i=1}^{n} 10^i - \sum_{i=1}^{n} 1 \right) \right)$

$\sum_{i=1}^{n} 10^i = \frac{10\left(10^n - 1\right)}{9}$ ***A1***

$\sum_{i=1}^{n} 1 = n$ ***A1***

$= \frac{1}{9}\left( \frac{10\left(10^n - 1\right)}{9} - n \right)$ OR $\frac{1}{9}\left( \frac{10\left(10^n - 1\right) - 9n}{9} \right)$ ***A1***

$= \frac{10\left(10^n - 1\right) - 9n}{81}$ ***AG***

***[4 marks]***

***Total [5 marks]***

## Section B

**9.** (a) attempt to find a difference ***(M1)***

$d = p - a,\ 2d = q - a,\ d = q - p$ OR $p = a + d,\ q = a + 2d,\ q = p + d$

correct equation ***A1***

$p - a = q - p$ OR $q - a = 2(p - a)$ OR $p = \frac{a+q}{2}$ (or equivalent)

$2p - q = a$ ***AG***

***[2 marks]***

(b) attempt to find a ratio ***(M1)***

$r = \frac{s}{a},\ r^2 = \frac{t}{a},\ r = \frac{t}{s}$ OR $s = ar,\ t = ar^2,\ t = sr$

correct equation ***A1***

$\left(\frac{s}{a}\right)^2 = \frac{t}{a}$ OR $\frac{s}{a} = \frac{t}{s}$ (or equivalent)

$s^2 = at$ ***AG***

***[2 marks]***

*continued…*

*Question 9 continued*

(c) **EITHER**

$2p-1=s^2$ (or equivalent) ***A1***

$\left(s^2>0\right) \Rightarrow 2p-1>0$ OR $s=\sqrt{2p-1} \Rightarrow 2p-1>0$ OR $p=\frac{s^2+1}{2}$ ( and $s^2>0$ ) ***R1***

**OR**

$2p-1=a$ and $s^2=a$ ***A1***

( $s^2>0$ , so ) $a>0 \Rightarrow 2p-1>0$ OR $p=\frac{a+1}{2}$ and $a>0$ ***R1***

$\Rightarrow p>\frac{1}{2}$ ***AG***

| **Note**: Do not award ***A0R1***. |
|---|

***[2 marks]***

*continued…*

*Question 9 continued*

(d) (i) $9, 5, 1, -3$ ***A1A1***

| ***Note:*** Award ***A1*** for each of 2nd term and 4th term |
|---|

(ii) $9, 3, 1, \frac{1}{3}$ ***A1A1***

| ***Note:*** Award ***A1*** for each of 2nd term and 4th term |
|---|

***[4 marks]***

(e) (i) attempt to find the difference between two consecutive terms ***(M1)***

$d = u_2 - u_1 = 5 + \ln 3 - 9 - \ln 9$ OR $d = u_3 - u_2 = 1 + \ln 1 - 5 - \ln 3$

$\ln 9 = 2\ln 3$ OR $\ln 1 = 0$ OR $\ln 3 - \ln 9 = \ln \frac{1}{3} \left(= \ln 3^{-1} = -\ln 3\right)$ (seen anywhere) ***(A1)***

$d = -4 - \ln 3$ ***A1***

*continued…*

*Question 9 continued.*

(ii) **METHOD 1**

attempt to substitute first term and their common difference into $S_{10}$ ***(M1)***

$\frac{10}{2}\left(2(9+\ln 9)+9\left(-4-\ln 3\right)\right)$ OR $\frac{10}{2}\left(2(9+2\ln 3)+9\left(-4-\ln 3\right)\right)$ (or equivalent ) ***A1***

$=5\left(-18-5\ln 3\right)$ (or equivalent in terms of $\ln 3$ ) ***A1***

$\sum_{i=1}^{10} u_i = -90-25\ln 3$ ***AG***

**METHOD 2**

$u_{10}=9+\ln 9+9\left(-4-\ln 3\right)\left(=-27+\ln 9-9\ln 3\right)$

attempt to substitute first term and their $u_{10}$ into $S_{10}$ ***(M1)***

$\frac{10}{2}\left(2(9+\ln 9)+9\left(-4-\ln 3\right)\right)$ OR $\frac{10}{2}\left(9+\ln 9-27+\ln 9-9\ln 3\right)$ OR

$\frac{10}{2}\left(2(9+2\ln 3)+9\left(-4-\ln 3\right)\right)$ OR $\frac{10}{2}\left(9+\ln 9-27-7\ln 3\right)$ (or equivalent) ***A1***

$=5\left(-18-5\ln 3\right)$ (or equivalent in terms of $\ln 3$ ) ***A1***

$\sum_{i=1}^{10} u_i = -90-25\ln 3$ ***AG***

***[6 marks]***

***Total [16 marks]***

**2.** (a) recognition that a $15\%$ loss leaves $85\%$ OR finding $15\%$ and subtracting from original ***(M1)***

$0.85 \times 35000$ OR $35000 - 0.15 \times 35000$

$= (\$)29750$ ***A1***

**Note**: Accept $(\$)29800$.

***[2 marks]***

(b) **EITHER**

$29750 \times 0.89^9$ ***(A1)***

**OR**

$\mathrm{N} = 9$

$\mathrm{I}\% = -11$

$\mathrm{PV} = \mp 29750$

***(A1)***

**THEN**

value $(FV) = (\$)10423$ ***A1***

**Note**: For this ***A1*** the answer must be rounded to the nearest dollar.
Accept $(\$)10441$ from using $3$ sf answer from part (a).

***[2 marks]***

*continued…*

*Question 2 continued*

(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750 \times 0.89^{n-1} < 3500$ OR table of values ***(M1)***

$19.3643...$ OR $(n = 19 \Rightarrow)\ 3651.80...$ OR $(n = 20 \Rightarrow)\ 3250.10...$ ***(A1)***

**Note**: For candidates using $(\$)29800$, $n > 19.3787...$, $(n = 19 \Rightarrow)\ 3657.93...$, $(n = 20 \Rightarrow)\ 3255.56...$.

$n = 20$ ***A1***

***[3 marks]***

**METHOD 2**

use of the finance app with $\mathrm{I\%} = -11, \mathrm{PV} = \mp 29750, \mathrm{FV} = \pm 3500$

OR $29750 \times 0.89^{N} < 3500$ (condone the use of $n$ or $x$) ***(M1)***

$(N =)\ 18.3643...$ ***(A1)***

**Note**: For candidates using $(\$)29800$, $N = 18.3787...$

$n = 20$ ***A1***

***[3 marks]***

***Total [7 marks]***

**4. METHOD 1**

attempt to expand $(2n+5)^2-(2n-5)^2$ ***M1***

**Note:** Award ***M0*** for invalid attempts such as $(2n+5)^2=4n^2+25$.

$=4n^2+20n+25-(4n^2-20n+25)$ or equivalent ***A1***

$=40n$ OR $20n+20n$ ***A1***

$=10(4n)$ OR $\frac{40}{10}=4$ OR $\frac{40n}{4}=10n$ OR $20n+20n=10(2n+2n)$ (or equivalent) ***R1***

so is a multiple of $10$ ***AG***

**Note:** Do not award the ***R1*** unless both ***A*** marks have been awarded.

**METHOD 2**

use of $a^2-b^2=(a+b)(a-b)$ where $a=2n+5, b=2n-5$ ***M1***

$=(2n+5+2n-5)(2n+5-2n+5)$ ***A1A1***

**Note:** Award ***A1*** for each correct bracket.

$=4n\times 10\ (=40n)$ ***R1***

so is a multiple of $10$ ***AG***

**Note:** Do not award the ***R1*** unless both ***A*** marks have been awarded.

***[4 marks]***

**6**. attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right]$ or $S_n = \frac{n}{2}\left[u_1 + u_n\right]$ to set up at least one equation in $u_1$ and $d$ ***(M1)***

$14 = u_1 + 9d$ and $200 = \frac{25}{2}\left[2u_1 + 24d\right]$ ***A1***

attempt to solve their two linear equations in $u_1$ and $d$ simultaneously (must eliminate one variable) ***(M1)***

$d = -2 \left(\Rightarrow u_1 = 32\right)$ ***(A1)***

attempt to solve $u_k = 0$ with their $d$ (or with their $d$ and $u_1$) ***(M1)***

$\Rightarrow k = 17$ ***A1***

***[6 marks]***

## Section B

**4. METHOD 1**

attempt to expand $(3n+2)^2-(3n-2)^2$ ***M1***

**Note:** Award ***M0*** for invalid attempts such as $(3n+2)^2=9n^2+4$.

$=9n^2+12n+4-(9n^2-12n+4)$ or equivalent ***A1***

$=24n$ OR $12n+12n$ ***A1***

$=12(2n)$ OR $\frac{24}{12}=2$ OR $\frac{24n}{2}=12n$ OR $12n+12n=12(n+n)$ (or equivalent) ***R1***

so is a multiple of $12$ ***AG***

**Note:** Do not award the ***R1*** unless both ***A*** marks have been awarded.

**METHOD 2**

use of $a^2-b^2=(a+b)(a-b)$ where $a=3n+2, b=3n-2$ ***M1***

$=(3n+2+3n-2)(3n+2-3n+2)$

$=6n\times 4$ ***A1***

$=24n$ ***A1***

$=12(2n)$ OR $\frac{24n}{12}=2n$ OR $\frac{24n}{2}=12n$ OR $12n+12n=12(n+n)$ (or equivalent) ***R1***

so is a multiple of $12$ ***AG***

**Note:** Do not award the ***R1*** unless both ***A*** marks have been awarded.

***[4 marks]***

**6.** attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right]$ or $S_n = \frac{n}{2}\left[u_1 + u_n\right]$ to set up at least one equation in $u_1$ and $d$ ***(M1)***

$16 = u_1 + 9d$ and $100 = \frac{25}{2}\left[2u_1 + 24d\right]$ ***(A1)***

attempt to solve their two linear equations in $u_1$ and $d$ simultaneously (must eliminate one variable) ***(M1)***

$d = -4 \left(\Rightarrow u_1 = 52\right)$ ***A1***

attempt to solve $u_k = 0$ with their $d$ (or with their $d$ and $u_1$) ***(M1)***

$\Rightarrow k = 14$ ***A1***

***[6 marks]***

## SECTION B

**2. EITHER**

attempt to form a product of binomial coefficient, a power of $2x$ and a power of $-5$ seen ***(M1)***

$^{11}C_8(2x)^8(-5)^3$ OR $^{11}C_3(2x)^8(-5)^3$ OR $165\times(2x)^8(-5)^3$ ***(A1)(A1)***

**Note:** Award ***A1*** for $^{11}C_8$ or $^{11}C_3$ or 165, ***A1*** for $(2x)^8(-5)^3$.

**OR**

attempt to use the general term ***(M1)***

$^{11}C_r(2x)^{11-r}(-5)^r$ and $r=3$ ***(A1)(A1)***

**THEN**

$-5280000$ (exact) ***A1***

**Note:** Award ***A0*** for a final answer of $-5280000x^8$.

***[4 marks]***

**2. EITHER**

attempt to form a product of binomial coefficient, a power of $2x$ and a power of $-5$ seen ***(M1)***

$^9C_3(2x)^6(-5)^3$ OR $^9C_6(2x)^6(-5)^3$ OR $84\times(2x)^6(-5)^3$ ***(A1)(A1)***

**Note:** Award ***A1*** for $^9C_6$ or $^9C_3$ or 84, ***A1*** for $(2x)^6(-5)^3$.

**OR**

attempt to use the general term ***(M1)***

$^9C_r(2x)^{9-r}(-5)^r$ and $r=3$ ***(A1)(A1)***

**THEN**

$-672000$ (exact) ***A1***

**Note:** Award ***A0*** for a final answer of $-672000x^6$.

***[4 marks]***

**3.** (a) $k = \frac{4}{400}\left(= \frac{1}{100} = 0.01\right)$ ***A1***

***[1 mark]***

(b) attempt to find binomial coefficients or multiply out brackets ***(M1)***

e.g. Pascal's triangle down to correct row OR $\left(1+2x+x^2\right)^2$ OR substitute into binomial expansion

$(1+x)^4$

$= 1 + 4x + 6x^2 + 4x^3 + x^4$ ***A1***

***[2 marks]***

(c) **METHOD 1**

recognition that the expansion can be used with $x$ replaced with $k$ ***(M1)***

$\left(1+\frac{1}{100}\right)^4$

$= 1 + \frac{4}{100} + \frac{6}{100^2} + ...\left(= 1 + 0.04 + 0.0006 + ...\right)$ ***(A1)***

multiplies by 1000 (seen anywhere) ***(M1)***

$1000\left(1+\frac{1}{100}\right)^4$

$= 1000 + 40 + 0.6 + ...\left(= 1040.6...\right)$

$= 1041$ (dinar) ***A1***

**METHOD 2**

attempt to find the value of $(1+k)^4$ by hand ***(M1)***

$(1.01)^4 = (1.0201)(1.01)^2 = (1.030301)(1.01)$

$= 1.0406...$ ***(A1)***

multiplies by 1000 (seen anywhere) ***(M1)***

$1000(1.01)^4$

$= 1040.6...$

$= 1041$ (dinar) ***A1***

***[4 marks]***

***Total [7 marks]***

**6.** (a) (i) for a sequence of areas, uses two consecutive terms to find a common ratio OR for sequences of both widths and heights uses two consecutive terms for both sequences to find both common ratios OR recognises that both widths and heights are geometric sequences with common ratio $\frac{3}{2}$ ***M1***

areas form a geometric sequence with first term 20 and common ratio $\frac{45}{20}$ ***A1***

OR area of picture frame $F_n$ is $4\left(\frac{3}{2}\right)^{n-1} \times 5\left(\frac{3}{2}\right)^{n-1}$

area of $F_n$ is $20\left(\frac{9}{4}\right)^{n-1}$ ***AG***

(ii) attempt to find the sum of the areas using $S_n = \frac{u_1\left(r^n - 1\right)}{r-1}$ ***(M1)***

sum of areas $\frac{20\left(\left(\frac{9}{4}\right)^{10} - 1\right)}{\frac{9}{4} - 1}\left(= 16\left(\left(\frac{9}{4}\right)^{10} - 1\right)\right)$ ***(A1)***

mean area $= \frac{1}{10}\left(\frac{20\left(\left(\frac{9}{4}\right)^{10} - 1\right)}{\frac{9}{4} - 1}\right)\left(= \frac{1}{10}\left(16\left(\left(\frac{9}{4}\right)^{10} - 1\right)\right)\right)$

$= \frac{16}{10}\left(\left(\frac{9}{4}\right)^{10} - 1\right)\left(= \frac{8}{5}\left(\left(\frac{9}{4}\right)^{10} - 1\right)\right)$ ***A1***

$p = \frac{8}{5},\ a = 10$

***[5 marks]***

*continued…*

*Question 6 continued*

(b) recognition that median is between 5th and 6th picture frame ***(M1)***

$$\text{median area} = \frac{20\left(\frac{9}{4}\right)^4 + 20\left(\frac{9}{4}\right)^5}{2} \quad \textit{(A1)}$$

$$= \frac{20\left(\frac{9}{4}\right)^4\left(1+\frac{9}{4}\right)}{2}$$

$$= \frac{65}{2}\left(\frac{9}{4}\right)^4 \quad \textbf{A1}$$

$$q = \frac{65}{2}$$

***[3 marks]***

***Total [8 marks]***

## Section B

**8.** (a) (i) **METHOD 1**

attempt to equate differences of consecutive terms ***(M1)***

$(3-2k)-(k-5)=(5k+3)-(3-2k)$ **OR** $(k-5)-(3-2k)=(3-2k)-(5k+3)$ ***A1***

$8-3k=7k$

$(k=)\ \frac{4}{5}$ ***A1***

**METHOD 2 (system of equations)**

**TWO** correct equations involving $k$ and $d$ ***A1***

$k-5+d=3-2k$ OR $3-2k+d=5k+3$ OR $k-5+2d=5k+3$

OR $\frac{3}{2}\left(2\left(k-5\right)+2d\right)=k-5+3-2k+5k+3$ (or equivalent)

valid attempt to solve their system of equations using substitution or elimination ***(M1)***

$\left(d=5.6\right)$

$(k=)\ \frac{4}{5}$ ***A1***

**METHOD 3 (in terms of k)**

$\frac{3}{2}\left(k-5+5k+3\right)=k-5+3-2k+5k+3$ (or equivalent) ***A1***

combining like terms ***(M1)***

$9k-3=4k+1$ **OR** $5k=4$ (or equivalent)

$(k=)\ \frac{4}{5}$ ***A1***

**METHOD 4 (arithmetic mean)**

attempt to find mean of $u_1$ and $u_3$ ***(M1)***

$\frac{(k-5)+(5k+3)}{2}=3-2k$ ***A1***

$3k-1=3-2k$

$(k=)\ \frac{4}{5}$ ***A1***

(ii) substituting their value of $k$ into expression for $u_3$ ***(A1)***

$\left(u_3=\right)\ 5\times\frac{4}{5}+3$

$=7$ ***A1***

***[5 marks]***

*continued…*

*Question 8 continued*

(b) (i) substituting $k = 12$ into $u_1$, $u_2$ or $u_3$ ***(M1)***

$(u_1 =)\ 7$ **AND** $(u_2 =) -21$ **AND** $(u_3 =)\ 63$ ***(A1)***

$\left(r =\right) \frac{-21}{7} = \frac{63}{-21}\ (= -3)$ **OR** $(-21)^2 = 7 \times 63$ **OR** $r = -3$ ***R1***

$u_1$, $u_2$ and $u_3$ are in geometric sequence ***AG***

(ii) since $\left| r \right| \geq 1$ ***R1***

hence not convergent ***AG***

***[4 marks]***

(c) (i) attempts to find ratios, in terms of $k$, of consecutive terms and equating ***(M1)***

$\frac{(3-2k)}{(k-5)} = \frac{(5k+3)}{(3-2k)}$ **OR** $(3-2k)^2 = (k-5)(5k+3)$ (or equivalent)

$9 - 12k + 4k^2 = 5k^2 - 22k - 15$ ***A1***

**Note:** Award ***A1*** for correct expansion of all brackets leading to given result.

$k^2 - 10k - 24 = 0$ ***AG***

(ii) recognizing need to factorize, complete the square or substitute into quadratic formula ***(M1)***

$(k+2)(k-12)\left(=0\right)$ **OR** $(k-5)^2 - 49\ (=0)$ **OR** $k = \frac{10 \pm \sqrt{196}}{2}$

$k = -2$ (accept $k = -2$ **and** $k = 12$) ***A1***

substituting their value of $k$ (other than $k = 12$) to find $u_1$, $u_2$ or $u_3$ ***(M1)***

$(u_1 =) -7$ **AND** $(u_2 =)\ 7$ **AND** $(u_3 =) -7$ ***A1***

(iii) $(S_{2m} =)\ 0$ ***A1***

***[7 marks]***

***Total [16 marks]***

**8.** (a) $15$ ***A1***

***[1 mark]***

(b) attempt to add 11 cards onto a stack with 3 rows OR attempt to consider all 4 rows ***(M1)***

valid diagram with 4 rows OR $t_4 = 15 + 11$ OR $t_4 = 2 + 5 + 8 + 11$

$= 26$ ***A1***

***[2 marks]***

(c) **METHOD 1**

recognition that $t_n$ is a sum of an arithmetic sequence ***(M1)***

$t_n = 2 + 5 + 8 + 11 + ...$

attempt to use formula for the sum of $n$ terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}\left(2\left(2\right) + 3(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}\left(3n + 1\right)$ ***AG***

**METHOD 2**

attempt to split $t_n$ into the total number of stacked and horizontal cards ***(M1)***

stacked $2 + 4 + 6 + ... = \frac{n}{2}\left(4 + 2(n-1)\right)\left( = \frac{n}{2}\left(2n + 2\right)\right)$ ***A1***

horizontal $0 + 1 + 2 + ... = \frac{n}{2}\left(0 + 1(n-1)\right)\left( = \frac{n}{2}\left(n - 1\right)\right)$ ***A1***

$t_n = \frac{n}{2}\left(4 + 2(n-1)\right) + \frac{n}{2}\left(0 + 1(n-1)\right)\left( = \frac{n}{2}\left(2n + 2\right) + \frac{n}{2}\left(n - 1\right)\right)$

$t_n = \frac{n}{2}\left(3n + 1\right)$ ***AG***

*continued…*

*Question 8 continued.*

**METHOD 3**

recognition that a stack with $n$ rows is made up of complete triangles with the bottom row of horizontal cards removed and that the numbers of complete triangle cards form an arithmetic sequence ***(M1)***

$t_n = (3+6+9+12+...+3n) - n$ OR $t_n = 3(1+2+3+4+...+n) - n$

attempt to use formula for the sum of $n$ terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}(2(3)+3(n-1)) - n$ OR $t_n = 3\times\frac{n}{2}(1+n) - n$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

***[3 marks]***

(d) attempt to solve $\frac{n(3n+1)}{2} \leq 14(52)(=728)$ (***M1***)

**Note:** Accept an attempt to solve an equation for (***M1***).

$21.8642...$ OR $n=21, t_n = 672$ and $n=22, t_n = 737$ ***(A1)***

max number of rows is 21 ***A1***

***[3 marks]***

*continued…*

*Question 8 continued.*

(e) **EITHER**

attempt to solve by listing at least six values of $t_n$ (***M1)***

$2, 7, 15, 26, 40, 57...$

**OR**

recognition that $\frac{\frac{1}{2}n(3n+1)}{52}$ must be an integer ***(M1)***

$\frac{1}{2}n(3n+1) = 52k \ \left(\text{where } k \text{ is an integer}\right)$

**THEN**

min number of rows is 13 ***A1***

**Note:** Award (***M1)A0*** for an answer of 5 packs.

Award ***M0A0*** for any answer resulting from solving $\frac{1}{2}n(3n+1) = 52$.

***[2 marks]***

*continued…*

*Question 8 continued.*

(f) **EITHER**

attempt to use Pythagoras's Theorem or trigonometry to find the height of an equilateral triangle with sides $88\text{mm}$ ***(M1)***

height= $\sqrt{88^2 - 44^2}$ OR $88\sin 60^\circ$ OR $88\cos 30^\circ$ OR $44\tan 60^\circ$ OR

$\frac{44}{\tan 30^\circ}$ OR $44\sqrt{3}\left(= 76.2102...\right)$ ***(A1)***

attempt to solve $44n\sqrt{3} > 2000$ OR $\text{their perpendicular height} \times n > 2000$ ***(M1)***

**Note:** Accept an attempt to solve an equation for (***M1)***.

**OR**

attempt to use trigonometry to find the side of an equilateral triangle with height $2000\text{mm}$ ***(M1)***

side= $\frac{2000}{\sin 60^\circ}$ OR $\frac{2000}{\cos 30^\circ}$ OR $\frac{4000}{\sqrt{3}}\left(= 2309.40...\right)$ ***(A1)***

attempt to solve $88n > 2309.40...$ OR $88n > \text{their side}$ ***(M1)***

**Note:** Accept an attempt to solve an equation for (***M1)***.

**THEN**

$n > 26.2431...$

so min number of rows is 27 ***(A1)***

$t_{27} = 1107$ ***A1***

***[5 marks]***

***Total [16 marks]***

**4. EITHER**

attempt to use the binomial expansion of $(2x+k)^{10}$ ***(M1)***

$^{10}C_0(2x)^{10} + {}^{10}C_1(2x)^9 k^1 + {}^{10}C_2(2x)^8 k^2 + \ldots$ OR $^{10}C_0 k^{10} + {}^{10}C_1 k^9 (2x)^1 + {}^{10}C_2 k^8 (2x)^2 + \ldots$

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example $^{10}C_r + (2x)^{10-r} + k^r$.

identifying the correct term ***(M1)***

$^{10}C_4(2x)^6 k^4$ OR $^{10}C_6 k^4 (2x)^6$

**OR**

attempt to use general term $^{10}C_r(2x)^{10-r}k^r$ OR $^{10}C_r k^{10-r}(2x)^r$ ***(M1)***

$r=4$ OR $r=6$ ***(A1)***

**THEN**

correct equation in terms of $k$ only ***(A1)***

$^{10}C_4 2^6 k^4 = 8.4\times10^6$ OR $^{10}C_6 k^4 2^6 = 8.4\times10^6 \ (=8400000)$

$k=\pm5$ ***A2***

**Note**: Award ***A1*** for $k=5$ only.

***Total [5 marks]***

**9.** (a) recognising geometric sequence with $r = \frac{2}{3}$ (seen anywhere) ***(M1)***

$10\left(\frac{2}{3}\right)^{6-1}$ OR $\frac{20}{3}\left(\frac{2}{3}\right)^{5-1}$ ***(A1)***

$1.31687...$

height after 5th bounce = $1.32$ (m) (accept $\frac{320}{243}$) ***A1***

***[3 marks]***

(b) recognition of the need to use $S_n$ (seen anywhere) ***(M1)***

$10+10\times\left(\frac{2}{3}\right)+10\times\left(\frac{2}{3}\right)^2+10\times\left(\frac{2}{3}\right)^3+10\times\left(\frac{2}{3}\right)^4$ OR $S_5$

recognition to double the height ***(M1)***

$10+2\times S_4$ OR $2\times S_5-10$ OR

$10+2\times10\times\left(\frac{2}{3}\right)+2\times10\times\left(\frac{2}{3}\right)^2+2\times10\times\left(\frac{2}{3}\right)^3+2\times10\times\left(\frac{2}{3}\right)^4$ (or equivalent)

$42.0987\ldots$

total distance travelled $= 42.1$ ***A1***

***[3 marks]***

(c) recognising the need to use $S_\infty$ formula (seen anywhere) ***(M1)***

$2S_\infty+u_1$ OR $2S_\infty-x$

$2\left(\frac{\frac{2}{3}x}{1-\frac{2}{3}}\right)+x$ OR $2\left(\frac{x}{1-\frac{2}{3}}\right)-x$ OR $\frac{x}{1-\frac{2}{3}}+\frac{\frac{2}{3}x}{1-\frac{2}{3}}$ ***A1***

$2(2x)+x$ OR $6x-x$ OR $3x+2x$ ***A1***

total distance travelled = $5x$ ***AG***

***[3 marks]***

*continued…*

*Question 9 continued.*

(d) $\delta_1 = 50$ (m) ***A1***

***[1 mark]***

(e) $\delta_2 \left(= 5 \times 9.56\right) = 47.8$

$d = 47.8 - 50\left(= -2.2\right)$ OR $d = 5 \times -0.44\left(= -2.2\right)$ ***(A1)***

attempt to find $n$ using $n$th term of an AP with their $d$ ***(M1)***

$50 + \left(n - 1\right)\left(-2.2\right) < 25$ (accept equation)

$12.3636\ldots$ ***(A1)***

Ball $13$ ***A1***

***[4 marks]***

***Total [14 marks]***

---

**3.** (a) 12 (terms) ***A1***

***[1 mark]***

(b) product of a binomial coefficient and a power of $k$ ***(M1)***

$^{11}\mathrm{C}_4 \times k^4 (\times x^7)$

evidence of correct term chosen ***(A1)***

$r = 4$ (or $r = 7$)

equating their coefficient to $1320$ or their term to $1320x^7$ ***(M1)***

$^{11}\mathrm{C}_4 \times k^4 = 1320 \left(\Rightarrow k^4 = 4\right)$

1.41421...

$(k =)\ 1.41\ \left(\sqrt{2}\ \text{exact}\right)$ ***A1***

***[4 marks]***

***Total [5 marks]***

**2.** (a) attempt to find the common difference ***(M1)***

$12 = 36 + 4d$ OR $\frac{12-36}{5-1}$

$d = -6$ ***(A1)***

attempt to use the term formula for an arithmetic sequence ***(M1)***

$u_{13} = u_1 + 12(-6)$ OR $u_5 + 8(-6)$

$u_{13} = -36$ ***A1***

***[4 marks]***

(b) **METHOD 1**

attempt to set $S_n = 0$ ***(M1)***

$\frac{n}{2}(2(36) + (n-1)(-6)) = 0$ OR $\frac{n(36+u_n)}{2} = 0 \Rightarrow u_n = -36$

$n = 13$ ***A1***

**METHOD 2**

attempt to recognize symmetry ***(M1)***

$36 + 30 + ... + 0 + ... - 30 - 36$

$n = 13$ ***A1***

***[2 marks]***

***Total [6 marks]***

**3.**

**METHOD 1**

attempt to use $u_n = u_1 + (n-1)d$ ***(M1)***

$u_1 + 6d = 6$ ***(A1)***

$u_1 + 5d + u_1 + 11d = 24$ ***(A1)***

$2u_1 + 16d = 24 \quad (u_1 + 8d = 12)$

attempt to solve their equations simultaneously ***(M1)***

$u_1 = -12$ **and** $d = 3$ ***A1***

**METHOD 2**

attempt to express $u_6$ and $u_{12}$ in terms of the 7th term and $d$ ***(M1)***

$u_6 = 6 - d$ ***(A1)***

$u_{12} = 6 + 5d$ ***(A1)***

setting their sum of $u_6$ and $u_{12}$ equal to $24$ ***(M1)***

$12 + 4d = 24$

$d = 3$ **and** $u_1 = -12$ ***A1***

***[5 marks]***

**4.** rewriting $\sqrt{x}$ as $x^{\frac{1}{2}}$ seen anywhere ***(A1)***

product of a binomial coefficient, a power of $\sqrt{x}$ and a power of $k$ seen ***(M1)***

evidence of correct term chosen ***(A1)***

$r = 3$ (or $r = 9$)

equating their coefficient to $-112\,640$ or their term to $-112\,640x^{\frac{3}{2}}$ ***(M1)***

${}^{12}C_9 k^9 = -112\,640$

$k = -2$ ***A1***

***Total [5 marks]***

**7.** (a) (i) $15.75$ ***A1***

(ii) $189$ ***A1***

(iii) $5000+189n$ ***A1***

(iv) $5000+189\times 5$ ***A1***

$=5945$ ***AG***

***[4 marks]***

(b) (i) **EITHER**

attempt to use formula $FV=PV\times\left(1+\frac{r}{100k}\right)^{kn}$ with $n=5$ ***(M1)***

$FV=5000\left(1+\frac{3}{100}\right)^{5}$ OR $5000(1+0.03)^{5}$

**OR**

attempt to use GDC app (at least two correct entries seen) ***(M1)***

$PV=5000$, $I=3\%, n=5, P/Y=C/Y=1$

**OR**

$5150,\ 5304.5,\ 5463.635,\ 5627.54\ldots,\ 5796.37\ldots$ ***(M1)***

**THEN**

$5796.37...$

$5796$ ***A1***

(ii) $796\left(=796.370...\right)$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued*

(c) (i) $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ or equivalent with $k = 1$ ***(M1)***

$5000(1 + 0.03)^n$

$5000(1.03)^n$ ***A1***

(ii) setting $5000(1.03)^n \geq 5000 + 189n$ (***M1)***

**Note:** Condone equality but do not accept $5000(1.03)^n \leq 5000 + 189n$ .

attempt to solve their inequality or equation to find $n$ ***(M1)***

$16.0097\ldots$ seen OR either $8023.53\ldots < 8024$ or $8264.23\ldots > 8213$

17 (years) ***A1***

***[5 marks]***

***Total [12 marks]***

APOLLO

# Functions

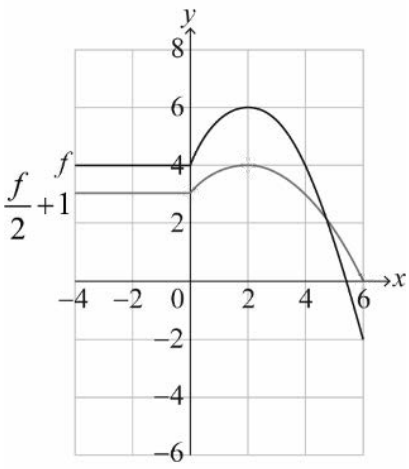
58 questions

**1.** (a) (i) $f(2)=6$ ***A1***

(ii) $(f\circ f)(2)=-2$ ***A1***

***[2 marks]***

(b)




***M1A1A1***

**Note:** Award ***M1*** for an attempt to apply any vertical stretch or vertical translation,
***A1*** for a correct horizontal line segment between $-4$ and 0
(located roughly at $y=3$),
***A1*** for a correct concave down parabola including max point at (2,4) and for correct end points at (0,3) and (6,0) (within circles). Points do not need to be labelled.

***[3 marks]***
***Total [5 marks]***

**7.** (a) **METHOD 1 (discriminant)**

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

recognizing $\Delta = 0$ (seen anywhere) ***M1***

$\Delta = (-3m)^2 - 4(m)(9)$ (do not accept only in quadratic formula for $x$) ***A1***

valid approach to solve quadratic for $m$ ***(M1)***

$9m(m-4) = 0$ OR $m = \dfrac{36 \pm \sqrt{36^2 - 4 \times 9 \times 0}}{2 \times 9}$

both solutions $m = 0, 4$ ***A1***

$m \neq 0$ with a valid reason ***R1***

the two graphs would not intersect OR $0 \neq -9$

$m = 4$ ***AG***

**METHOD 2 (equating slopes)**

$mx^2 - 2mx = mx - 9$ (seen anywhere) ***(M1)***

$f'(x) = 2mx - 2m$ ***A1***

equating slopes, $f'(x) = m$ (seen anywhere) ***M1***

$2mx - 2m = m$

$x = \dfrac{3}{2}$ ***A1***

substituting their $x$ value ***(M1)***

$\left(\dfrac{3}{2}\right)^2 m - 2m \times \dfrac{3}{2} = m \times \dfrac{3}{2} - 9$

$\dfrac{9}{4}m - \dfrac{12}{4}m = \dfrac{6}{4}m - 9$ ***A1***

$\dfrac{-9m}{4} = -9$

$m = 4$ ***AG***

*continued…*

*Question 7(a) continued*

**METHOD 3 (using $\frac{-b}{2a}$)**

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

attempt to find $x$-coord of vertex using $\frac{-b}{2a}$ ***(M1)***

$\frac{-(-3m)}{2m}$ ***A1***

$x = \frac{3}{2}$ ***A1***

substituting their $x$ value ***(M1)***

$\left(\frac{3}{2}\right)^2 m - 3m \times \frac{3}{2} + 9 = 0$

$\frac{9}{4}m - \frac{9}{2}m + 9 = 0$ ***A1***

$-9m = -36$

$m = 4$ ***AG***

***[6 marks]***

(b) $4x(x-2)$ ***(A1)***

$p = 0$ and $q = 2$ OR $p = 2$ and $q = 0$ ***A1***

***[2 marks]***

(c) attempt to use valid approach ***(M1)***

$\frac{0+2}{2}$, $\frac{-(-8)}{2\times 4}$, $f(1)$, $8x - 8 = 0$ OR $4(x^2 - 2x + 1 - 1)\left(= 4(x-1)^2 - 4\right)$

$h = 1,\ k = -4$ ***A1A1***

***[3 marks]***

(d) **EITHER**

recognition $x = h$ to 2 (may be seen on sketch) ***(M1)***

**OR**

recognition that $f(x) < 0$ and $f'(x) > 0$ ***(M1)***

**THEN**

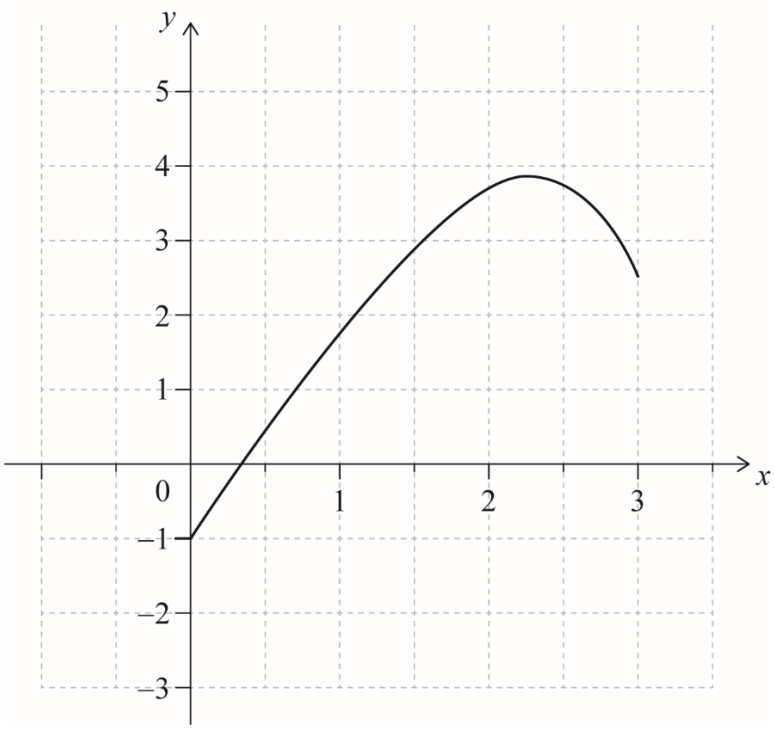
$1 < x < 2$ ***A1A1***

**Note:** Award ***A1*** for two correct values, ***A1*** for correct inequality signs.

***[3 marks]***

***Total [14 marks]***

**2.** (a)




***A1A1A1***

**Note:** Award ***A1*** for a smooth concave down curve with generally correct shape. If first mark is awarded, award ***A1*** for local maximum and *x*-intercept in approximately correct position, award ***A1*** for endpoints at $x = 0$ and $x = 3$ with approximately correct y-coordinates.

***[3 marks]***

(b) recognizing that $f'(x) = 0$ at local maximum ***(M1)***

$x = 2.33084...$

$x = 2.33$ ***A1***

***[2 marks]***

***Total [5 marks]***

**5.** (a) attempting to find the vertex ***(M1)***

$x = 1$ OR $y = -5$ OR $f(x) = 6(x-1)^2 - 5$

range is $y \geq -5$ ***A1***

***[2 marks]***

(b) **METHOD 1**

$(g \circ f)(x) = -(6x^2 - 12x + 1) + c \ \left(= -\left(6(x-1)^2 - 5\right) + c\right)$ ***(A1)***

**EITHER**

relating to the range of $f$ OR attempting to find $g(-5)$ ***(M1)***

$5 + c \leq 0$ ***(A1)***

**OR**

attempting to find the discriminant of $(g \circ f)(x)$ ***(M1)***

$144 + 24(c-1) \leq 0 \ (120 + 24c \leq 0)$ ***(A1)***

**THEN**

$c \leq -5$ ***A1***

***[4 marks]***

**METHOD 2**

vertical reflection followed by vertical shift ***(M1)***

new vertex is $(1, 5 + c)$ ***(A1)***

$5 + c \leq 0$ ***(A1)***

$c \leq -5$ ***A1***

***[4 marks]***

***Total [6 marks]***

**6.** (a) $100 = A_0 e^0$ ***A1***

$A_0 = 100$ ***AG***

***[1 mark]***

(b) correct substitution of values into exponential equation ***(M1)***

$50 = 100e^{-5730k}$ OR $e^{-5730k} = \frac{1}{2}$

**EITHER**

$-5730k = \ln\frac{1}{2}$ ***A1***

$\ln\frac{1}{2} = -\ln 2$ OR $-\ln\frac{1}{2} = \ln 2$ ***A1***

**OR**

$e^{5730k} = 2$ ***A1***

$5730k = \ln 2$ ***A1***

**THEN**

$k = \frac{\ln 2}{5730}$ ***AG***

**Note:** There are many different ways of showing that $k = \frac{\ln 2}{5730}$ which involve showing different steps. Award full marks for at least two correct algebraic steps seen.

***[3 marks]***

*continued…*

*Question 6 continued*

(c) if $25\,\%$ of the carbon-14 has decayed, $75\,\%$ remains ie, $75$ units remain ***(A1)***

$75 = 100\mathrm{e}^{-\frac{\ln 2}{5730}t}$

**EITHER**

using an appropriate graph to attempt to solve for $t$ ***(M1)***

**OR**

manipulating logs to attempt to solve for $t$ ***(M1)***

$\ln 0.75 = -\frac{\ln 2}{5730}t$

$t = 2378.164...$

**THEN**

$t = 2380$ (years) (correct to the nearest 10 years) ***A1***

***[3 marks]***

***Total [7 marks]***

## Section B

**1.** (a) (i) setting $f(x) = 0$ ***(M1)***

$x = 1$, $x = -3$ (accept $(1,0), (-3,0)$) ***A1***

(ii) **METHOD 1**

$x = -1$ ***A1***

substituting their $x$-coordinate into $f$ ***(M1)***

$y = 8$ ***A1***

$(-1,8)$

**METHOD 2**

attempt to complete the square ***(M1)***

$-2\left((x+1)^2 - 4\right)$

$x = -1$, $y = 8$ ***A1A1***

$(-1,8)$

***[5 marks]***

(b) $h = -1$ ***A1***

$k = 8$ ***A1***

***[2 marks]***

***Total [7 marks]***

**3.** (a) (i) $x = 3$ ***A1***

(ii) $y = -2$ ***A1***

***[2 marks]***

(b) (i) $(-2, 0)$ (accept $x = -2$) ***A1***

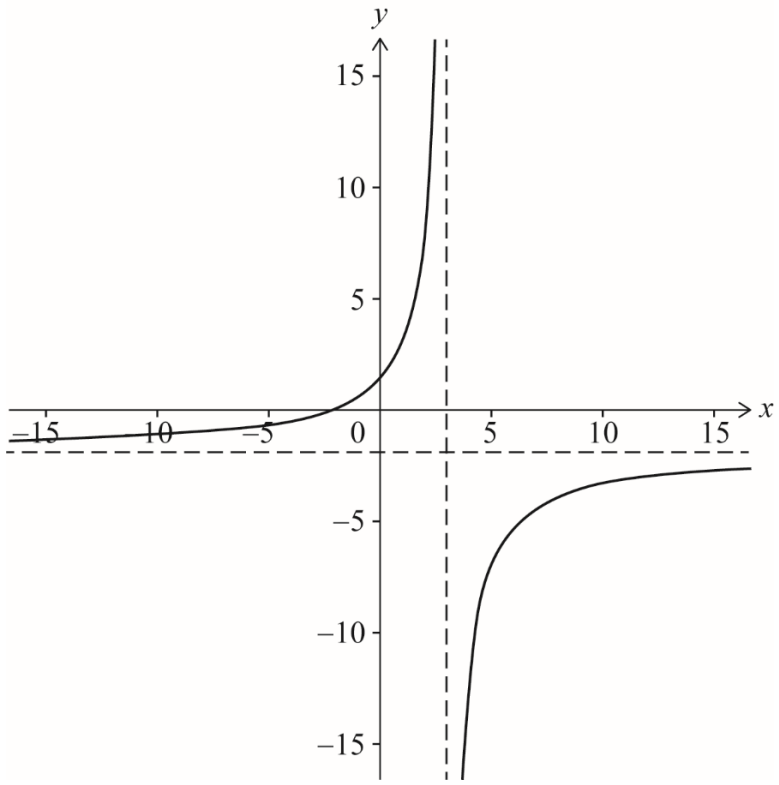
(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) ***A1***

***[2 marks]***

*continued…*

*Question 3 continued.*

(c)




***A1***

**Note:** Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

***[1 mark]***

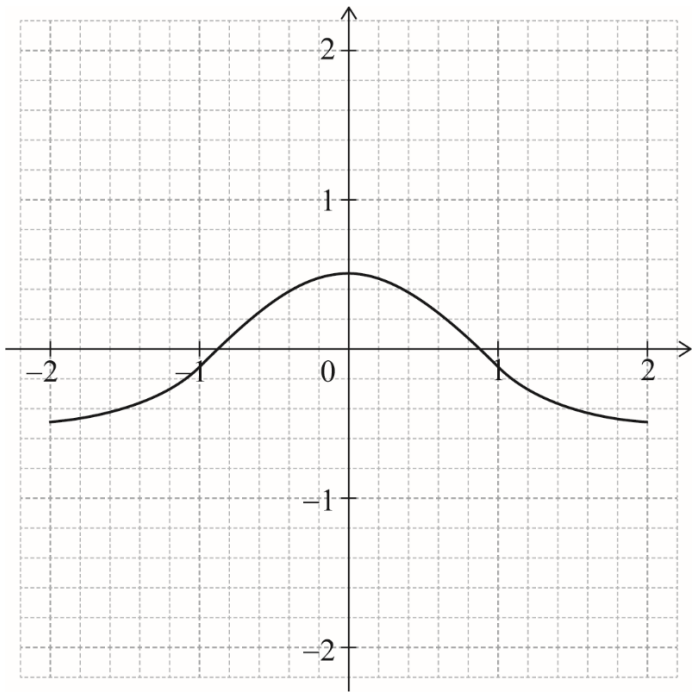
***Total [5 marks]***

**2.** (a) $x = -0.832554..., x = 0.832554...$

$x = -0.833, x = 0.833$ ***A1A1***

***[2 marks]***

(b)




***A1A1A1***

**Note**: Award ***A1*** for approximately correct shape. Only if this mark is awarded, award ***A1*** for approximately correct roots and maximum point and ***A1*** for approximately correct endpoints.
Allow $-1 < x \leq -0.8,\ 0.8 \leq x < 1$ for roots, $x = 0,\ 0.4 \leq y \leq 0.6$ for maximum and $x = \pm 2$, $-0.6 \leq y \leq -0.4$ for endpoints.

***[3 marks]***

***Total [5 marks]***

**5.** (a) attempt to form composition ***M1***

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ ***A1***

$(g\circ f)(x)=2x+11$ ***AG***

***[2 marks]***

(b) attempt to substitute 4 (seen anywhere) ***(M1)***

correct equation $a=2\times4+11$ ***(A1)***

$a=19$ ***A1***

***[3 marks]***

***Total [5 marks]***

**1.** (a) $m_{\text{BC}} = \frac{12-6}{-14-4} \left(= -\frac{1}{3}\right)$ ***(A1)***

finding $m_L = \frac{-1}{m_{\text{BC}}}$ using their $m_{\text{BC}}$ ***(M1)***

$m_L = 3$

$y - 20 = 3(x+2)$, $y = 3x + 26$ ***A1***

**Note:** Do not accept $L = 3x + 26$.

***[3 marks]***

(b) substituting $(k, 2)$ into their *L* ***(M1)***

$2 - 20 = 3(k+2)$ OR $2 = 3k + 26$

$k = -8$ ***A1***

***[2 marks]***

***Total [5 marks]***

**6.** (a) translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***

translation (shift) by $q$ upwards/positive vertical direction ***A1***

**Note**: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

**Do not accept** 'move' for translation/shift.

***[2 marks]***

(b) **METHOD 1**

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is $-4$ (may be seen in sketch) ***(M1)***

$-4+2.5+q\geq 7$

$q\geq 8.5$ (accept $q=8.5$) ***A1***

substituting $x=0$ and their $q$ $(=8.5)$ to find $r$ ***(M1)***

$(r=)\quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ ***(A1)***

smallest value of $r$ is $15$ ***A1***

*continued…*

*Question 6 continued*

**METHOD 2**

substituting $x = 0$ to find an expression (for $r$) in terms of $q$ ***(M1)***

$\left(g(0) = r =\right) \quad 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + q$

$\left(r =\right) \quad 6.5 + q$ ***A1***

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is $-4$ ***(M1)***

$-4 + 2.5 + q \geq 7$

$-4 + 2.5 + (r - 6.5) \geq 7$ (accept =) ***(A1)***

smallest value of $r$ is $15$ ***A1***

**METHOD 3**

$4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q = 4\cos x + 2.5 + q$ ***A1***

$y$-intercept of $4\cos x + 2.5 + q$ is a maximum ***(M1)***

amplitude of $g(x)$ is $4$ ***(A1)***

attempt to find least maximum ***(M1)***

$r = 2 \times 4 + 7$

smallest value of $r$ is $15$ ***A1***

***[5 marks]***

***Total [7 marks]***

## Section B

**1.** (a) $g(0) = -2$ ***A1***

***[1 mark]***

(b) evidence of using composite function ***(M1)***

$f(g(0))$ OR $f(-2)$

$(f \circ g)(0) = 8$ ***A1***

***[2 marks]***

(c) $x = 3$ ***A2***

***[2 marks]***

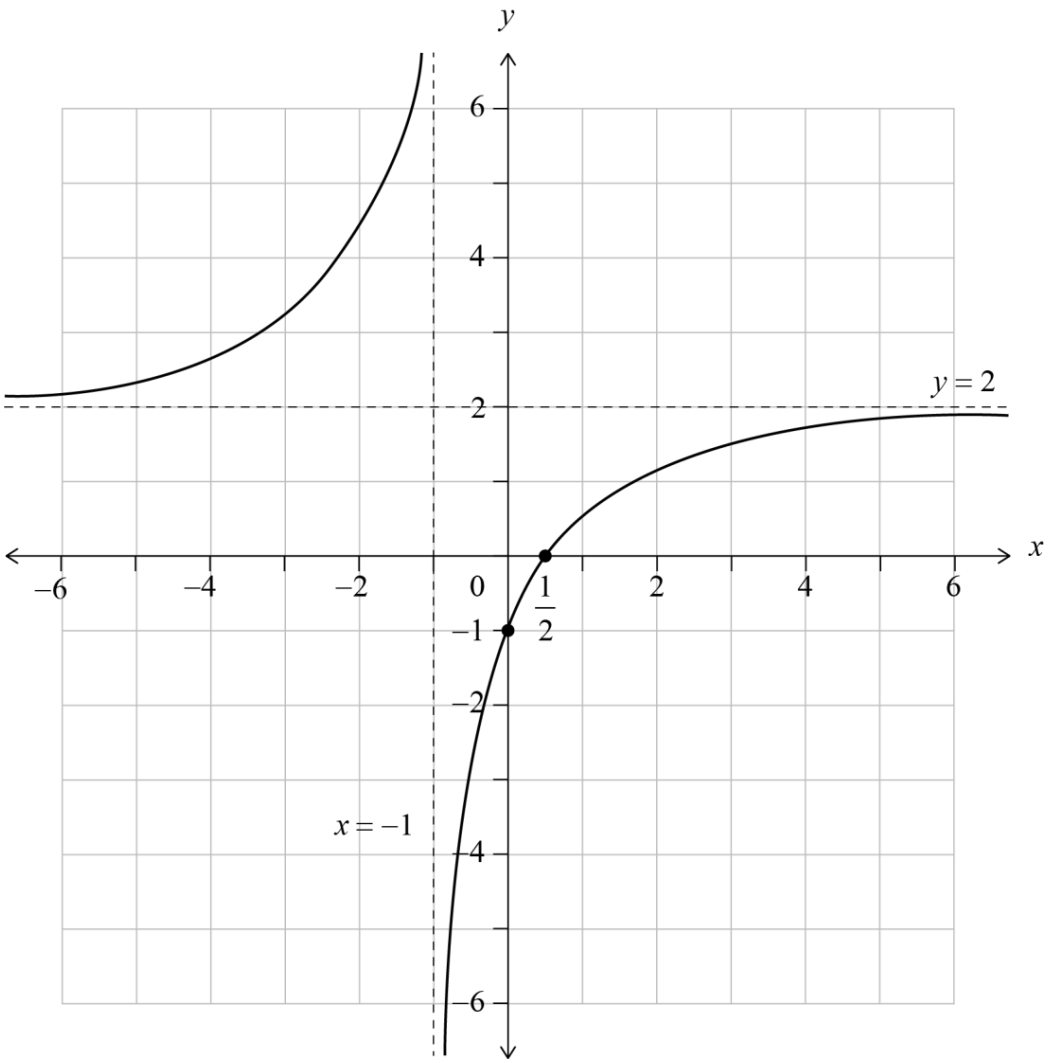
***Total [5 marks]***

**4.** (a) (i) $x = -1$ ***A1***

(ii) $y = 2$ ***A1***

***[2 marks]***

(b)




*continued…*

*Question 4 continued*

rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

**Note:** The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x=-1$ and $y=2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x=\frac{1}{2}$ and $y=-1$ ***A1A1***

***[3 marks]***

(c) $x>\frac{1}{2}$ ***A1***

**Note:** Accept correct alternative correct notation, such as $\left(\frac{1}{2},\infty\right)$ and $\left]\frac{1}{2},\infty\right[$.

***[1 mark]***

***Total [6 marks]***

**8.** (a) (i) $x = -4$ ***A1***

(ii) attempt to substitute into $y = \frac{a}{c}$ OR table with large values of $x$ OR sketch of $f$ showing asymptotic behaviour ***(M1)***

$y = 4$ ***A1***

***[3 marks]***

(b) (i) $y = \frac{4x+1}{x+4}$

attempt to interchange $x$ and $y$ (seen anywhere) ***M1***

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$ ***(A1)***

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$ ***(A1)***

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$) ***A1***

*continued…*

*Question 8 continued*

(ii) reflection in $y$-axis given by $f(-x)$ ***(M1)***

$f(-x) = \dfrac{-4x+1}{-x+4}$ ***(A1)***

reflection of their $f(-x)$ in $x$-axis given by $-f(-x)$ accept "now $-f(x)$" ***M1***

$\left(-f(-x) = \right) -\dfrac{-4x+1}{-x+4}$

$= \dfrac{-4x+1}{x-4}$ OR $\dfrac{4x-1}{-x+4}$ ***A1***

$= \dfrac{1-4x}{x-4}$ $\left(= f^{-1}(x)\right)$ ***AG***

**Note:** If the candidate attempts to show the result using a particular coordinate on the graph of $f$ rather than a general coordinate on the graph of $f$, where appropriate, award marks as follows:
***M0A0* for eg** $(2,3) \to (-2,3)$
***M0A0* for** $(-2,3) \to (-2,-3)$

***[8 marks]***

*continued…*

*Question 8 continued*

(c) (i) attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically ***(M1)***

$p = -1$ AND $q = 1$ ***A1***

**Note:** Award ***(M1)A0*** if only one correct value seen.

(ii) attempt to set up an integral to find area between $f$ and $f^{-1}$ ***(M1)***

$\int_{-1}^{1}\left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4}\right)\mathrm{d}x$ ***(A1)***

$= 0.675231...$

$= 0.675$ ***A1***

***[5 marks]***

***Total [16 marks]***

**1.** (a) gradient of $g$ is $-2$ (may be seen in function, do not accept $-2x+3$) ***(A1)***

$g(x)=-2x$ ***A1***

***[2 marks]***

(b) gradient is $\frac{1}{2}$ (may be seen in function) ***(A1)***

attempt to substitute **their** gradient and $(-1,2)$ into any form of equation for straight line ***(M1)***

$y-2=\frac{1}{2}(x+1)$ OR $2=\frac{1}{2}\cdot(-1)+c$

$h(x)=\frac{1}{2}(x+1)+2 \left(=\frac{1}{2}x+\frac{5}{2}\right)$ ***A1***

***[3 marks]***

(c) $(g\circ h)(x)=-2\left(\frac{1}{2}x+\frac{5}{2}\right)$ OR $h(0)=\frac{5}{2}$ OR $g\left(\frac{5}{2}\right)$ ***(A1)***

$(g\circ h)(0)=-5$ ***A1***

***[2 marks]***

***Total [7 marks]***

**7.** (a) correct substitution of $h = 3$ and $k = 2$ into $f(x)$ ***(A1)***

$f(x) = a(x-3)^2 + 2$

correct substitution of $(5, 0)$ ***(A1)***

$0 = a(5-3)^2 + 2 \quad \left(a = -\frac{1}{2}\right)$

**Note**: The first two A marks are independent.

$f(x) = -\frac{1}{2}(x-3)^2 + 2$ ***A1***

***[3 marks]***

(b) (i) **METHOD 1**

correct substitution of $(1, 4)$ ***(A1)***

$p + (t-1) - p = 4$

$t = 5$ ***A1***

substituting their value of $t$ into $9p - 3(t-1) - p = 4$ ***(M1)***

$8p - 12 = 4$

$p = 2$ ***A1***

**METHOD 2**

correct substitution of ONE of the coordinates $(-3,4)$ or $(1,4)$ ***(A1)***

$9p-3(t-1)-p=4$ OR $p+(t-1)-p=4$

valid attempt to solve their two equations ***(M1)***

$p=2,\ t=5$ ***A1A1***

$\left(g(x)=2x^2+4x-2\right)$

(ii) attempt to find the $x$-coordinate of the vertex ***(M1)***

$x=\frac{-3+1}{2}(=-1)$ OR $\frac{-4}{2\times 2}$ OR $4x+4=0$ OR $2(x+1)^2-4$

$y$-coordinate of the vertex $=-4$ ***(A1)***

correct range ***A1***

$[-4,+\infty[$ OR $y\geq -4$ OR $g\geq -4$ OR $[-4,\infty)$

***[7 marks]***

(c) equating the two functions or equations ***(M1)***

$g(x) = j(x)$ OR $px^2 + (t-1)x - p = -x + 3p$

$px^2 + tx - 4p = 0$ ***(A1)***

attempt to find discriminant (do not accept only in quadratic formula) ***(M1)***

$\Delta = t^2 + 16p^2$ ***A1***

$\Delta = t^2 + 16p^2 > 0$, because $t^2 \geq 0$ and $p^2 > 0$, therefore the sum will be positive ***R1R1***

**Note**: Award ***R1*** for recognising that $\Delta$ is positive and ***R1*** for the reason.

There are two distinct points of intersection between the graphs of $g$ and $j$. ***AG***

***[6 marks]***

***Total [16 marks]***

**8**. (a) (i) valid approach to find the required logarithm ***(M1)***

$2^x = \frac{1}{16}$ OR $2^x = 2^{-4}$ OR $\frac{1}{16} = 2^{-4}$ OR $\log_2 1 - \log_2 16$

$\log_2 \frac{1}{16} = -4$ ***A1***

(ii) valid approach to find the required logarithm ***(M1)***

$9^x = 3$ OR $3^{2x} = 3$ OR $3 = 9^{\frac{1}{2}}$ OR $\frac{\log_3 3}{\log_3 9}$

$\log_9 3 = \frac{1}{2}$ ***A1***

(iii) $\left(\sqrt{3}\right)^x = 81$ OR $\frac{\log_3 81}{\log_3 \sqrt{3}}$ ***(A1)***

$(3)^{\frac{x}{2}} = 3^4$ OR $\frac{x}{2} = 4$ OR $\frac{4}{\frac{1}{2}}$ ***(A1)***

$x = 8$ ***A1***

***[7 marks]***

*continued…*

*Question 8 continued*

(b) (i)

**Note:** There are many valid approaches to the question, and the steps may be seen in different ways. Some possible methods are given here, but candidates may use a combination of one or more of these methods.

In all methods, the final ***A*** mark is awarded for working which leads directly to the ***AG***.

**METHOD 1**

$(ab)^3 = a$ ***(A1)***

attempt to isolate $b$ or a power of $b$ ***(M1)***

correct working ***(A1)***

$b = \frac{a}{a^3b^2}$ OR $b^3 = a^{-2}$ OR $b^{-1} = (ab)^2$ OR $b^3 = \frac{1}{a^2}$

$b = \frac{1}{a^2b^2}$ OR $b = (ab)^{-2}$ OR $3\log_{ab} b = -2\log_{ab} a$ OR $-\log_{ab} b = 2\log_{ab} ab$ ***A1***

$\log_{ab} b = -2$ ***AG***

**METHOD 2**

$(ab)^3 = a$ ***(A1)***

taking logarithm to base $ab$ on both sides ***(M1)***

$\log_{ab}(ab)^3 = \log_{ab} a$ OR $\log_{ab} a^3b^3 = \log_{ab} a$

correct application of log rules leading to equation in terms of $\log_{ab}$ ***(A1)***

$3\log_{ab} a + 3\log_{ab} b = \log_{ab} a$ OR $3\log_{ab} b = -2\log_{ab} a$ OR $\log_{ab} b^3 = \log_{ab} a^{-2}$

$\log_{ab} b = \log_{ab} a^{-\frac{2}{3}}$ OR $\log_{ab} b = -\frac{2}{3}\log_{ab} a$ OR $\log_{ab} b = -\frac{2}{3}(3)$ ***A1***

$\log_{ab} b = -2$ ***AG***

**Note:** Candidates may substitute $\log_{ab} a = 3$ at any point in their working.

*continued…*

*Question 8 continued*

**METHOD 3**

$\log_{ab} a = 3$

writing in terms of base $a$ ***(M1)***

$\frac{\log_a a}{\log_a ab} (= 3)$

correct application of log rules ***(A1)***

$\frac{\log_a a}{\log_a a + \log_a b} (= 3)$ OR $\frac{1}{1 + \log_a b} (= 3)$ OR $3\log_a b = -2$ OR

$\log_a b = -\frac{2}{3}$

writing $\log_{ab} b$ in terms of base $a$ ***(A1)***

$\log_{ab} b = \frac{\log_a b}{\log_a a + \log_a b}$

correct working ***A1***

$\log_{ab} b = \frac{-\frac{2}{3}}{1 - \frac{2}{3}}$ OR $\frac{\left(-\frac{2}{3}\right)}{\left(\frac{1}{3}\right)}$

$\log_{ab} b = -2$ ***AG***

*continued…*

*Question 8 continued*

**METHOD 4**

$\log_{ab} ab = 1$ ***A2***

$\log_{ab} a + \log_{ab} b = 1$ ***(A1)***

$3 + \log_{ab} b = 1$ ***A1***

$\log_{ab} b = -2$ ***AG***

(ii) applying the quotient rule or product rule for logs

$\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} - \log_{ab} \sqrt{b}$ OR $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} + \log_{ab} \frac{1}{\sqrt{b}}$ ***(A1)***

correct working ***(A1)***

$= \frac{1}{3}\log_{ab} a - \frac{1}{2}\log_{ab} b$ OR $\log_{ab} ab - \log_{ab} \sqrt{b}$

$= \frac{1}{3} \cdot 3 - \frac{1}{2}(-2)$ ***(A1)***

$= 2$ ***A1***

**Note:** Award ***A1A0A0A1*** for a correct answer with no working.

***[8 marks]***

***Total [15 marks]***

**5.** recognition that initial population is $15000$ (seen anywhere) ***(A1)***

$P(0) = 15000$ OR $0.11 \times 15000$ OR $0.89 \times 15000$

population after 11% decrease is $15000 \times 0.89 (= 13350)$ ***(A1)***

recognizing that $t = 8$ on 1 January 2022 (seen anywhere) ***(A1)***

substitution of their value of $t$ for 1 January 2022 and their value of $P(8)$ into the model ***(M1)***

$15000 \times 0.89 = 15000\mathrm{e}^{8k}$ OR $13350 = 15000\mathrm{e}^{8k}$

$k = \frac{\ln 0.89}{8}$ $(-0.014566)$ ***(A1)***

substitution of $t = 2041 - 2014 (= 27)$ and their value for $k$ into the model ***(M1)***

$P(27) = 15000\, e^{-0.0145... \times 27}$

$10122.3\ldots$

$P(27) = 10100$ $(10122)$ ***A1***

***Total [7 marks]***

**1.** (a) $\mathrm{M}\left(6,-3\right)$ ***A1A1***

***[2 marks]***

(b) gradient of [PQ] $=-\frac{5}{9}$ ***(A1)***

gradient of $L=\frac{9}{5}$ ***A1***

***[2 marks]***

(c) $y+3=\frac{9}{5}(x-6)$ OR $y=\frac{9}{5}x-\frac{69}{5}$ (or equivalent) ***A1***

**Note**: Do not accept $L=\frac{9}{5}x-\frac{69}{5}$.

***[1 mark]***

***Total [5 marks]***

**2.** (a) recognizing $f(x)=0$ ***(M1)***

$x=-1$ ***A1***

***[2 marks]***

(b) (i) $x=2$ (must be an equation with $x$) ***A1***

(ii) $y=\frac{7}{2}$ (must be an equation with $y$) ***A1***

***[2 marks]***

(c) **EITHER**

interchanging $x$ and $y$ ***(M1)***

$2xy-4x=7y+7$

correct working with $y$ terms on the same side: $2xy-7y=4x+7$ ***(A1)***

**OR**

$2yx-4y=7x+7$

correct working with $x$ terms on the same side: $2yx-7x=4y+7$ ***(A1)***

interchanging $x$ and $y$ OR making $x$ the subject $x=\frac{4y+7}{2y-7}$ ***(M1)***

**THEN**

$f^{-1}(x)=\frac{4x+7}{2x-7}$ (or equivalent) $\left(x\neq\frac{7}{2}\right)$ ***A1***

***[3 marks]***

***Total [7 marks]***

**5.** recognition of quadratic in $e^x$ ***(M1)***

$\left(e^x\right)^2 - 3e^x + \ln k\,(=0)$ OR $A^2 - 3A + \ln k\,(=0)$

recognizing discriminant ≥ 0 (seen anywhere) (***M1)***

$(-3)^2 - 4(1)(\ln k)$ OR $9 - 4\ln k$ ***(A1)***

$\ln k \le \frac{9}{4}$ ***(A1)***

$e^{9/4}$ (seen anywhere) ***A1***

$0 < k \le e^{9/4}$ ***A1***

***[6 marks]***

**3.** (a) (i) $x = 2$ ***A1***

(ii) $y = 1$ ***A1***

***[2 marks]***

(b) (i) $\left(0, \frac{3}{2}\right)$ ***A1***

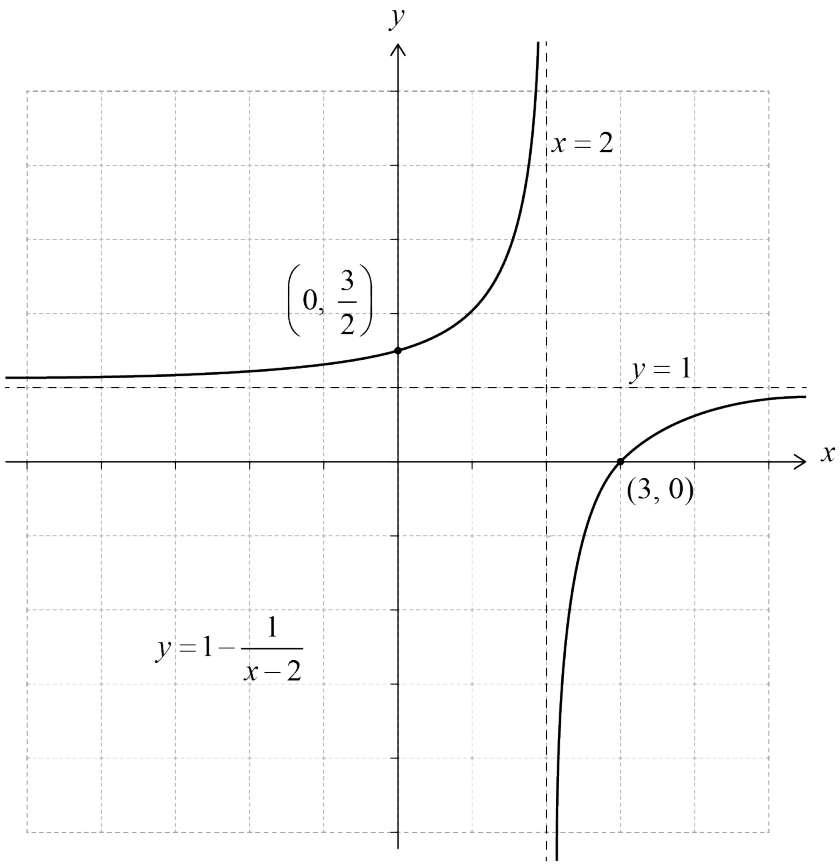
(ii) $(3, 0)$ ***A1***

***[2 marks]***

*continued…*

*Question 3 continued*

(c)




two correct branches with correct asymptotic behaviour and intercepts clearly shown ***A1***

***[1 mark]***

***Total [5 marks]***

**6.** attempts to form $(g \circ f)(x)$ ***(M1)***

$[f(x)]^2 + f(x) + 3$ OR $(ax+b)^2 + ax + b + 3$

$a^2x^2 + 2abx + b^2 + ax + b + 3 \left(= 4x^2 - 14x + 15\right)$ ***(A1)***

equates their corresponding terms to form at least one equation ***(M1)***

$a^2x^2 = 4x^2$ OR $a^2 = 4$ OR $2abx + ax = -14x$ OR $2ab + a = -14$ OR $b^2 + b + 3 = 15$

$a = \pm 2$ (seen anywhere) ***A1***

attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) ***(M1)***

$f(x) = 2x - 4$ (accept $a = 2$ with $b = -4$), $f(x) = -2x + 3$ (accept $a = -2$ with $b = 3$) ***A1A1***

***[7 marks]***

## Section B

**7.** (a) $x = -2$ (must be an equation) ***A1***

***[1 mark]***

(b) $h = -2$, $k = -5$ ***A1A1***

***[2 marks]***

(c) substituting $x = 0$ into $f(x)$ ***(M1)***

$y = \frac{1}{4}(0+2)^2 - 5$

$y = -4$ (accept $\mathrm{P}(0, -4)$) ***A1***

***[2 marks]***

(d) $f'(x) = \frac{1}{2}(x+2)\left(= \frac{1}{2}x + 1\right)$ ***(A1)***

substituting $x = 0$ into their derivative ***(M1)***

$f'(0) = 1$

gradient of normal is $-1$ (may be seen in their equation) ***A1***

$y = -x - 4$ (accept $a = -1$, $b = -4$) ***A1***

**Note:** Award ***A0*** for $L = -x - 4$ (without the $y =$).

***[4 marks]***

*continued…*

*Question 7 continued*

(e) equating their $f(x)$ to their $L$ ***(M1)***

$\frac{1}{4}(x+2)^2 - 5 = -x - 4$

$\frac{1}{4}x^2 + 2x = 0$ (or equivalent) ***(A1)***

valid attempt to solve their quadratic ***(M1)***

$\frac{1}{4}x(x+8) = 0$ OR $x(x+8) = 0$

$x = -8$ ***A1***

**Note:** Accept both solutions $x = -8$ and $x = 0$ here, $x = -8$ may be seen in working to find coordinates of $\mathrm{Q}$ or distance.

substituting their value of $x$ (not $x = 0$) into their $f(x)$ or their $L$ ***(M1)***

$y = -(-8) - 4$ or $y = \frac{1}{4}(-8+2)^2 - 5$

$\mathrm{Q}(-8, 4)$ ***A1***

correct substitution into distance formula ***(A1)***

$\sqrt{(-8-0)^2 + (4-(-4))^2}$

$\text{distance} = \sqrt{128}\ (= 8\sqrt{2})$ ***A1***

***[8 marks]***

***Total [17 marks]***

**3.** (a) attempt to substitute $g$ into $f$ ***(M1)***

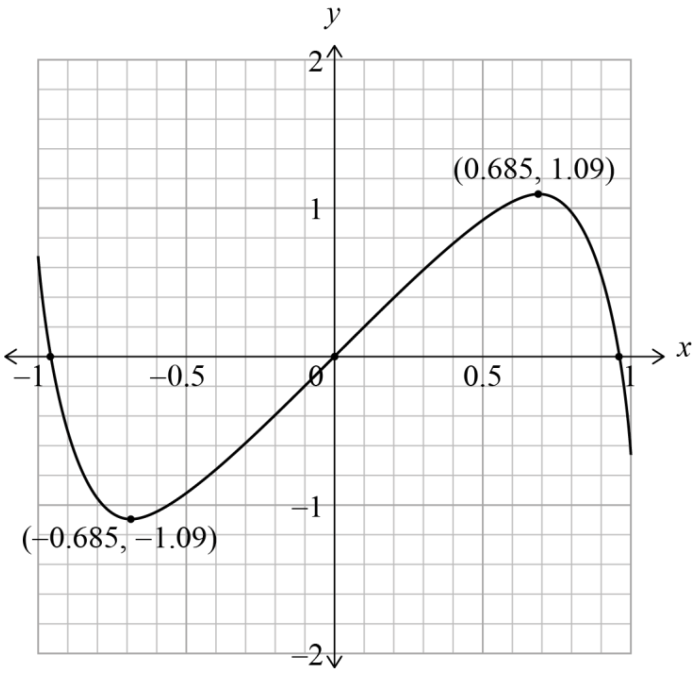
$(f \circ g)(x) = 2\tan x - \tan^3 x$ ***A1***

***[2 marks]***

*continued…*

*Question 3 continued*

(b)




***A1A1A1***

**Note**: ***A1*** for approximately correct odd function passing through the origin with a maximum above $y = 1$ and a minimum below $y = -1$.

***A1*** for endpoints at $x = \pm 1$ and $y$ in the intervals $[0.6, 0.8]$ and $[-0.8, -0.6]$

***A1*** for maximum in approximately correct position and labelled $(0.685, 1.09)$ AND minimum in approximately correct position and labelled $(-0.685, -1.09)$. For approximate position, allow $-0.8 \le x \le -0.6$, $-1.2 \le y \le -1$ for minimum and $0.6 \le x \le 0.8$, $1 \le y \le 1.2$ for maximum. If the candidate gives the coordinates of extrema below their sketch, only award this mark if extrema are marked in the correct interval (eg by a dot).

***[3 marks]***

***Total [5 marks]***

**3.** (a) $A(0) = 500\,(\text{mg})$ ***A1***

***[1 mark]***

(b) $280 = 500\text{e}^{-3k}$ ***(A1)***

$k = 0.193272...$

$k = 0.193 \left(= -\frac{1}{3}\ln\left(\frac{280}{500}\right)\right)$ ***A1***

***[2 marks]***

(c) $500\text{e}^{-0.193272...T} = 140$ ***(A1)***

$T = 6.58636...$

$T = 6.59$ (h) ***A1***

***[2 marks]***

***Total [5 marks]***

**7.** (a) (i) swapping $x$ and $y$, or $h\left(h^{-1}(x)\right)=x$ ***(M1)***

$h^{-1}(x)=\frac{x^2+2}{4}$ ***A1***

recognizing range of $h$ is domain of $h^{-1}$ ***(M1)***

Domain: $x \geq 0$ ***A1***

(ii) range of $h^{-1}$ is $y \geq \frac{1}{2}$ ***A1***

***[5 marks]***

(b) $\sqrt{4x-2}=\frac{x^2+2}{4}$ OR $\sqrt{4x-2}=x$ OR $\frac{x^2+2}{4}=x$ ***(M1)***

$x=0.585786...,\ x=3.414213...\left(=2+\sqrt{2}\right)$

$x=0.586,\ x=3.41$ ***A1A1***

***[3 marks]***

(c) attempt to form integral of the difference between $h(x)$ and their $h^{-1}$, using their limits from part (b) ***(M1)***

$\int_{0.585786...}^{3.414213...}\left(h(x)-h^{-1}(x)\right)\mathrm{d}x$ OR $\int_{0.585786...}^{3.414213...}\left(\sqrt{4x-2}-\frac{x^2+2}{4}\right)\mathrm{d}x$ OR

$6.5996632...-4.7140452...$

$1.88561...$

$\text{area}=1.89$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued*

(d) attempt to use chain rule or power rule ***(M1)***

$h'(x) = 4 \cdot \frac{1}{2}(4x-2)^{-\frac{1}{2}}$

$h'(x) = \frac{2}{\sqrt{4x-2}}$ ***A1***

***[2 marks]***

(e) **EITHER**

$\left(h^{-1}\right)'(x) = \frac{x}{2}$ ***(A1)***

equating their $h'(x)$ to the derivative of their $h^{-1}(x)$ and attempting to solve for $x$ ***(M1)***

$\frac{2}{\sqrt{4x-2}} = \frac{x}{2}$

**OR**

finding intersection of graphs of their derivatives ***(M2)***

**THEN**

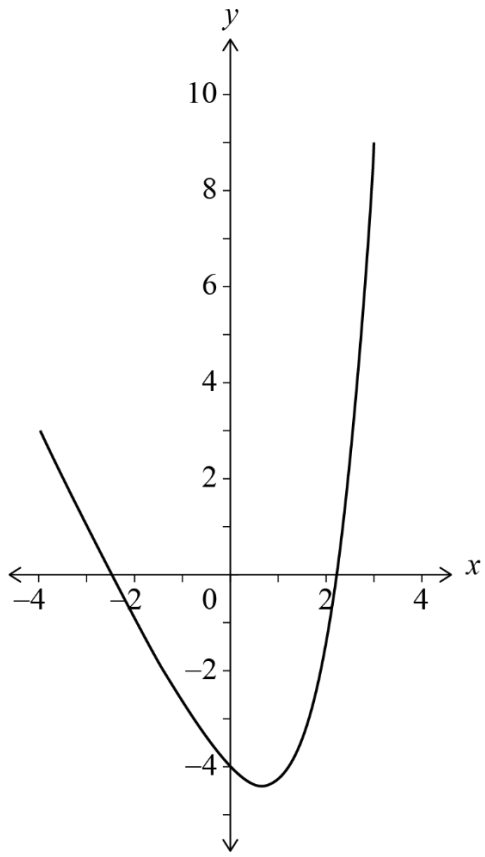
$1.772776...$

$x = 1.77$ ***A1***

***[3 marks]***

***Total [15 marks]***

**3.** (a)




***A1A1A1***

**Note:** Award marks as follows:

***A1*** for approximately correct roots, in the intervals $-3 < x < -2$ and $2 < x < 3$.

***A1*** for y-intercept AND local minimum in approximately correct positions. Allow for $y$-intercept $-4.5 < y < -3.5$, and for local minimum $0.2 < x < 1.2$, $-5 < y < -4$.

***A1*** for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5$, $2.5 < y < 3.5$ and the right end in the intervals $2.5 < x < 3.5$, $8.5 < y < 9.5$.

***[3 marks]***

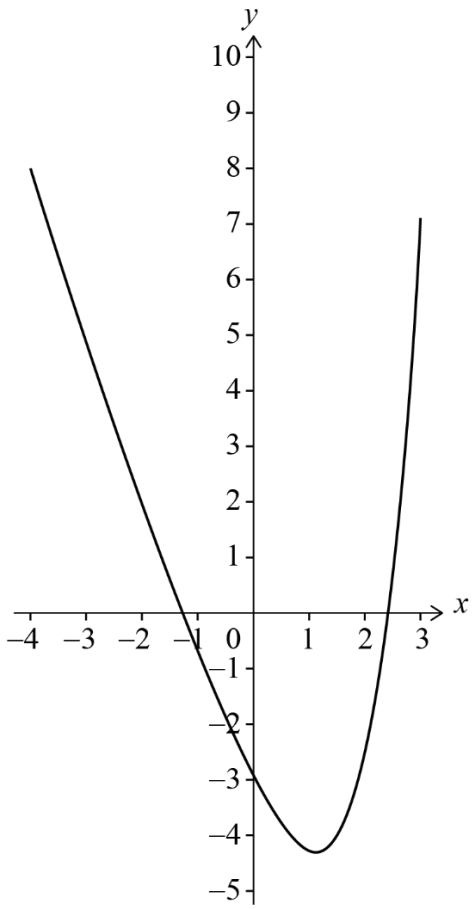
(b) $k = \frac{1}{3}$ ***A1***

$c = -2$ (accept translate/shift 2 (units) down) ***A1***

***[2 marks]***

***Total [5 marks]***

**3.** (a)




***A1A1A1***

**Note:** Award marks as follows:

***A1*** for approximately correct roots, in the intervals $-2 < x < -1$ and $2 < x < 3$.

***A1*** for y-intercept AND local minimum in approximately correct positions. Allow for $y$-intercept $-3.5 < y < -2.5$, and for local minimum $0.5 < x < 1.5$, $-5 < y < -4$.

***A1*** for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5,\ 7.5 < y < 8.5$ and the right end in the intervals $2.5 < x < 3.5,\ 6.5 < y < 7.5$.

***[3 marks]***

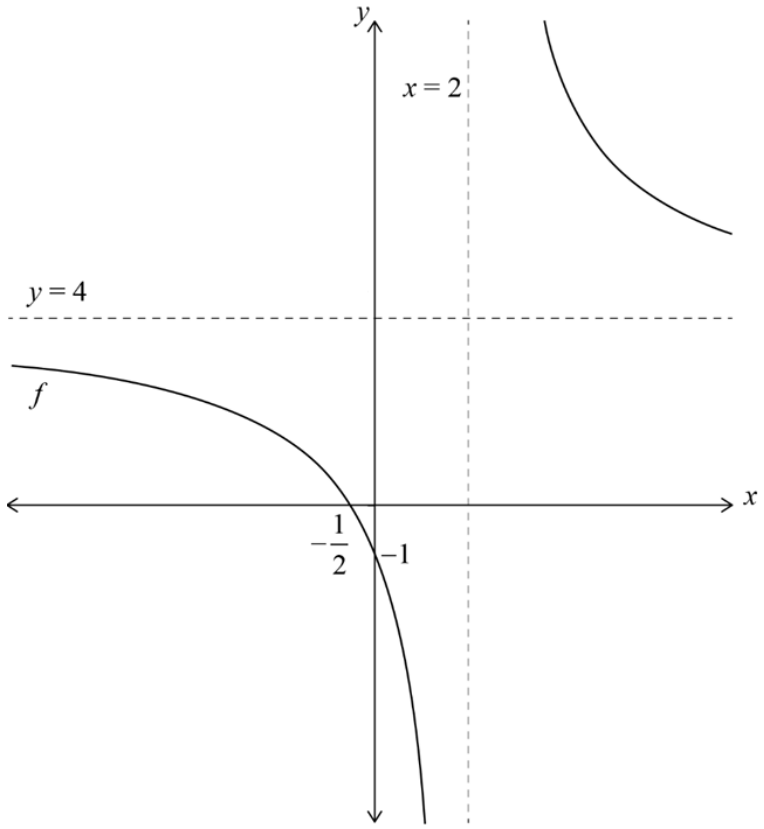
(b) $k = \frac{1}{2}$ ***A1***

$c = -3$ (accept translate/shift 3 (units) down) ***A1***

***[2 marks]***

***Total [5 marks]***

**8.** (a)




vertical asymptote $x = 2$ sketched and labelled with correct equation ***A1***

horizontal asymptote $y = 4$ sketched and labelled with correct equation ***A1***

For an approximate rational function shape:

labelled intercepts $-\frac{1}{2}$ on $x$-axis, $-1$ on $y$-axis ***A1A1***

two branches in correct opposite quadrants with correct asymptotic behaviour ***A1***

**Note:** These marks may be awarded independently.

***[5 marks]***

(b) $y \neq 4$ (or equivalent) ***A1***

***[1 mark]***

*continued…*

*Question 8 continued*

(c) $2+\frac{5}{2}$ OR $\left(-\frac{1}{2}\right)+2\times\frac{5}{2}$ OR $\frac{-\frac{1}{2}+p}{2}=2$ OR $-4=-p+\frac{1}{2}$ ***A1***

$p=\frac{9}{2}$ ***AG***

***[1 mark]***

(d) **METHOD 1**

attempt to substitute both roots to form a quadratic ***(M1)***

**EITHER**

$\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ OR $x^2-\left(-\frac{1}{2}+\frac{9}{2}\right)x+\left(-\frac{1}{2}\times\frac{9}{2}\right)$

$=x^2-4x-\frac{9}{4}$ ***A1A1***

$\left(b=-4, c=-\frac{9}{4}\right)$

**Note:** Award ***A1*** for each correct value. They may be embedded or stated explicitly.

**OR**

$(2x+1)(2x-9)=4\left(x^2-4x-\frac{9}{4}\right)$

$b=-4, c=-\frac{9}{4}$ ***A1A1***

**Note:** Award ***A1*** for each correct value. They must be stated explicitly.

**METHOD 2**

$-\frac{b}{2}=2$ OR $4+b=0\Rightarrow b=-4$ ***A1***

attempt to form a valid equation to find $c$ using their $b$ ***(M1)***

$\left(-\frac{1}{2}\right)^2+-4\left(-\frac{1}{2}\right)+c=0$ OR $\left(\frac{9}{2}\right)^2+-4\left(\frac{9}{2}\right)+c=0$

$c=-\frac{9}{4}$ ***A1***

*continued…*

*Question 8 continued*

**METHOD 3**

attempt to form two valid equations in $b$ and $c$ ***(M1)***

$\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c = 0$ , $\left(\frac{9}{2}\right)^2 + b\left(\frac{9}{2}\right) + c = 0$

$b = -4, c = -\frac{9}{4}$ ***A1A1***

**METHOD 4**

attempt to write $g(x)$ in the form $(x-h)^2 + k$ and substitute for $x, h$ and $g(x)$ ***(M1)***

$\left(-\frac{1}{2} - 2\right)^2 + k = 0 \Rightarrow k = -\frac{25}{4}$

$(x-2)^2 - \frac{25}{4}$

$= x^2 - 4x - \frac{9}{4}$ ***A1A1***

$\left(b = -4, c = -\frac{9}{4}\right)$

**Note:** Award ***A1*** for each correct value. They may be embedded or stated explicitly.

***[3 marks]***

(e) attempt to substitute $x = 2$ into their $g(x)$ OR

complete the square on their $g(x)$ (may be seen in part (d)) ***(M1)***

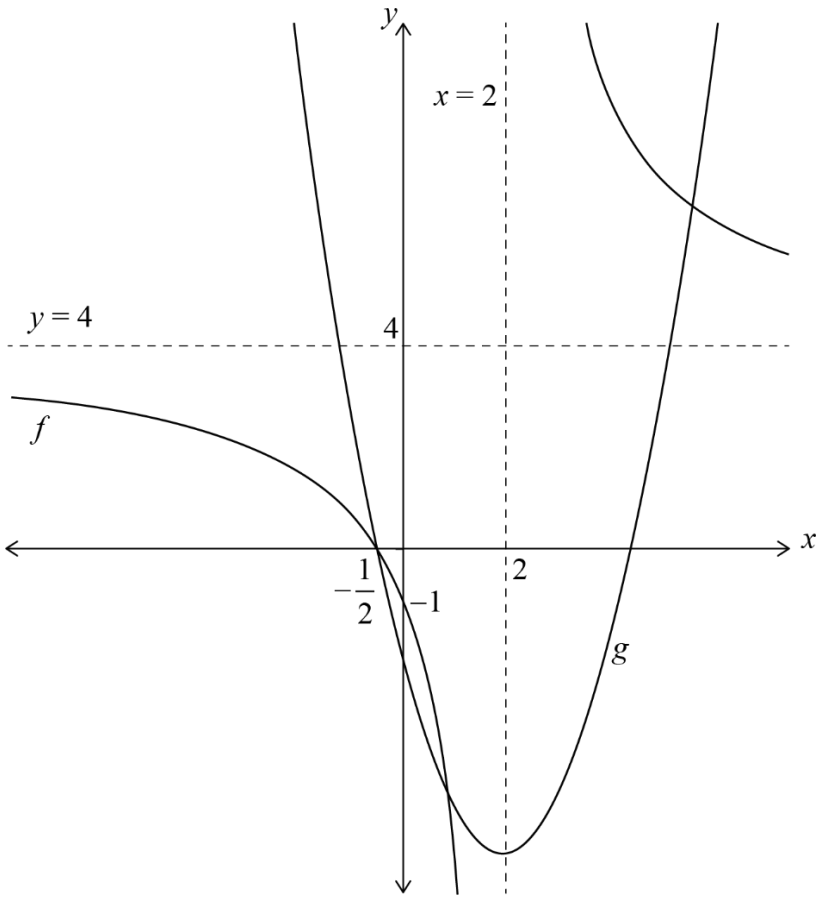
$y = -\frac{25}{4}$ ***A1***

***[2 marks]***

*continued…*

*Question 8 continued*

(f)




both graphs sketched on same axes and identifying points of intersection ***(M1)***

3 solutions ***A1***

**Note**: Exception to ***FT***: If the candidate's graph in part (a) is incorrect, the ***M1*** may be awarded for a sketch of their graph from part (a) and $g(x)$. Do not award the final ***A1*** in this case.

***[2 marks]***

***Total [14 marks]***

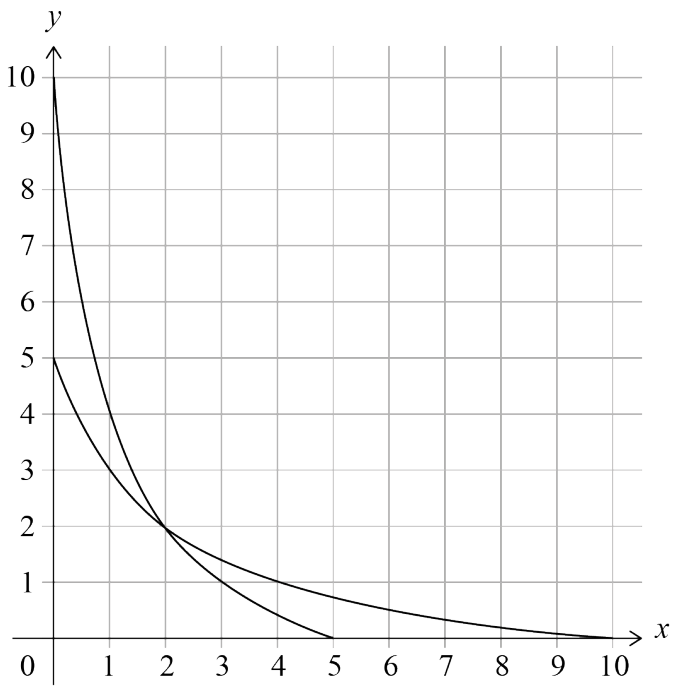
**1.** (a) (i) $f(4)=1$ ***A1***

(ii) $f \circ f(4)=3$ ***A1***

(iii) $f^{-1}(3)=1$ ***A1***

***[3 marks]***

(b)




concave up curve with $y$ intercept at $(0,10)$ and $x$ intercept at $(5,0)$ ***A1***

curve passes through $(2,2)$ OR through (1, 4) and (3, 1) ***A1***

| |
|---|
| ***Note:*** Do not award the second mark unless the first mark has been awarded.(Do not award ***A0A1***). |

***[2 marks]***

***Total [5 marks]***



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**3.** (a) valid attempt to solve a quadratic equation (factorising, use of formula, completing square, or otherwise) ***(M1)***

$(3m-1)(m+2)=0$ OR $m=\dfrac{-5\pm\sqrt{25+24}}{6}$ (or equivalent) ***(A1)***

$m=\dfrac{1}{3}$ , $m=-2$ ***A1***

***[3 marks]***

(b) setting their $m$ -value(s) $=3^x$ OR recognising a quadratic in $3^x$ ***(M1)***

$3^x=\dfrac{1}{3}$ ( or $3^x=-2$ ) OR $3\times\left(3^x\right)^2+5\times3^x-2=0$

$x=-1$ ***A1***

**Note:** Award the final ***A1*** if candidate's answer includes $x=-1$ and $x=\log_3(-2)$. Award ***A0*** if other incorrect answers are given.

***[2 marks]***

***Total [5 marks]***

**4.** (a) (i) $\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$ (accept $x = \frac{9}{2}$ and $y = \frac{3\sqrt{3}}{2}$) ***A1***

(ii) using $m = \frac{\text{change in } y}{\text{change in } x}$ with their midpoint OR gradient perpendicular to AC

OR $m = \tan 30^\circ$ ***(M1)***

$m = \frac{\sqrt{3}}{3}$ ***(A1)***

$y = \frac{\sqrt{3}}{3}x$ OR $y - \frac{3\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x - \frac{9}{2}\right)$ (must be written as an equation) ***A1***

***[4 marks]***

(b) substituting $x = 6$ into their equation ***(M1)***

so at B $y = 2\sqrt{3}$ ***(A1)***

area of triangle $\text{OAB} = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$

area of quadrilateral $\text{OABC} = 12\sqrt{3}$ ***A1***

***[3 marks]***

***Total [7 marks]***

**6.** (a) $y = \frac{2}{3}$ (must be written as equation with $y =$,) ***A1***

***[1 mark]***

(b) (i) $2$ ***A1***

(ii) **EITHER**

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x + 6 = 3mx^2 + 6mx + 3x + 6$

$3mx^2 + (6m+1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1***

recognition that discriminant $\Delta = 0$ for one solution ***(M1)***

$(6m+1)^2 = 0$

*continued…*

*Question 6 continued.*

**OR**

$\frac{2(x+3)}{3(x+2)} = mx+1$

attempt to expand to obtain a quadratic equation ***(M1)***

$2x+6 = 3mx^2+6mx+3x+6$

$3mx^2+(6m+1)x = 0$ OR $3mx^2+6mx+x = 0$ ***A1***

attempt to solve their quadratic for $x$ and equating their solutions ***(M1)***

$x(3mx+6m+1) = 0$

$x = 0$ OR $x = -\frac{6m+1}{3m}(=0)$

$-\frac{6m+1}{3m} = 0$

**OR**

attempt to find $f'(x)$ using the quotient rule ***(M1)***

$f'(x) = \frac{2}{3}\left(\frac{(x+2)-(x+3)}{(x+2)^2}\right) = \left(\frac{-2}{3(x+2)^2}\right)$ OR $\frac{2(3x+6)-3(2x+6)}{(3x+6)^2}$ or equivalent ***A1***

recognition that $m$ is the derivative of $f(x)$ at $x = 0$ (***M1)***

**THEN**

$\Rightarrow m = -\frac{1}{6}$ ***A1***

*continued…*

*Question 6 continued.*

(iii)

**Note:** In this part, FT may be awarded only for values of $m$ between $-1$ and 0.

$$-\frac{1}{6} < m < 0$$ ***A2***

**Note:** Award ***A1*** for only $m > -\frac{1}{6}$ . Award ***A1*** for only $m < 0$ .

***[7 marks]***

***Total [8 marks]***

**SECTION B**

**7.** (a) **METHOD 1**

attempt to interchange $x$ and $y$ ***M1***

**Note:** This ***M1*** may be awarded at any stage in the working.

attempt to rearrange using definition of natural log or take the natural log of both sides ***M1***

$\frac{2x}{3} = \mathrm{e}^{y-2} \Rightarrow \ln\left(\frac{2x}{3}\right) = y-2$ OR $x = \frac{3}{2}\mathrm{e}^{y-2} \Rightarrow \ln(x) = \ln\left(\frac{3}{2}\right) + y - 2$ ***A1***

$y = 2 + \ln\left(\frac{2x}{3}\right)$

so $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$ ***AG***

***[3 marks]***

**METHOD 2**

attempt to verify that $\left(f \circ f^{-1}\right)(x) = x$ ***M1***

$\left(f \circ f^{-1}\right)(x) = \frac{3}{2}\mathrm{e}^{\ln\left(\frac{2x}{3}\right)+2-2}\left(= \frac{3}{2}\mathrm{e}^{\ln\left(\frac{2x}{3}\right)}\right)$

attempt to use definition of natural log ***M1***

$\left(f \circ f^{-1}\right)(x) = \frac{3}{2} \times \frac{2x}{3}$ ***A1***

$\left(f \circ f^{-1}\right)(x) = x$ ***AG***

***[3 marks]***

*continued…*

*Question 7 continued*

(b) $(0.264456\ldots, 0.264456\ldots)$ AND $(2.51799\ldots, 2.51799\ldots)$ ***(A1)***

**Note:** Award ***A1*** for $0.264456\ldots$ and $2.51799\ldots$ seen.

attempt to put their values in distance formula or use of the isosceles right-angled triangle ***(M1)***

$\sqrt{(2.51799\ldots - 0.264456\ldots)^2 + (2.51799\ldots - 0.264456\ldots)^2}$ OR

$\sqrt{2} \times (2.51799\ldots - 0.264456\ldots)$

$= 3.18689\ldots$

$= 3.19$ ***A1***

***[3 marks]***

(c) (i) $g(x) = -\frac{3}{2}e^{x-2} + 5$ OR $g(x) = -f(x) + 5$ ***A1A1***

**Note:** Award ***A1*** for each correct term.

(ii) $0 \leq x \leq 4$ ***A1***

***[3 marks]***

(d) $\frac{3}{2}e^{x-2} = -\frac{3}{2}e^{x-2} + 5$ OR $f(x) = -f(x) + 5$

attempt to collect together terms in $e^{x-2}$ or $f(x)$ ***(M1)***

$3e^{x-2} = 5$ OR $2f(x) = 5$

$e^{x-2} = \frac{5}{3}$ OR $x = f^{-1}\left(\frac{5}{2}\right)$ ***(A1)***

$x = 2 + \ln\left(\frac{5}{3}\right)$ ***A1***

$\left(a = 2, b = \frac{5}{3}\right)$

**Note:** Award ***A1*** for each correct term given in exact form.

***[3 marks]***

***Total [12 marks]***

**4.** (a) $I = 2\times10^{-6}\left(=\frac{1}{500000}\right)$ (units) ***A1***

***[1 mark]***

(b) substitutes their doubled $I$-value from part (a) into $L$ ***(M1)***

$L = 10\log_{10}\left(2\times10^{-6}\times10^{12}\right)\ \left(=63.0102...\right)$

$=63.0$ (decibels) ***A1***

**Note:** Accept $60+10\log_{10}2$ (decibels) as a final answer.
Do not award the final ***A1*** for $L=0$ (from $I=10^{-12}$).

***[2 marks]***

(c) $115 = 10\log_{10}\left(I\times10^{12}\right)$ ***(A1)***

attempts to solve for $I$ ***(M1)***

$I=\frac{10^{11.5}}{10^{12}}$ (or equivalent) $\left(=0.316227...\right)$

$I=0.316$ (units) ***A1***

**Note:** Accept exact final answers such as $10^{-0.5}$ and $\frac{1}{\sqrt{10}}$.

***[3 marks]***

***Total [6 marks]***

**8.** (a) (i) substitution of $x = 2$ in part (i) or $x = \frac{1}{8}$ in part (ii) **(*M1*)**

$\log_2(8 \times 2)$ OR $\log_2(16)$

$\log_2(16) = 4$ ***A1***

(ii) $f\left(\frac{1}{8}\right) = 0$ ***A1***

***[3 marks]***

(b) swap $x$ and $y$ **(*M1*)**

$x = \log_2(8y)$ OR $x = 3 + \log_2 y$

attempt to write as exponential **(*M1*)**

$2^x = 8y$ **(*A1*)**

$\left(f^{-1}(x) =\right)\frac{2^x}{8}$ $\left(= 2^{x-3}\right)$ $\left(\text{accept } y = \frac{2^x}{8} \text{ or } y = 2^{x-3}\right)$ ***A1***

***[4 marks]***

(c) $\frac{1}{8}$ ***A1***

***[1 mark]***

*continued…*

*Question 8 continued*

(d) **METHOD 1**

$f(4x^2) = \log_2(8 \times 4x^2)$

attempt to use addition rule for logs **(*M1*)**

$\log_2 8 + \log_2 4 + \log_2 x^2$ OR $\log_2 32 + \log_2 x^2$ (or equivalent) **(*A1*)**

attempt to use exponent property for logarithms **(*M1*)**

$f(4x^2) = 5 + 2\log_2 x$ (or equivalent) ***A1***

the graph of $g$ must be vertically stretched (dilated) by a scale factor of $2$ and then vertically translated (shifted) $5$ units upwards. ***A2***

**METHOD 2**

$f(4x^2) = \log_2(8 \times 4x^2)$

attempt to write argument as a power **(*M1*)**

$\log_2\left(32x^2\right) = \log_2\left(\left(\sqrt{32}x\right)^2\right)$ (or equivalent) **(*A1*)**

attempt to use exponent property for logarithms **(*M1*)**

$f(4x^2) = 2\log_2\left(\sqrt{32}x\right)$ (or equivalent) ***A1***

**EITHER**

the graph of $g$ must be vertically stretched (dilated) by a scale factor of $2$ and stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$. ***A1***

**OR**

the graph of $g$ must be stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$ and vertically stretched (dilated) by a scale factor of $2$. ***A1***

**Note**: In this method, the final mark is ***A1***, as the question specifically asks for a translation and a stretch.

***[6 marks]***

***[Total: 14 marks]***

**8.** (a) (i) substitution of $x = 8$ in part (i) or $x = \frac{1}{4}$ in part (ii) **(*M1*)**

$\log_2(4 \times 8)$ OR $\log_2(32)$

$\log_2(32) = 5$ ***A1***

(ii) $f\left(\frac{1}{4}\right) = 0$ ***A1***

***[3 marks]***

(b) swap $x$ and $y$ ***(M1)***

$x = \log_2(4y)$ OR $x = 2 + \log_2 y$

attempt to write as exponential ***(M1)***

$2^x = 4y$ ***(A1)***

$\left(f^{-1}(x) =\right)\frac{2^x}{4}$ $\left(= 2^{x-2}\right)$ ***A1***

***[4 marks]***

(c) $\frac{1}{4}$ ***A1***

***[1 mark]***

*continued…*

*Question 8 continued.*

(d) **METHOD 1**

$f(16x^3) = \log_2(4 \times 16x^3)$

attempt to use addition rule for logs ***(M1)***

$\log_2 4 + \log_2 16 + \log_2 x^3$ OR $\log_2 64 + \log_2 x^3$ (or equivalent) ***(A1)***

attempt to use exponent property for logarithms ***(M1)***

$f(16x^3) = 6 + 3\log_2 x$ (or equivalent) ***A1***

the graph of $g$ must be vertically stretched (dilated) by a scale factor of $3$ and then vertically translated (shifted) $6$ units upwards. ***A2***

**METHOD 2 (5 marks maximum)**

$f(16x^3) = \log_2(4 \times 16x^3)$

attempt to write argument as a power ***(M1)***

$\log_2\left(64x^3\right) = \log_2\left((4x)^3\right)$ (or equivalent) ***(A1)***

attempt to use exponent property for logarithms ***(M1)***

$f(16x^3) = 3\log_2(4x)$ (or equivalent) ***A1***

**EITHER**

the graph of $g$ must be vertically stretched (dilated) by a scale factor of $3$ and stretched (dilated) horizontally by a scale factor of $\frac{1}{4}$. ***A1***

**OR**

the graph of $g$ must be stretched (dilated) horizontally by a scale factor of $\frac{1}{4}$ and vertically stretched (dilated) by a scale factor of $3$. ***A1***

**Note**: In this method, the final mark is ***A1***, as the question specifically asks for a translation and a stretch.

***[6 marks]***

***[Total: 14 marks]***

**1.** (a) (i) $f(0) = -7$ ***A1***

(ii) $-12.7782...$

$f(60) = -12.8 \left(= 7\sqrt{60} - 67 = 14\sqrt{15} - 67\right)$ ***A1***

***[2 marks]***

(b) attempt to find at least one root ***(M1)***

$x = 1.46098...$ and $x = 33.5390...$

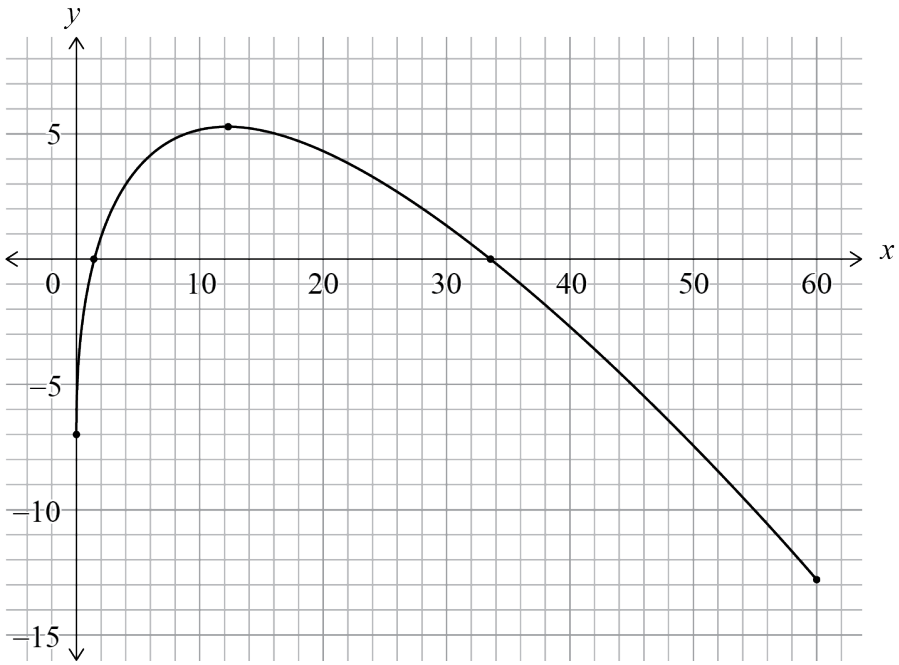
$x = 1.46$ and $x = 33.5$ ***A1***

***[2 marks]***

*continued…*

*Question 1 continued*

(c)




***A1A1A1***

**Note:** Follow through from their part (a).

Award ***A1*** for endpoints at approximately $(0,-7)$ and $(60,-12.8)$. Allow for $y$-intercept $-8 < y < -6$ and for the right endpoint in the interval $x = 60,\ -13 < y < -12$.

The following two ***A*** marks may only be awarded if the approximate shape is correct.

Award ***A1*** for $x$-intercepts at approximately $x = 1.5$ and $x = 33.5$ and award ***A1*** for maximum at approximately $(12.2,5.3)$. Allow for $x$-intercepts in the intervals $0 < x < 2,\ 32 < x < 34$, and maximum in the intervals $10 < x < 14,\ 4.5 < y < 6$.

***[3 marks]***

***Total [7 marks]***

**6. METHOD 1**

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1***

(discriminant = ) $(-r)^2 - 4(3)(r-2)$ $\left(= r^2 - 12r + 24\right)$ (seen anywhere) ***(A1)***

$2.53589...$ $\left(= 6 - 2\sqrt{3}\right)$ AND $9.46410...$ $\left(= 6 + 2\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $3x^2 - rx + r - 2$ is less than zero ***(M1)***

$2.54 < r < 9.46$ $\left(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3}\right)$ ***A1***

**Note:** Accept $2.54 \le r \le 9.46$.

**METHOD 2**

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1***

**EITHER**

minimum when $x = \frac{r}{6} \Rightarrow (y =) 3\left(\frac{r}{6}\right)^2 - r\left(\frac{r}{6}\right) + r - 2$ $(> 0)$ ***(A1)***

attempt to solve their inequality for $y$ (must be in terms of $r$ and $r^2$) ***(M1)***

**OR**

$x < 1 \Rightarrow r > \frac{3x^2 - 2}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{3x^2 - 2}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{3x^2 - 2}{x - 1}$ ***(M1)***

**THEN**

$(r >) 2.53589...$ $\left(= 6 - 2\sqrt{3}\right)$ AND $(r <) 9.46410...$ $\left(= 6 + 2\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$2.54 < r < 9.46$ $\left(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3}\right)$ ***A1***

**Note:** Accept $2.54 \le r \le 9.46$.

***[5 marks]***

## Section B

**1.** (a) (i) $f(0) = -11$ ***A1***

(ii) $-1.80650...$

$f(20) = -1.81 \left(= 11\sqrt{20} - 51 = 22\sqrt{5} - 51\right)$ ***A1***

***[2 marks]***

(b) attempt to find at least one root ***(M1)***

$x = 1.72622...$ and $x = 17.5237...$

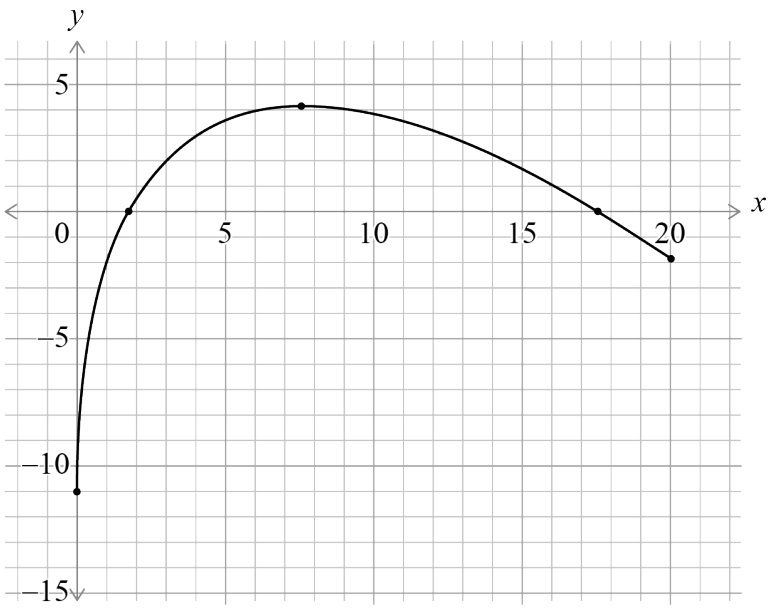
$x = 1.73$ and $x = 17.5$ ***A1***

***[2 marks]***

*continued…*

*Question 1 continued.*

(c)




***A1A1A1***

**Note:** Follow through from their part (a).

Award ***A1*** for endpoints at approximately $(0,-11)$ and $(20,-1.8)$. Allow for $y$-intercept $-12 < y < -10$ and for the right endpoint in the interval $x = 20, -2 < y < -1$.

The following two ***A*** marks may only be awarded if the approximate shape is correct.

Award ***A1*** for $x$-intercepts at approximately $x = 1.7$ and $x = 17.5$ and award ***A1*** for maximum at approximately $(7.6, 4.1)$. Allow for $x$-intercepts in the intervals $1 < x < 2$, $17 < x < 18$, and maximum in the intervals $6.5 < x < 8.5$, $3.5 < y < 5$.

***[3 marks]***

***Total [7 marks]***

**6. METHOD 1**

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

(discriminant = ) $(-r)^2 - 4(4)(r-1)$ $\left(= r^2 - 16r + 16\right)$ (seen anywhere) ***(A1)***

$1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero ***(M1)***

$1.07 < r < 14.9$ $(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3})$ ***A1***

**Note:** Accept $1.08 \leq r \leq 14.9$.

**METHOD 2**

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

**EITHER**

minimum when $x = \frac{r}{8} \Rightarrow (y =) 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1$ $(> 0)$ ***(A1)***

attempt to solve their inequality for $y$ (must be in terms of $r$ and $r^2$) ***(M1)***

**OR**

$x < 1 \Rightarrow r > \frac{4x^2 - 1}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{4x^2 - 1}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{4x^2 - 1}{x - 1}$ ***(M1)***

**THEN**

$(r >) 1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $(r <) 14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$1.07 < r < 14.9$ $\left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

**Note:** Accept $1.08 \leq r \leq 14.9$.

***[5 marks]***

**Section B**

**1.** (a) $\ln 12$ ***A1***

***[1 mark]***

(b) $\ln 2^3$ ***(A1)***

$= \ln 8$ ***A1***

***[2 marks]***

(c) $\ln\left(\frac{1}{2}\right)^{-1}$ OR $-(\ln 1 - \ln 2)$ OR $-\ln 2^{-1}$ ***(A1)***

$= \ln 2$ ***A1***

***[2 marks]***

***Total [5 marks]***

**8.** (a) **METHOD 1**

$a = 5$ ***(A1)***

attempt to use roots and symmetry to find $h$ ***(M1)***

$h = \frac{(-1)+(-3)}{2}$ OR half the distance between the roots $\frac{(-1)-(-3)}{2} = 1$ (may be seen on a diagram)

$h = -2$ (accept $x = -2$) ***(A1)***

$f(x) = 5(x-(-2))^2 - 5\left(= 5(x+2)^2 - 5\right)$ ***A1***

$(a = 5,\ h = -2,\ k = -5)$

**METHOD 2**

$a = 5$ ***(A1)***

attempt to expand

$(x+1)(x+3) = x^2 + 4x + 3$ OR $5(x+1)(x+3) = 5x^2 + 20x + 15$

**EITHER**

uses their expansion to attempt to complete the square to the form ***(M1)***

$p(x+q)^2 + r$ ,where q is half the coefficient of their $x$ term

$= (x+2)^2 - 2^2 + 3\left(= (x+2)^2 - 1\right)$ OR $5\left[(x+2)^2 - 2^2 + 3\right](= 5(x+2)^2 - 5)$ ***(A1)***

**OR**

uses their expansion to attempt to differentiate and sets equal to zero ***(M1)***

$\frac{dy}{dx} = 2x + 4 = 0$ OR $\frac{dy}{dx} = 10x + 20 = 0$

$h = -2$ (accept $x = -2$) ***(A1)***

**OR**

uses their expansion to attempt to find axis of symmetry using $h = \frac{-b}{2a}$ ***(M1)***

$h = \frac{-4}{2}$ OR $h = \frac{-20}{10}$

$h = -2$ (accept $x = -2$) ***(A1)***

**THEN**

$f(x) = 5(x-(-2))^2 - 5\left(= 5(x+2)^2 - 5\right)$ ***A1***

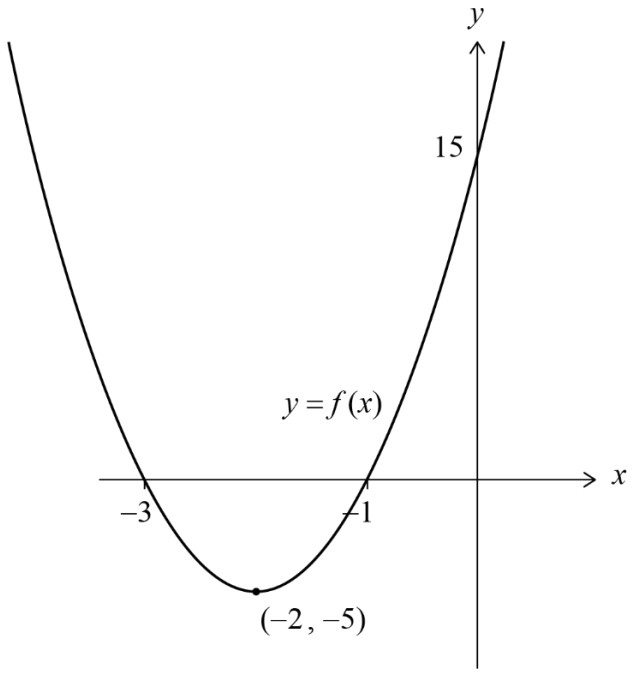
$(a = 5,\ h = -2,\ k = -5)$

***[4 marks]***

*continued…*

*Question 8 continued.*

(b)




***M1A1A1A1***

award ***M1*** for a roughly symmetric curve which is concave up

award ***A1*** for $x$ intercepts at $-3$ and $-1$

award ***A1*** for $y$ intercept at $15$

award ***A1*** for vertex at $(-2,-5)$

***[4 marks]***

(c) $5(x+2)^2-5\le 40$ OR $5(x+1)(x+3)\le 40$ OR $(x+1)(x+3)\le 8$ leading to

$(x+2)^2\le 9$ OR $5x^2+20x-25\le 0$ OR $x^2+4x-5\le 0$ ***(A1)***

valid attempt to find the critical values for their quadratic inequality ***(M1)***

$x+2=\pm 3$ OR $(x+5)(x-1)=0$

$x=-5,\ x=1$ ***(A1)***

$-5\le x\le 1$ ***A1***

**Note:** Accept $(x\in)[-5,1]$ or equivalent.

***[4 marks]***

*continued…*

*Question 8 continued.*

(d) (i) $(f \circ g)(x) = 5(\ln x + 1)(\ln x + 3)$ OR $5(\ln x + 2)^2 - 5$ OR $5(\ln x)^2 + 20\ln x + 15$ ***A1***

(ii) attempt to replace $x$ with $\ln x$ using their solution to part (c) ***(M1)***

$-5 \leq \ln x \leq 1$

$e^{-5} \leq x \leq e$ ***A1***

**Note:** Accept $(x \in)[e^{-5}, e]$ or equivalent.

***[3 marks]***

***Total [15 marks]***

**3.**

(a) $-2$ (accept $(0,-2)$) ***A1***

***[1 mark]***

(b) $y=\frac{3}{2}$ (must be an equation) ***A1***

**Note**: Do not accept $\neq$ sign.

***[1 mark]***

(c) attempt to reflect the graph of $f$ in the $x$-axis OR one correct value seen ***(M1)***

both correct values $-\frac{3}{2}$ and $2$ (seen anywhere) ***A1***

$-\frac{3}{2}<y\leq 2$ ***A1***

***[3 marks]***

***Total [5 marks]***

**5.**

(a) recognising $\Delta > 0$ (seen anywhere) ***(M1)***

$\Delta = k^2 - 4(15-k)$ $\left(= k^2 + 4k - 60\right)$ ***A1***

valid attempt to solve quadratic (in)equality ***(M1)***

$(k-6)(k+10)$ OR $k = \dfrac{-4 \pm \sqrt{4^2 - 4(-60)}}{2}$

two correct values $-10$ and $6$ (seen anywhere) ***A1***

$k < -10$, $k > 6$ ***A1***

***[5 marks]***

(b) recognising constant or product of roots is positive ***(M1)***

$15 - k > 0 \Rightarrow k < 15$

$k < -10$, $6 < k < 15$ ***A1***

***[2 marks]***

***Total [7 marks]***

**9.**

(a) $4^x = 8$ OR $f^{-1}(x) = \log_4 x$ OR $f^{-1}(8) = \log_4 8$ ***(A1)***

attempt to use indices with same base OR change of base of logs ***(M1)***

$2^{2x} = 2^3$, $4^x = 4^{\frac{3}{2}}$ OR $f^{-1}(8) = \log_4 4^{\frac{3}{2}}$ OR $f^{-1}(8) = \dfrac{\log_2 8}{\log_2 4}$

$f^{-1}(8) = \dfrac{3}{2}$ ***A1***

***[3 marks]***

(b) (i) interchanging $x$ and $y$ (seen anywhere) ***(M1)***

$x = 1 + \log_2 y$ OR $y - 1 = \log_2 x$

$x - 1 = \log_2 y$ OR $2^{y-1} = x$

$g^{-1}(x) = 2^{x-1}$ (or equivalent) ***A1***

(ii) **METHOD 1**

a horizontal translation/shift by 1 unit to the left (do not accept 'move')

followed by a horizontal stretch/dilation with scale factor $\frac{1}{2}$ (accept horizontal compression by a factor of 2)

one correct transformation ***A1***

two correct transformations in the correct order ***A1***

**METHOD 2**

horizontal stretch/dilation with scale factor $\frac{1}{2}$ (accept horizontal compression by a factor of 2) ***A1***

vertical stretch/dilation with scale factor $2$ ***A1***

**Note:** ***A1***s can be awarded independently here as order is not important.

***[4 marks]***

*continued…*

*Question 9 continued.*

(c) attempt to find composite function (in any order) ***M1***

$f\left(1+\log_2 x\right)\left(=4^{1+\log_2 x}\right)$

$4\times 4^{\log_2 x}$ OR $4\times 2^{2\log_2 x}$ OR $4^{\log_2 2x}$ OR $2^{(2+2\log_2 x)}$ OR $4^{\left(\log_4 4+2\log_4 x\right)}$ ***(A1)***

$=4\times 2^{\log_2 x^2}$ OR $2^{\log_2 (2x)^2}$ OR $4^{\log_4\left(4x^2\right)}$ ***A1***

$=4x^2$ ***AG***

***[3 marks]***

(d) (i) $2x-1+\dfrac{1}{2x+1}=\dfrac{(2x-1)(2x+1)+1}{2x+1}$ ***(A1)***

$=\dfrac{4x^2-1+1}{2x+1}$ ***A1***

$2x-1+\dfrac{1}{2x+1}=\dfrac{4x^2}{2x+1}$ ***AG***

**Note:** Accept working from RHS to LHS.

(ii) **METHOD 1**

enclosed area is $\displaystyle\int_1^3\left(2x-1+\frac{1}{2x+1}\right)\mathrm{d}x$

attempt to integrate (at least one correct term or $\ln(2x+1)$ seen) ***(M1)***

$=x^2-x+\dfrac{1}{2}\ln|2x+1|(+c)$ ***A1A1***

**Note:** Award ***A1*** for $x^2-x$ and ***A1*** for $\dfrac{1}{2}\ln|2x+1|$.

Accept $\dfrac{1}{2}\ln(2x+1)$.

substitute correct limits into their integrated expression and subtract ***(M1)***

$=\left(9-3+\dfrac{1}{2}\ln 7\right)-\left(1-1+\dfrac{1}{2}\ln 3\right)$

$A=6+\dfrac{1}{2}\ln\dfrac{7}{3}$ ***A1***

*continued…*

*Question 9 continued.*

**METHOD 2**

attempt to use integration by substitution ***(M1)***

let $u = 2x+1$ OR $u = 2x-1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$

$= \frac{1}{2}\int \left(u - 2 + \frac{1}{u}\right)\mathrm{d}u$ OR $\frac{1}{2}\int \frac{(u-1)^2}{u}\mathrm{d}u$ OR $\frac{1}{2}\int \left(u + \frac{1}{u+2}\right)\mathrm{d}u$ ***A1***

correct integration

$= \frac{1}{4}u^2 - u + \frac{1}{2}\ln|u|(+c)$ OR $\frac{1}{4}u^2 + \frac{1}{2}\ln|u+2|(+c)$ ***A1***

substitution of their limits into their integrated expression and subtract ***(M1)***

$= \left(\frac{49}{4} - 7 + \frac{1}{2}\ln 7\right) - \left(\frac{9}{4} - 3 + \frac{1}{2}\ln 3\right)$ OR $\left(\frac{25}{4} + \frac{1}{2}\ln 7\right) - \left(\frac{1}{4} + \frac{1}{2}\ln 3\right)$

$A = 6 + \frac{1}{2}\ln\frac{7}{3}$ (or equivalent) ***A1***

***[7 marks]***

***Total [17 marks]***

**2.** (a) correct application of $\log_a xy = \log_a x + \log_a y$ or $\log_a x^m = m\log_a x$ ***(M1)***

correct expression in terms of $\log_{10} 2$ AND $\log_{10} 3$ (arguments must be $2$ and $3$) ***(A1)***

$3\log_{10} 2 + \log_{10} 3$ OR $\log_{10} 2 + \log_{10} 2 + \log_{10} 2 + \log_{10} 3$

$3p + q$ ***A1***

***[3 marks]***

(b) $\left(\log_3 8 = \right) \dfrac{\log_{10} 8}{\log_{10} 3} \left( = \dfrac{3\log_{10} 2}{\log_{10} 3} \right)$ ***(A1)***

$= \dfrac{3p}{q}$ ***A1***

***[2 marks]***

***Total [5 marks]***

**3.** (a) $f(-3) = -1$ ***A1***

***[1 mark]***

(b) $-3 \leq x \leq 5$ ***A1***

**Note:** Award ***A1*** for answers using interval notation $[-3, 5]$.

***[1 mark]***

(c) $\left(f^{-1}(2x-7) = -3 \Rightarrow\right) 2x - 7 = f(-3)$ **OR** $f^{-1}(-1) = -3$ ***(M1)***

$2x - 7 = -1$ ***(A1)***

$x = 3$ ***A1***

***[3 marks]***

***Total [5 marks]***

**5.** (a) equating $y = mx - 3$ and $y = x^2 - x - 1$ ***M1***

$x^2 - x - 1 - (mx - 3) = 0$ **OR** $x^2 - mx - x + 2 = 0$ ***A1***

$x^2 - (m+1)x + 2 = 0$ ***AG***

***[2 marks]***

(b) **METHOD 1 (discriminant)**

correct substitution into discriminant (do not award if seen only in quadratic formula) ***(A1)***

$(-(m+1))^2 - 4(1)(2)$ **OR** $(m+1)^2 - 4(1)(2)$

discriminant equals $0$ (seen anywhere) ***A1***

$(m+1)^2 = 8$ **OR** $m^2 + 2m - 7 = 0$ ***A1***

$(m =)\ -1 - 2\sqrt{2},\ -1 + 2\sqrt{2}$ ***A1A1***

**METHOD 2 (derivative)**

$\frac{dy}{dx} = 2x - 1$ ***A1***

substituting $m = 2x - 1$ or $x = \frac{m+1}{2}$ into ***AG*** from part (a) ***(A1)***

$x^2 - (2x - 1 + 1)x + 2 = 0$ **OR** $x^2 - 2x^2 + 2 = 0$ **OR** $\left(\frac{m+1}{2}\right)^2 - (m+1)\frac{m+1}{2} + 2 = 0$

$x = \pm\sqrt{2}$ **OR** $-m^2 - 2m + 7 = 0$ **OR** $m^2 + 2m - 7 = 0$ ***A1***

$(m =)\ 2\sqrt{2} - 1,\ -2\sqrt{2} - 1$ ***A1A1***

***[5 marks]***

***Total [7 marks]***

**9.** (a) (i) recognizing need to factorise, complete the square or substitute into quadratic formula ***(M1)***

$(5+x)(1-x)$ **OR** $9-(x+2)^2$ **OR** $\frac{4\pm\sqrt{(-4)^2-4\times(-1)\times5}}{2\times(-1)}$

$x=-5,\ x=1$ ***A1***

(ii) sign diagram or sketch of $y=5-4x-x^2$ with $-5$ and $1$ indicated ***(M1)***

$-5<x<1$ ***A1***

**Note:** Award ***A1*** for answers using interval notation $(-5, 1)$.

***[4 marks]***

(b) (i) $(a=)-5$ ***A1***

(ii) $(b=)1$ ***A1***

***[2 marks]***

(c) $\left(\log_k(5-4x-x^2)=0\Rightarrow\right)5-4x-x^2=1$ ***(A1)***

$x^2+4x-4=0$ **OR** $4-4x-x^2=0$

attempting to solve their quadratic equation set to $0$ ***(M1)***

$x=\frac{-4\pm\sqrt{4^2-4\times1\times(-4)}}{2\times1}$ **OR** $(x+2)^2-8\ (=0)$ ***A1***

$x=-2\pm2\sqrt{2}$ ***A1***

***[4 marks]***

*continued…*

*Question 9 continued*

(d) **EITHER**

recognizing that maximum of $f$ is when $5-4x-x^2$ is a maximum or when $f'(x)=0$ ***(M1)***

substituting into $x=-\frac{b}{2a}$ **OR** $-4-2x=0$ **OR** $\frac{-4-2x}{(5-4x-x^2)\ln k}=0$

**OR**

use of symmetry to find $x$-coordinate of maximum ***(M1)***

$\frac{a+b}{2}$ **OR** $x=\frac{\text{sum of their answers to (c)}}{2}$

**THEN**

$x=-2$ (seen anywhere) ***A1***

maximum of quadratic is $9$ **OR** argument of logarithm is $9$ ***(A1)***

$\log_k(9)=2$ **OR** $\log_k(9)=\log_k\left(k^2\right)$ **OR** $k^2=5-4(-2)-(-2)^2$ (or equivalent) ***A1***

valid approach to solve ***(M1)***

$k^2=9$ OR $k=\pm 3$

$k=3$ ***A1***

***[6 marks]***

***Total [16 marks]***

**9.** (a) **METHOD 1**

attempt to rearrange and swap $x$ and $y$ (at any stage) ***(M1)***

$x = \frac{2-2y}{y+2} \Rightarrow xy + 2x = -2y + 2$

$xy + 2y = 2 - 2x$ OR $y(x+2) = 2 - 2x$ ***A1***

$f^{-1}(x) = \frac{2-2x}{x+2}$ OR $y = \frac{2-2x}{x+2}$ ***A1***

So $f^{-1}(x) = f(x)$ ***AG***

**METHOD 2**

attempt to find $f(f(x))$ ***(M1)***

$$f(f(x)) = \frac{2-2\left(\frac{2-2x}{x+2}\right)}{\left(\frac{2-2x}{x+2}\right)+2}$$

$= \frac{2(x+2)-2(2-2x)}{(2-2x)+2(x+2)} \left(= \frac{6x}{6}\right)$ ***A1***

$= x$ ***A1***

so $f^{-1}(x) = f(x)$ ***AG***

***[3 marks]***

*continued…*

*Question 9 continued.*

(b) (i) **EITHER**

OP $= \sqrt{k^2 + \left(\frac{2-2k}{k+2}\right)^2}$ ***(A1)***

attempt to minimise distance ***(M1)***

**OR**

recognition that P lies on $y = x$ due to symmetry ***(M1)***

$k = -4.44948...$ or $k = 0.449489....$ ***(A1)***

**THEN**

minimum when $k = 0.449489....$

$k = 0.449$ ***A1***

**Note:** If no working seen, award ***(M0)(A0)A0*** for $0.45$.

(ii) coordinates of P $(0.449, 0.449)$ ***A1***

**Note:** Award ***A1FT*** for $f(k)$, using their $k$.

***[4 marks]***

(c) (i) $x = -\frac{d}{c}$ ***A1***

(ii) $y = -\frac{3}{c}$ ***A1***

**Note:** Answers in (i) and (ii) must be correct equations.

***[2 marks]***

(d) $g^{-1}(x) = \frac{2-dx}{cx+3}$ OR $-\frac{d}{c} = -\frac{3}{c}$ ***(A1)***

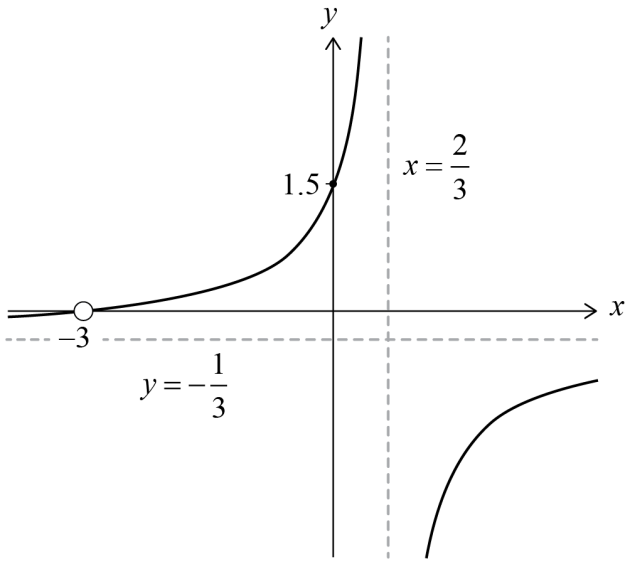
$d = 3$ ***A1***

***[2 marks]***

*continued…*

*Question 9 continued.*

(e)




Only if shape is approximately correct for this rational function (i.e. two branches in opposite corners and displaying appropriate asymptotic behavior), award the following marks independently.

asymptotes $x = \frac{2}{3}$ and $y = -\frac{1}{3}$ drawn with correct labels ***A1A1***

labelled point discontinuity (or $x$-intercept) at $(-3, 0)$, labelled $y$-intercept at $\left(0, \frac{3}{2}\right)$ ***A1A1***

**Note:** Award ***A1FTA1FT***, as appropriate, for their value of $d$ (i.e. $x$-intercept= $-d$, $y$-intercept $\frac{d}{2}$).

***[4 marks]***

***Total [15 marks]***

**1.**

| Function | Letter |
|---|---|
| $y = f(x+1)$ | **B** |
| $y = f(x)-1$ | **A** |
| $y = f(-x)$ | **F** |
| $y = f(2x)$ | **D** |

***A1A1A1A1***

***Total [4 marks]***

**5.** (a) attempt to factorize at least one expression ***(M1)***

$\frac{x}{(x-7)(x-1)} \times \frac{(x-1)(x+1)}{x+1}$ **OR** $\frac{x(x^2-1)}{(x-7)(x^2-1)}$ **OR** $\frac{x(x^2-1)}{(x-7)(x-1)(x+1)}$ ***A1***

$\frac{x}{x-7}$ ***AG***

***[2 marks]***

(b) **METHOD 1** (combines given log terms)

attempt to use the subtraction/quotient property of logs ***(M1)***

$\log_2\left(\frac{x\left(x^2-1\right)}{\left(x^2-8x+7\right)(x+1)}\right)$

valid attempt to rewrite equation with (or without) logs on both sides (seen anywhere) ***(M1)***

$2 = \frac{x}{x-7}$ **OR** $\log_2\left(\frac{x}{x-7}\right) = \log_2 2$ **OR** $2^{\log_2\left(\frac{x}{x-7}\right)} = 2^1$ ***(A1)***

$x = 14$ ***A1***

**METHOD 2** (uses log laws with LHS)

attempt to use $1 = \log_2 2$ (seen anywhere) ***(M1)***

attempt to use the subtraction/quotient property of logs ***(M1)***

$\log_2\left(\frac{x^3-x}{2}\right) = \log_2\left(\left(x^2-8x+7\right)(x+1)\right)$ **OR** $\frac{x\left(x^2-1\right)}{2} = (x-1)(x-7)(x+1)$ ***(A1)***

$\frac{x}{2} = x-7$

$x = 14$ ***A1***

***[4 marks]***

***Total [6 marks]***

**6.** (a) **METHOD 1**

$x$-coordinate of the vertex at $\mathrm{A}$ is $2$ ***(A1)***

attempt to substitute their $x$-coordinate into equation of line ***(M1)***

$2(2)-1(=3)$

$3=-(2)^2+4(2)+p$ ***A1***

$p=-1$ ***AG***

**METHOD 2**

$x$-coordinate of the vertex at $\mathrm{A}$ is $2$ ***(A1)***

attempt to substitute their $x$-coordinate into $f(x)$ ***(M1)***

$f(2)=p+4$

$p+4=2(2)-1$ ***A1***

$p=-1$ ***AG***

**METHOD 3**

attempt to complete the square on $f(x)$ ***(M1)***

$-(x-2)^2+p+4$

$x_{\mathrm{A}}=2$ **OR** $y_{\mathrm{A}}=p+4$ ***(A1)***

$p+4=2(2)-1$ ***A1***

$p=-1$ ***AG***

**Note**: Award no marks if $x=0$ is substituted into $-x^2+4x+p=2x-1$ to show $p=-1$.

***[3 marks]***

*continued…*

*Question 6 continued.*

(b) **METHOD 1** (equating $g(x)$ and $2x-1$)

$x$-coordinate is $\frac{-q}{2}$ (seen anywhere) ***(A1)***

attempt to find intersection points, equating $g(x)$ and $2x-1$ ***(M1)***

$x^2+qx-1=2x-1$

attempt to substitute their $x$-coordinate ***(M1)***

$\left(\frac{-q}{2}\right)^2+q\left(\frac{-q}{2}\right)-1=2\left(-\frac{q}{2}\right)-1$

$q^2-4q=0$ ***(A1)***

$q=4$ (since $q\neq 0$) ***A1***

**METHOD 2** (find coordinates of B in terms of $q$)

$x$-coordinate is $\frac{-q}{2}$ ***(A1)***

**EITHER**

attempt to substitute their $x$-coordinate into $y=2x-1$ ***(M1)***

$2\left(\frac{-q}{2}\right)-1 \Rightarrow \mathrm{B}\left(\frac{-q}{2},-q-1\right)$

attempt to substitute their coordinates of B into $g(x)$ ***(M1)***

$-q-1=\left(\frac{-q}{2}\right)^2+q\left(\frac{-q}{2}\right)-1$

**OR**

attempt to substitute their $x$-coordinate into $g(x)$ ***(M1)***

$\left(\frac{-q}{2}\right)^2+q\left(\frac{-q}{2}\right)-1 \Rightarrow \mathrm{B}\left(\frac{-q}{2},-\frac{q^2}{4}-1\right)$

attempt to substitute their coordinates of B into the line ***(M1)***

$-\frac{q^2}{4}-1=2\left(\frac{-q}{2}\right)-1$

*continued…*

*Question 6 continued.*

**THEN**

$q^2 - 4q = 0$ ***(A1)***

$q = 4$ (since $q \neq 0$) ***A1***

**METHOD 3** (completing the square)

attempt to complete the square on $g(x)$ ***(M1)***

$\left(x + \frac{q}{2}\right)^2 - \left(\frac{q}{2}\right)^2 - 1$

$x_B = \frac{-q}{2}$ **OR** $y_B = -\frac{q^2}{4} - 1$ ***(A1)***

attempt to substitute their coordinates of $B$ into equation of line ***(M1)***

$-\frac{q^2}{4} - 1 = 2\left(\frac{-q}{2}\right) - 1$

$q^2 - 4q = 0$ ***(A1)***

$q = 4$ (since $q \neq 0$) ***A1***

***[5 marks]***

***Total [8 marks]***

**Section B**

**7.** (a) (i) $7$ ***A1***

(ii) attempt to interchange $x$ and $y$ (seen anywhere) ***(M1)***

$f^{-1}(x) = \frac{x-3}{2}$ (accept $(y =)\frac{x-3}{2}$) ***A1***

***[3 marks]***

(b) $h(x) = \frac{1}{2}x - 1$ (or equivalent) ***A1A1***

**Note:** Award ***A1*** for $\frac{1}{2}x$ and ***A1*** for $-1$.

***[2 marks]***

**Special note:** The following parts (c), (d) and (e) may use their expression for $f^{-1}$ and/or their expression for $h$. Please ensure to apply ***FT*** to all these parts, as appropriate.

(c) **METHOD 1**

recognition that $h^{-1}(x) = -2 \Rightarrow h(-2) = x$ ***(M1)***

$x = -2$ ***A1***

**METHOD 2**

attempt to find inverse of $h$ ***(M1)***

$h^{-1}(x) = 2(x+1)$

$2(x+1) = -2$

$x = -2$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued.*

(d) **METHOD 1**

attempt to use properties of inverse functions ***(M1)***

$f(h(x))$ OR $f(f^{-1}(g(x)))$ OR $f\left(\frac{1}{2}x-1\right)=g(x)$

$2\left(\frac{1}{2}x-1\right)+3=g(x)$ **OR** $2\left(\frac{1}{2}x-1\right)+3=mx+c$ (or equivalent) ***(A1)***

attempt to solve for $g(x)$ OR equate terms (one of $m=2\times\frac{1}{2}$ or $c=-2+3$)

OR substituting $x=0$ into their equation to solve for $c$ ***(M1)***

$m=1$, $c=1$ (accept $g(x)=x+1$) ***A1***

**METHOD 2**

attempt to compose $f^{-1}$ and $g$ (in any order) ***(M1)***

$\frac{g(x)-3}{2}$ OR $\frac{1}{2}(mx+c)-\frac{3}{2}$

$\frac{g(x)-3}{2}=\frac{1}{2}x-1$ **OR** $\frac{1}{2}(mx+c)-\frac{3}{2}=\frac{1}{2}x-1$ (or equivalent) ***(A1)***

attempt to solve for $g(x)$ OR equate terms (one of $\frac{1}{2}m=\frac{1}{2}$ or $\frac{1}{2}c-\frac{3}{2}=-1$)

OR substituting $x=0$ into their equation to solve for $c$ ***(M1)***

$m=1$, $c=1$ (accept $g(x)=x+1$) ***A1***

**Note:** If $g\circ f^{-1}\left(=\frac{mx}{2}-\frac{3m}{2}+c\right)$ is seen, then ***M1A0M1A0*** may be awarded for attempting to equate their terms.

***[4 marks]***

*continued…*

*Question 7 continued.*

(e) attempt to compose functions OR recognition that $k(x) = h^{-1}(x)$ ***(M1)***

$\frac{1}{2}(k(x)) - 1 = x$

$k(x) = 2x + 2$ ***A1***

***[2 marks]***

(f) reflection in the line $y = x$ ***A1***

***[1 mark]***

***Total [14 marks]***

**5.** (a) substitute $\mathrm{P}(-10,1)$ or $\mathrm{Q}(3,-12)$ into $y=\frac{Ax+B}{x-4}$ ***(M1)***

$-14=-10A+B$ (or equivalent)

$12=3A+B$ (or equivalent)

attempt to solve their equations simultaneously ***(M1)***

$A=2,\ B=6$ ***A1***

***[3 marks]***

(b) **EITHER**

attempt to express $y$ in the form $P+\frac{Q}{x-4}$ ***(M1)***

**OR**

attempt to find one or both asymptotes using $x-4=0$ or $y=\frac{a}{c}$ (dividing coefficients) ***(M1)***

**THEN**

recognizing vertical asymptote at $x=4$ (may be seen anywhere, including labeled on graph or in part (a)) OR horizontal shift $4$ units to the right ***A1***

recognizing horizontal asymptote at $y=2$ (may be seen anywhere, including labeled on graph or in part (a)) OR vertical shift $2$ units up ***A1***

$\frac{2x+6}{x-4}=2+\frac{14}{x-4}$ ***(A1)***

**EITHER**

horizontal translation $4$ units to the right;
vertical stretch scale factor $14$ followed by vertical translation $2$ up. ***A1***

**Note:** Horizontal translation may be seen anywhere; $y$ -axis transformations must be seen in the order above.

**OR**

vertical stretch scale factor $14$; followed by translation (through/by) $\begin{pmatrix}4\\2\end{pmatrix}$ ***A1***

**Note:** In this question, the transformations may be described using terms such as *translate* or *shift*, and *dilate* or *stretch*. Do not accept "*move*" to describe a transformation.

***[5 marks]***
***Total [8 marks]***

**6.**

**Note**: Candidates may approach this problem in different ways and may do their steps in many different orders.
The ***M*** marks are independent and may be awarded in any order. Use the description of each mark to determine when the mark may be awarded.

$3\log_8 10x - \log_4 x = 1$

attempt to apply power rule, quotient rule, or product rule (seen anywhere) ***(M1)***

attempt to apply change of base rule (seen anywhere) ***(M1)***

correct equation with same base in all logarithms ***A1***

$\log_2 10x - \log_2 x^{\frac{1}{2}} = 1$ or $\log_2 10 + \log_2 x - \frac{1}{2}\log_2 x = 1$ or $\frac{\log_4 (10x)^3}{\log_4 8} - \log_4 x = 1$ (or equivalent)

correct equation without logarithms ***A1***

$10\sqrt{x} = 2$ or $\sqrt{x} = \frac{2}{10}\left(= \frac{1}{5}\right)$ (or equivalent)

$x = \frac{1}{25}$ ***A1***

***[5 marks]***

## Section B

APOLLO

# Geometry and trigonometry

72 questions

**6.** (a)

**Note:** Do not award the final ***A1*** for proofs which work from both sides to find a common expression other than $2\sin x\cos x - 2\sin^2 x$.

**METHOD 1 (LHS to RHS)**

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

LHS= $2\sin x\cos x + \cos 2x - 1$ OR

$\sin 2x + 1 - 2\sin^2 x - 1$ OR

$2\sin x\cos x + 1 - 2\sin^2 x - 1$

$= 2\sin x\cos x - 2\sin^2 x$ ***A1***

$\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$ =RHS ***AG***

**METHOD 2 (RHS to LHS)**

RHS $= 2\sin x\cos x - 2\sin^2 x$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

$= \sin 2x + 1 - 2\sin^2 x - 1$ ***A1***

$= \sin 2x + \cos 2x - 1 =$ LHS ***AG***

***[2 marks]***

(b) attempt to factorise ***M1***

$(\cos x - \sin x)(2\sin x + 1) = 0$ ***A1***

recognition of $\cos x = \sin x \Rightarrow \dfrac{\sin x}{\cos x} = \tan x = 1$ OR $\sin x = -\dfrac{1}{2}$ ***(M1)***

one correct reference angle seen anywhere, accept degrees ***(A1)***

$\dfrac{\pi}{4}$ OR $\dfrac{\pi}{6}$ (accept $-\dfrac{\pi}{6}, \dfrac{7\pi}{6}$)

**Note*:*** This ***(M1)(A1)*** is independent of the previous ***M1A1.***

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{\pi}{4}, \dfrac{5\pi}{4}$ ***A2***

**Note:** Award ***A1*** for any two correct (radian) answers.
Award ***A1A0*** if additional values given with the four correct (radian) answers.

Award ***A1A0*** for four correct answers given in degrees.

***[6 marks]***

***Total [8 marks]***

**1.** (a) minor arc AB has length $r$ ***(A1)***

recognition that perimeter of shaded sector is $3r$ ***(A1)***

$3r = 12$

$r = 4$ ***A1***

***[3 marks]***

(b) **EITHER**

$\theta = 2\pi - \text{A}\hat{\text{O}}\text{B}\left(= 2\pi - 1\right)$ ***(M1)***

Area of non-shaded region $= \frac{1}{2}\left(2\pi - 1\right)\left(4^2\right)$ ***(A1)***

**OR**

area of circle - area of shaded sector ***(M1)***

$16\pi - \left(\frac{1}{2} \times 1 \times 4^2\right)$ ***(A1)***

**THEN**

$\text{area} = 16\pi - 8\ \left(= 8\left(2\pi - 1\right)\right)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**3.** (a) **METHOD 1**

correct substitution of $\cos^2 x = 1 - \sin^2 x$ ***A1***

$2(1 - \sin^2 x) + 5\sin x = 4$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

**METHOD 2**

correct substitution using double-angle identities ***A1***

$(2\cos^2 x - 1) + 5\sin x = 3$

$1 - 2\sin^2 x + 5\sin x = 3$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

***[1 mark]***

*continued…*

*Question 3 continued*

(b) **EITHER**

attempting to factorise ***M1***

$(2\sin x - 1)(\sin x - 2)$ ***A1***

**OR**

attempting to use the quadratic formula ***M1***

$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left( = \frac{5 \pm 3}{4} \right)$ ***A1***

**THEN**

$\sin x = \frac{1}{2}$ ***(A1)***

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ ***A1A1***

***[5 marks]***

***Total [6 marks]***

**6. METHOD 1**

attempt to use the cosine rule to find the value of $x$ ***(M1)***

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ ***A1***

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}\ \left(= 5\sqrt{2}\right)$ ***A1***

attempt to find $\sin\hat{C}$ (seen anywhere) ***(M1)***

$\sin^2\hat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin\hat{C} = \frac{\sqrt{7}}{4}$ ***(A1)***

**Note:** The marks for finding $\sin\hat{C}$ may be awarded independently of the first three marks for finding $x$.

correct substitution into the area formula using their value of $x$ (or $x^2$) and their value of $\sin\hat{C}$ ***(M1)***

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

*continued…*

*Question 6 continued*

**METHOD 2**

attempt to find the height, $h$, of the triangle in terms of $x$ ***(M1)***

$h^2+\left(\frac{3}{4}x\right)^2=x^2$ OR $h^2+\left(\frac{5}{4}x\right)^2=10^2$ OR $h=\frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either $h^2$ or $h$ ***(M1)***

$x^2-\left(\frac{3}{4}x\right)^2=10^2-\left(\frac{5}{4}x\right)^2$ OR $\sqrt{100-\frac{25}{16}x^2}=\frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2=50$ OR $x=\sqrt{50}\ \left(=5\sqrt{2}\right)$ ***A1***

correct substitution into the area formula using their value of $x$ (or $x^2$) ***(M1)***

$A=\frac{1}{2}\times 2\sqrt{50}\times\frac{\sqrt{7}}{4}\sqrt{50}$ OR $A=\frac{1}{2}\left(2\times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A=\frac{25\sqrt{7}}{2}$ ***A1***

***Total [7 marks]***

**Section B**

**4** . amplitude is $\frac{110}{2} = 55$ ***(A1)***

$a = -55$ ***A1***

$c = 65$ ***A1***

$\frac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ ***(M1)***

$b = \frac{\pi}{10}\left(= 0.314\right)$ ***A1***

***Total [5 marks]***

**8.** (a) $r = \frac{28}{\theta}$ ***A1***

***[1 mark]***

(b) recognising sum of area of sector and area of triangle required ***(M1)***

$\frac{1}{2}r^2\theta + \frac{1}{2}r \times r \times \sin(\pi - \theta)$ $\left( = \frac{r^2}{2}(\theta + \sin(\pi - \theta)) \right)$ ***A1***

$\sin(\pi - \theta) = \sin\theta$ (substitution seen anywhere) ***A1***

$\frac{1}{2}\left(\frac{28}{\theta}\right)^2 \theta + \frac{1}{2}\left(\frac{28}{\theta}\right)^2 \sin\theta$ OR $\frac{1}{2}\left(\frac{28}{\theta}\right)^2 (\theta + \sin\theta)$ ***A1***

$\text{area} = \frac{392}{\theta^2}(\theta + \sin\theta)$ ***AG***

***[4 marks]***

(c) $\frac{392}{\theta^2}(\theta + \sin\theta) = 460$ ***(M1)***

$\theta = 1.43917...$

$\theta = 1.44$ ***A1***

***[2 marks]***

(d) $\frac{\pi - (\pi - \theta)}{2}$ OR $\frac{\theta}{2}$ ***(M1)***

$\hat{DAE} = 0.719588...$

$\hat{DAE} = 0.720$ ***A1***

***[2 marks]***

(e) (i) $\hat{ABC} = 195 - 180 + 90$ ***(A1)***

$= 105°$ ***A1***

(ii) choosing sine rule ***(M1)***

$\frac{BC}{\sin\hat{DAE}} = \frac{800}{\sin 105}$ OR $\frac{BC}{\sin\hat{DAE}} = \frac{800}{\sin 1.83}$ ***A1***

$BC = 546\ (m)$ ***A1***

***[5 marks]***

***Total [14 marks]***

**7.** (a) $\tan 0.6 = \frac{h}{12}$ ***(M1)***

$8.20964...$

$8.21$ (m) ***A1***

***[2 marks]***

(b) $\tan B = \frac{8.2096...}{5}$ OR $\tan^{-1} 1.6419...$ ***(A1)***

$1.02375\ldots$

$1.02$ (radians) (accept $58.7°$) ***A1***

***[2 marks]***

(c) $x + 1.8 + 2.5 = 8.20964...$ (or equivalent) ***(A1)***

$3.90964\ldots$

$3.91$ (m) ***A1***

***[2 marks]***

(d) **METHOD 1**

recognition that blade length = amplitude, $p = \frac{\max - \min}{2}$ ***(M1)***

$p = 3.91$ ***A1***

centre of windmill = vertical shift, $q = \frac{\max + \min}{2}$ ***(M1)***

$q = 8.21$ ***A1***

**METHOD 2**

attempting to form two equations in terms of $p$ and $q$ ***(M1) (M1)***

$12.1192... = p\cos\left(\frac{3\pi}{10} \cdot 0\right) + q$, $4.3000... = p\cos\left(\frac{3\pi}{10} \cdot \frac{10}{3}\right) + q$

$p = 3.91$ ***A1***

$q = 8.21$ ***A1***

***[4 marks]***

(e) appropriate working towards finding the period ***(M1)***

$\text{period} = \dfrac{2\pi}{\frac{3\pi}{10}} \left(= 6.6666...\right)$

rotations per minute $= \dfrac{60}{\text{their period}}$ ***(M1)***

$n = 9$ (must be an integer) (accept $n = 10$, $n = 18$, $n = 19$) ***A1***

***[3 marks]***

***Total [13 marks]***

**6.** (a) **METHOD 1**

attempt to write all LHS terms with a common denominator of $x-1$ ***(M1)***

$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1}$ OR $\frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$

$=\frac{2x^2-2x-3x+3-6}{x-1}$ OR $\frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$ ***A1***

$=\frac{2x^2-5x-3}{x-1}$ ***AG***

**METHOD 2**

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder $-6$ ***A1***

$=2x-3-\frac{6}{x-1}$ as required. ***AG***

***[2 marks]***

*continued…*

*Question 6 continued.*

(b) consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$

**EITHER**

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ ***(M1)***

**Note:** Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

**OR**

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$

**THEN**

$\sin 2\theta = -\frac{1}{2}$ or $\sin 2\theta = 3$ ***(A1)***

***Note:*** Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

***Note:*** Award ***A0*** if additional answers given.

***[5 marks]***

***Total [7 marks]***

## Section B

**3. EITHER**

attempt to use cosine rule ***(M1)***

$12^2 + \text{AB}^2 - 2\times 12\times\cos 25^\text{o}\times\text{AB} = 7^2$ OR $\text{AB}^2 - 21.7513...\text{AB} + 95 = 0$ ***(A1)***

at least one correct value for $\text{AB}$ ***(A1)***

$\text{AB} = 6.05068...$ OR $\text{AB} = 15.7007...$

using their smaller value for $\text{AB}$ to find minimum perimeter ***(M1)***

$12 + 7 + 6.05068...$

**OR**

attempt to use sine rule ***(M1)***

$\frac{\sin \text{B}}{12} = \frac{\sin 25^\text{o}}{7}$ OR $\sin \text{B} = 0.724488...$ OR $\hat{\text{B}} = 133.573...^\text{o}$ OR $\hat{\text{B}} = 46.4263...^\text{o}$ ***(A1)***

at least one correct value for $C$ ***(A1)***

$\hat{C} = 21.4263...^\text{o}$ OR $\hat{C} = 108.573...^\text{o}$

using their acute value for $\hat{C}$ to find minimum perimeter ***(M1)***

$12 + 7 + \sqrt{12^2 + 7^2 - 2\times 12\times 7\cos 21.4263...^\circ}$ OR $12 + 7 + \frac{7\sin 21.4263...^\text{o}}{\sin 25^\text{o}}$

**THEN**

$25.0506\ldots$

minimum perimeter = $25.1$. ***A1***

***Total [5 marks]***

**5.** (a) attempt to find the area of either shaded region in terms of $r$ and $\theta$ ***(M1)***

> **Note*:*** Do not award ***M1*** if they have only copied from the booklet and not applied to the shaded area.

Area of segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ ***A1***

Area of triangle $= \frac{1}{2}r^2\sin(\pi - \theta)$ ***A1***

correct equation in terms of $\theta$ only ***(A1)***

$\theta - \sin\theta = \sin(\pi - \theta)$

$\theta - \sin\theta = \sin\theta$ ***A1***

$\theta = 2\sin\theta$ ***AG***

> **Note*:*** Award a maximum of ***M1A1A0A0A0*** if a candidate uses degrees (i.e., $\frac{1}{2}r^2\sin(180° - \theta)$), even if later work is correct.

> **Note*:*** If a candidate directly states that the area of the triangle is $\frac{1}{2}r^2\sin\theta$, award a maximum of ***M1A1A0A1A1***.

***[5 marks]***

(b) $\theta = 1.89549...$

$\theta = 1.90$ ***A1***

> **Note:** Award ***A0*** if there is more than one solution. Award ***A0*** for an answer in degrees.

***[1 mark]***

***Total [6 marks]***

**8.** (a) $12 = \frac{2\pi}{b}$ OR $b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG***

***[1 mark]***

(b) $a = \frac{6.8 - 2.2}{2}$ OR $a = \frac{\max - \min}{2}$ ***(M1)***

$= 2.3$ (m) ***A1***

***[2 marks]***

(c) $d = \frac{6.8 + 2.2}{2}$ OR $d = \frac{\max + \min}{2}$ ***(M1)***

$= 4.5$ (m) ***A1***

***[2 marks]***

*continue…*

*Question 8 continued.*

(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for $H$ ***(A1)***

$$6.8 = 2.3\sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$$

attempt to solve their equation ***(M1)***

$c = 1.5$ ***A1***

**METHOD 2**

using horizontal translation of $\frac{12}{4}$ ***(M1)***

$4.5 - c = 3$ ***(A1)***

$c = 1.5$ ***A1***

**METHOD 3**

$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right)$ ***(A1)***

attempts to solve their $H'(4.5) = 0$ for $c$ ***(M1)***

$$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$$

$c = 1.5$ ***A1***

***[3 marks]***

(e) attempt to find $H$ when $t = 12$ or $t = 0$, graphically or algebraically ***(M1)***

$H = 2.87365...$

$H = 2.87\,(\text{m})$ ***A1***

***[2 marks]***

*continue…*

*Question 8 continued.*

(f) attempt to solve $5 = 2.3\sin\left(\frac{\pi}{6}(t-1.5)\right)+4.5$ ***(M1)***

times are $t = 1.91852...$ and $t = 7.08147...$, $(t = 13.9185..., t = 19.0814...)$ ***(A1)***

total time is $2\times(7.081...-1.919...)$

$10.3258...$

$= 10.3$ (hours) ***A1***

**Note:** Accept $10$.

***[3 marks]***

***Total [13 marks]***

**1.** (a) valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

***[3 marks]***

(b) $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

$\text{area} = 4\sqrt{11}$ ***A1***

***[2 marks]***

***Total [5 marks]***

**6.** (a) attempting to use the change of base rule ***M1***

$\log_9(\cos 2x+2)=\dfrac{\log_3(\cos 2x+2)}{\log_3 9}$ ***A1***

$=\dfrac{1}{2}\log_3(\cos 2x+2)$ ***A1***

$=\log_3\sqrt{\cos 2x+2}$ ***AG***

***[3 marks]***

(b) $\log_3(2\sin x)=\log_3\sqrt{\cos 2x+2}$

$2\sin x=\sqrt{\cos 2x+2}$ ***M1***

$4\sin^2 x=\cos 2x+2$ (or equivalent) ***A1***

use of $\cos 2x=1-2\sin^2 x$ ***(M1)***

$6\sin^2 x=3$

$\sin x=(\pm)\dfrac{1}{\sqrt{2}}$ ***A1***

$x=\dfrac{\pi}{4}$ ***A1***

**Note:** Award ***A0*** if solutions other than $x=\dfrac{\pi}{4}$ are included.

***[5 marks]***

***Total [8 marks]***



## Section B

**2.** (a) ***METHOD 1***

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2 + 4^2 - 5^2}{2 \times 4 \times 4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

***[3 marks]***

***METHOD 2***

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

***[3 marks]***

(b) attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35...)$ ***A1***

$= 39.5\ (\text{cm}^2)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**7.** (a) $\frac{4.2}{60} \times 45$ ***A1***

$\mathrm{AB} = 3.15\ (\mathrm{km})$ ***A1***

***[2 marks]***

(b) (i) $66^\circ$ or $(180 - 114)$ ***A1***

$35 + 66$ ***A1***

$\mathrm{A\hat{B}C} = 101^\circ$ ***AG***

(ii) attempt to use cosine rule ***(M1)***

$\mathrm{AC}^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101^\circ$ (or equivalent) ***A1***

$\mathrm{AC} = 6.05\ (\mathrm{km})$ ***A1***

***[5 marks]***

(c) valid approach to find angle $\mathrm{BCA}$ ***(M1)***

*eg* sine rule

correct substitution into sine rule ***A1***

*eg* $\frac{\sin(\mathrm{B\hat{C}A})}{3.15} = \frac{\sin 101}{6.0507...}$

$\mathrm{B\hat{C}A} = 30.7^\circ$ ***A1***

***[3 marks]***

(d) $\mathrm{B\hat{A}C} = 48.267$ (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

*eg* $48.267 + 35$

bearing $= 83.3^\circ$ (accept $083^\circ$) ***A1***

***[3 marks]***

(e) attempt to use time $= \frac{\text{distance}}{\text{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or $0.065768\ \mathrm{km/min}$ ***(A1)***

$t = 93$ (minutes) ***A1***

***[3 marks]***

***Total [16 marks]***

**9.** (a) correct approach ***A1***

*eg* $\frac{\pi}{6} = \frac{2\pi}{period}$ (or equivalent)

$\text{period} = 12$ ***A1***

***[2 marks]***

(b) (i) valid approach ***(M1)***

*eg* $\frac{\max + \min}{2}$ $b = \max - \text{amplitude}$

$\frac{21.8 + 10.2}{2}$, or equivalent

$b = 16$ ***A1***

*continued…*

*Question 9 continued*

(ii) attempt to substitute into **their** function ***(M1)***

$5.8\sin\left(\frac{\pi}{6}(6+1)\right)+16$

$f(6)=13.1$ ***A1***

***[4 marks]***

(c) valid attempt to set up a system of equations ***(M1)***

two correct equations ***A1***

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q=2.5,\ p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q=15.1$

valid attempt to solve system ***(M1)***

$p=8.4;\ q=6.7$ ***A1A1***

***[5 marks]***

(d) attempt to use $|f(x)-g(x)|$ to find maximum difference ***(M1)***

$x=1.64$ ***A1***

***[2 marks]***

***Total [13 marks]***

**4.** (a) $(f \circ g)(x) = f(2x)$ ***(A1)***

$f(2x) = \sqrt{3}\sin 2x + \cos 2x$ ***A1***

***[2 marks]***

(b) $\sqrt{3}\sin 2x + \cos 2x = 2\cos 2x$

$\sqrt{3}\sin 2x = \cos 2x$

recognising to use $\tan$ or $\cot$ ***M1***

$\tan 2x = \frac{1}{\sqrt{3}}$ OR $\cot 2x = \sqrt{3}$ (values may be seen in right triangle) ***(A1)***

$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ ***A1A1***

| **Note:** Do not award the final ***A1*** if any additional solutions are seen.<br>Award ***A1A0*** for correct answers in degrees.<br>Award ***A0A0*** for correct answers in degrees with additional values. |
|---|

***[5 marks]***

***Total [7 marks]***

**5.** determines $\frac{\pi}{4}$ (or $45°$) as the first quadrant (reference) angle ***(A1)***

attempts to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}$ ***(M1)***

**Note:** Award ***M1*** for attempting to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}, \frac{7\pi}{4}(,...)$

$\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4} \Rightarrow x<0$ and so $\frac{\pi}{4}$ is rejected ***(R1)***

$\frac{x}{2}+\frac{\pi}{3}=2\pi-\frac{\pi}{4}\left(=\frac{7\pi}{4}\right)$ ***A1***

$x=\frac{17\pi}{6}$ (must be in radians) ***A1***

***[5 marks]***

**3.** (a) valid approach to find area of segment by finding area of sector – area of triangle ***(M1)***

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2\sin\theta$ ***(A1)***

area = $2\theta - 2\sin\theta$ ***A1***

***[3 marks]***

(b) **EITHER**

area of logo = area of rectangle – area of segments ***(M1)***

$5\times 4 - 2\times(2\theta - 2\sin\theta) = 13.4$ ***(A1)***

**OR**

area of one segment = $\frac{20-13.4}{2}$ $(=3.3)$ ***(M1)***

$2\theta - 2\sin\theta = 3.3$ ***(A1)***

**THEN**

$\theta = 2.35672...$

$\theta = 2.36$ (do not accept an answer in degrees) ***A1***

**Note:** Award ***(M1)(A1)A0*** if there is more than one solution.
Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

***[3 marks]***

***Total [6 marks]***

**7.** (a) attempt to use the distance formula to find $\mathrm{AV}$ ***(M1)***

$\sqrt{(1-(-1))^2+(5-1)^2+(0-6)^2}$

$=7.48331...$

$=7.48\ (\text{cm})\ \left(=\sqrt{56}\ \text{or}\ 2\sqrt{14}\right)$ ***A1***

***[2 marks]***

(b) **METHOD 1**

attempt to apply cosine rule OR sine rule to find $\mathrm{AB}$ ***(M1)***

$\left(\mathrm{AB}=\right)\ \sqrt{7.48...^2+7.48...^2-2\times 7.48...\times 7.48...\cos\left(40^\circ\right)}$ OR $\frac{\mathrm{AB}}{\sin 40^\circ}=\frac{\sqrt{56}}{\sin 70^\circ}$ ***(A1)***

$=5.11888...$

$=5.12\ (\text{cm})$ ***A1***

**METHOD 2**

Let $\mathrm{M}$ be the midpoint of $[\mathrm{AB}]$

attempt to apply right-angled trigonometry on triangle $\mathrm{AVM}$ ***(M1)***

$=2\times 7.48...\times\sin(20^\circ)$ ***(A1)***

$=5.11888...$

$=5.12\ (\text{cm})$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued*

(c) **METHOD 1**

equating volume of pyramid formula to $57.2$ ***(M1)***

$\frac{1}{3} \times 5.11...^2 \times h = 57.2$ ***(A1)***

$h = 6.54886...$

$h = 6.55 \text{ (cm)}$ ***A1***

**METHOD 2**

Let $\text{M}$ be the midpoint of $[\text{AB}]$

$\text{AV}^2 = \text{AM}^2 + \text{MX}^2 + \text{XV}^2$ ***(M1)***

$\Rightarrow \text{XV} = \sqrt{7.48...^2 - \left(\frac{5.11...}{2}\right)^2 - \left(\frac{5.11...}{2}\right)^2}$ ***(A1)***

$h = 6.54886...$

$h = 6.55 \text{ (cm)}$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued*

(d) $V = x \times 2x \times y = 57.2$ ***(A1)***

$S = 2\left(2x^2 + xy + 2xy\right)$ ***A1***

**Note:** Condone use of $A$.

attempt to substitute $y = \frac{57.2}{2x^2}$ into their expression for surface area ***(M1)***

$\left(S(x) =\right)\ 4x^2 + 6x\left(\frac{57.2}{2x^2}\right)$

**EITHER**

attempt to find minimum turning point on graph of area function ***(M1)***

**OR**

$\frac{\mathrm{d}S}{\mathrm{d}x} = 8x - 171.6x^{-2} = 0$ OR $x = 2.77849...$ ***(M1)***

**THEN**

$92.6401...$

minimum surface area $= 92.6\ (\mathrm{cm}^2)$ ***A1***

***[5 marks]***

***Total [13 marks]***

**1.** (a) **EITHER**

uses the cosine rule ***(M1)***

$\mathrm{AB}^2 = 5^2 + 5^2 - 2\times 5\times 5\times \cos 1.9$ ***(A1)***

**OR**

uses right-angled trigonometry ***(M1)***

$\frac{\frac{\mathrm{AB}}{2}}{5} = \sin 0.95$ ***(A1)***

**OR**

uses the sine rule ***(M1)***

$\alpha = \frac{1}{2}(\pi - 1.9)\left(= 0.6207...\right)$

$\frac{\mathrm{AB}}{\sin 1.9} = \frac{5}{\sin 0.6207...}$ ***(A1)***

**THEN**

$\mathrm{AB} = 8.13415...$

$\mathrm{AB} = 8.13$ (m) ***A1***

***[3 marks]***

*continued…*

*Question 1 continued*

(b) let the shaded area be $A$

**METHOD 1**

attempt at finding reflex angle ***(M1)***

$\text{AÔB} = 2\pi - 1.9 \ (= 4.3831...)$

substitution into area formula ***(A1)***

$A = \frac{1}{2} \times 5^2 \times 4.3831...$ OR $\left(\frac{2\pi - 1.9}{2\pi}\right) \times \pi\left(5^2\right)$

$= 54.7898...$

$= 54.8$ (m²) ***A1***

**METHOD 2**

let the area of the circle be $A_C$ and the area of the unshaded sector be $A_U$

$A = A_C - A_U$ ***(M1)***

$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \ \left(= 78.5398... - 23.75\right)$ ***(A1)***

$= 54.7898...$

$= 54.8$ (m²) ***A1***

***[3 marks]***

***Total [6 marks]***

**7.** (a) $\hat{ABC} = 27°$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$175^2 + 230^2 - 2(175)(230)\cos 27°$ ***(A1)***

$108.62308\ldots$

$AC = 109\ (\text{m})$ ***A1***

***[4 marks]***

(b) correct substitution into area formula ***(A1)***

$\frac{1}{2} \times 175 \times 230 \times \sin 27°$

$9136.55...$

$\text{area} = 9140\left(\text{m}^2\right)$ ***A1***

***[2 marks]***

(c) attempt to substitute into sine rule or cosine rule ***(M1)***

$\frac{\sin 27°}{108.623...} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623...)^2 + 230^2 - 175^2}{2 \times 108.623... \times 230}$ ***(A1)***

$47.0049\ldots$

$C\hat{A}B = 47.0°$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued*

(d) **METHOD 1**

recognizing that for areas to be equal, $\mathrm{AD}=\mathrm{DC}$ ***(M1)***

$\mathrm{AD}=\frac{1}{2}\mathrm{AC}=54.3115...$ ***A1***

attempt to substitute into cosine rule to find $\mathrm{BD}$ ***(M1)***

correct substitution into cosine rule ***(A1)***

$\mathrm{BD}^2=230^2+54.3115^2-2(230)(54.3115)\cos 47.0049°$

$\mathrm{BD}=197.009...$

$\mathrm{BD}=197(\mathrm{m})$ ***A1***

***[5 marks]***

*continued…*

*Question 7 continued*

**METHOD 2**

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2}\times \text{BD}\times 230\times \sin x°$ and $\frac{1}{2}\times \text{BD}\times 175\times \sin(27-x)°$ OR

$\frac{1}{2}\times \text{BD}\times 230\times \sin(27-x)°$ and $\frac{1}{2}\times \text{BD}\times 175\times \sin x°$

correct equation in terms of $x$ ***(A1)***

$175\sin(27-x)=230\sin x$ or $175\sin x=230\sin(27-x)$

$x=11.6326...$ or $x=15.3673...$ ***(A1)***

substituting their value of $x$ into equation to solve for BD ***(M1)***

$\frac{1}{2}\times \text{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times \text{BD}\times 175\times \sin 15.3673...$ or

$\frac{1}{2}\times \text{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times 9136.55...$

$\text{BD}=197(\text{m})$ ***A1***

***[5 marks]***

***Total [14 marks]***

**3.** (a) (i) attempt to find midpoint of A and B ***(M1)***

centre $(-1, 3, -2)$ (accept vector notation and/or missing brackets) ***A1***

(ii) attempt to find AB or half of AB or distance between the centre and A (or B) ***(M1)***

$\frac{\sqrt{4^2+2^2+4^2}}{2}$ or $\sqrt{2^2+1^2+2^2}$

$= 3$ ***A1***

***[4 marks]***

(b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) ***(M1)***

$\sqrt{0^2+4^2+2^2}$ OR $\sqrt{(\text{their slant height})^2-(\text{their radius})^2}$

$= \sqrt{20}\left(= 2\sqrt{5}\right)$ ***(A1)***

Volume $= \frac{1}{3}\pi 3^2\sqrt{20}$

$= 3\pi\sqrt{20}\left(= 6\pi\sqrt{5}\right)$ ***A1***

***[3 marks]***

***Total [7 marks]***

**9.** (a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function ***(M1)***

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$ OR $1 - \sin^2 x - 3\sin^2 x = 0$ OR $\cos^2 x - 3\left(1 - \cos^2 x\right) = 0$

OR $\cos 2x - 1 + \cos 2x = 0$

correct equation ***(A1)***

$\tan^2 x = \frac{1}{3}$ OR $\cos^2 x = \frac{3}{4}$ OR $\sin^2 x = \frac{1}{4}$ OR $\cos 2x = \frac{1}{2}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ OR $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\sin x = (\pm)\frac{1}{2}$ OR $2x = \frac{\pi}{3}\left(, \frac{5\pi}{3}\right)$ ***(A1)***

$x = \frac{\pi}{6},\ x = \frac{5\pi}{6}$ ***A1A1***

**Note:** Award ***M1A1A0A1A0*** for candidates who omit the $\pm$ (for tan or cos) and give only $x = \frac{\pi}{6}$.

Award ***M1A1A0A0A0*** for candidates who omit the $\pm$ (for tan or cos) and give only $x = 30°$.

Award ***M1A1A1A1A0*** for candidates who give both answers in degrees.

Award ***M1A1A1A1A0*** for candidates who give both correct answers in radians, but who include additional solutions outside the domain.

Award a maximum of ***M1A0A0A1A1*** for correct answers with no working.

***[5 marks]***

*continued…*

*Question 9 continued*

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

At least 2 correct $x$-coordinates (may be seen in coordinates) ***(A1)***

$x = 0,\ x = \frac{\pi}{2},\ x = \pi$

**Note:** Accept additional correct solutions outside the domain.

Award ***A0*** if any additional incorrect solutions are given.

correct coordinates (may be seen in graph for part (c)) ***A1A1A1***

$(0,1),\ (\pi,1),\ \left(\frac{\pi}{2},-3\right)$

**Note:** Award a maximum of ***M1A1A1A1A0*** if any additional solutions are given.

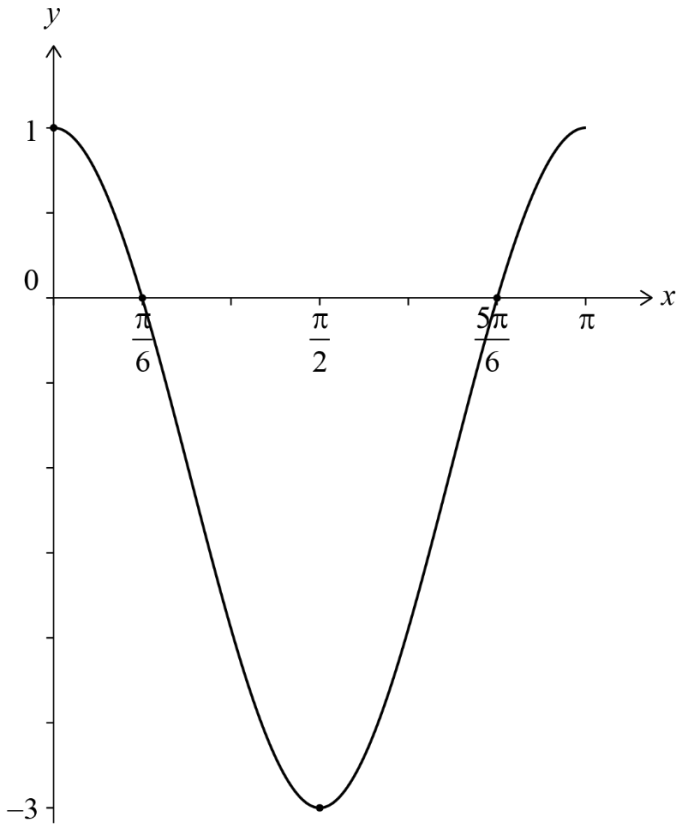
**Note:** If candidates do not find at least two correct x-coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as ***M1A0A0A1A0.***

***[7 marks]***

*continued…*

*Question 9 continued*

(c)




**Note:** In this question do not award follow through from incorrect values found in earlier parts.

approximately correct smooth curve with minimum at $\left(\frac{\pi}{2}, -3\right)$ ***A1***

**Note:** If candidates do not gain this mark then award no further marks.

endpoints at $(0,1)$, $(\pi,1)$, $x$-intercepts at $\frac{\pi}{6}$, $\frac{5\pi}{6}$ ***A1***

correct concavity clearly shown at $(0,1)$ and $(\pi,1)$ ***A1***

**Note:** The final two marks may be awarded independently of each other.

***[3 marks]***

***Total [15 marks]***

**2.** (a) attempt to use sine rule ***(M1)***

$\frac{24}{\sin 113^\circ} = \frac{17}{\sin \hat{BAC}}$ OR $\left(\sin \hat{BAC} =\right) 0.652024...$ ***(A1)***

$40.6943\ldots$

$\hat{BAC} = 40.7^\circ$ ***A1***

***[3 marks]***

(b) **METHOD 1** (cosine rule with $A\hat{B}C$ or $B\hat{A}C$)

attempt to use the cosine rule ***(M1)***

$24^2 = AB^2 + 17^2 - 2 \cdot 17 \cdot AB \cdot \cos 113°$ $\left(AB^2 + 13.2848...AB - 287 = 0\right)$ OR

$17^2 = AB^2 + 24^2 - 2 \cdot 24 \cdot AB \cdot \cos 40.6943...°$ $\left(AB^2 - 36.3935...AB + 287 = 0\right)$ ***(A1)***

$11.5543\ldots$

$AB = 11.6$ ***A1***

**METHOD 2** (cosine rule with $B\hat{C}A$)

attempt to use cosine rule ***(M1)***

correct substitution ***(A1)***

$AB^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos 26.3056...°$ OR $AB^2 = 133.502...$

$11.5543\ldots$

$AB = 11.6$ ***A1***

*continued…*

*Question 2 continued*

**METHOD 3** (sine rule)

attempt to use sine rule ***(M1)***

correct substitution ***(A1)***

$\frac{\text{AB}}{\sin 26.3056...^\circ} = \frac{24}{\sin 113^\circ} = \frac{17}{\sin 40.6943...^\circ}$ OR $\text{AB} = \frac{24 \cdot \sin\left(180^\circ - 113^\circ - 40.6943...^\circ\right)}{\sin 113^\circ}$

$11.5543\ldots$

$\text{AB} = 11.6$ ***A1***

***[3 marks]***

***Total [6 marks]***

**8.**

**Note:** In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

(a) attempt to use right angled trigonometry or sine rule to find $\mathrm{AE}$ in terms of $r$ and $\alpha$ ***(M1)***

$\tan\alpha = \frac{r}{\mathrm{AE}}$ OR $\frac{\mathrm{AE}}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{r}{\sin\alpha}$

$\mathrm{AE} = \frac{r}{\tan\alpha}$ OR $\mathrm{AE} = \frac{r\sin\left(\frac{\pi}{2}-\alpha\right)}{\sin\alpha}$ OR $\mathrm{AE} = \frac{r\cos\alpha}{\sin\alpha}$ ***A1***

valid approach to find the area of $\mathrm{ADOE}$ ***(M1)***

2x area of triangle AOE OR area of triangle AED + area of triangle OED OR $\mathrm{OE}\times\mathrm{AE}$

Area $\mathrm{ADOE}$ = $2\left(\frac{1}{2}\cdot\frac{r}{\tan\alpha}\cdot r\right)$ OR $r\times\mathrm{AE}$ ***A1***

Area $\mathrm{ADOE} = \frac{r^2}{\tan\alpha}$ ***AG***

***[4 marks]***

*continued…*

*Question 8 continued*

(b) (i) recognizing that the sum of the angles of a kite is $2\pi$ ***(M1)***

$\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 2\pi$ OR $2\alpha + 2 \cdot \frac{\pi}{2} + \mathrm{D\hat{O}E} = 2\pi$

$\mathrm{D\hat{O}E} = \pi - 2\alpha$ ***A1***

**Note:** Award ***M1A0*** if candidate uses degrees (i.e. $\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 360°$ or $2\alpha + 2 \cdot \frac{\pi}{2} + \mathrm{D\hat{O}E} = 360°$ ) and obtains $\mathrm{D\hat{O}E} = 180° - 2\alpha$ .

(ii) valid approach to find the area of $\mathrm{R}$ ***(M1)***

area of kite – area of sector OR 2(area of triangle $\mathrm{AOE} - 0.5$ area of sector OED)

Area of sector = $\frac{1}{2}r^2.\mathrm{D\hat{O}E}\left(= \frac{1}{2}r^2(\pi - 2\alpha)\right)$ seen anywhere ***(A1)***

Area of R = $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha)$ ***A1***

**Note:** Accept $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2.\mathrm{D\hat{O}E}$ .

***[5 marks]***

*continued…*

*Question 8 continued*

(c) equating their area formula to $\pi r^2$ ***(M1)***

$$\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha) = \pi r^2$$

correct equation in terms of $\alpha$ ***A1***

$$\frac{1}{\tan\alpha} - \frac{1}{2}(\pi - 2\alpha) = \pi$$

valid approach to solve the equation ***(M1)***

$\alpha = 0.218979...$

$\alpha = 0.219$ ***A1***

***[4 marks]***

***Total [13 marks]***

**4.** (a) $1-2\sin^2 x = \sin x$ ***A1***

$2\sin^2 x + \sin x - 1 = 0$ ***AG***

***[1 mark]***

(b) valid attempt to solve quadratic ***(M1)***

$(2\sin x - 1)(\sin x + 1)$ OR $\frac{-1 \pm \sqrt{1-4(2)(-1)}}{2(2)}$

recognition to solve for $\sin x$ ***(M1)***

$\sin x = \frac{1}{2}$ OR $\sin x = -1$

any correct solution from $\sin x = -1$ ***A1***

any correct solution from $\sin x = \frac{1}{2}$ ***A1***

**Note:** The previous two marks may be awarded for degree or radian values, irrespective of domain.

$x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ ***A1***

**Note:** If no working shown, award no marks for a final value(s).
Award ***A0*** for $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ if additional values also given.

***[5 marks]***

***Total [6 marks]***

**6.** (a) recognition that period is $4m$ OR substitution of a point on $f$ (except the origin) ***(M1)***

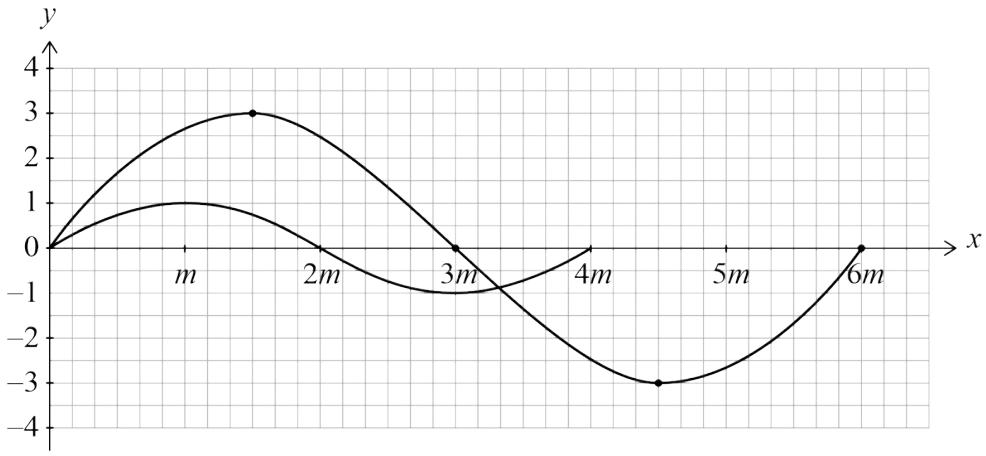
$4m = \frac{2\pi}{q}$ OR $1 = \sin qm$

$m = \frac{\pi}{2q}$ ***A1***

***[2 marks]***

(b) horizontal scale factor is $\frac{3}{2}$ (seen anywhere) ***(A1)***

**Note:** This ***(A1)*** may be earned by seeing a period of $6m$, half period of $3m$ or the correct $x$-coordinate of the maximum/minimum point.




***A1A1A1***

**Note:** Curve must be an approximate sinusoidal shape (sine or cosine).
Only in this case, award the following:
***A1*** for correct amplitude.
***A1*** for correct domain.
***A1*** for correct max and min points **and** correct $x$-intercepts.

***[4 marks]***

***Total [6 marks]***

## Section B

**1.** (a) attempts to find perimeter ***(M1)***

$\text{arc} + 2 \times \text{radius}$ OR $10 + 4 + 4$

$= 18 \text{ (cm)}$ ***A1***

***[2 marks]***

(b) $10 = 4\theta$ ***(A1)***

$\theta = \frac{10}{4} \left( = \frac{5}{2}, 2.5 \right)$ ***A1***

***[2 marks]***

(c) area $= \frac{1}{2}\left(\frac{10}{4}\right)\left(4^2\right) \left(= 1.25 \times 16\right)$ ***(A1)***

$= 20 \text{ (cm}^2\text{)}$ ***A1***

***[2 marks]***

***Total [6 marks]***

**8.** (a) (i) attempt to use Pythagoras ***(M1)***

$\sin^2\theta+\left(\frac{2}{3}\right)^2=1$ OR $x^2+2^2=3^2$ OR right triangle with side 2 and hypotenuse 3

$\sin\theta=\frac{\sqrt{5}}{3}$ ***A1***

(ii) attempt to substitute into double-angle identity using their value of $\sin\theta$ ***(M1)***

$\sin 2\theta=2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\sin 2\theta=\frac{4\sqrt{5}}{9}$ ***A1***

***[4 marks]***

(b) **METHOD 1 (using values from part (a))**

$\frac{b}{\sin\theta}=\frac{a}{\sin 2\theta}$

attempt to use sine rule with their values from part (a) ***(M1)***

$\frac{b}{\left(\frac{\sqrt{5}}{3}\right)}=\frac{a}{\left(\frac{4\sqrt{5}}{9}\right)}$ OR $\frac{\left(\frac{\sqrt{5}}{3}\right)}{b}=\frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$

correct working that leads to ***AG*** ***A1***

$\frac{\sqrt{5}}{3}a=\frac{4\sqrt{5}}{9}b$ OR $\frac{3b}{\sqrt{5}}=\frac{9a}{4\sqrt{5}}$ OR $\frac{a}{3}=\frac{4b}{9}$ (or equivalent)

$b=\frac{3a}{4}$ ***AG***

*continued…*

*Question 8 continued*

**METHOD 2 (double-angle identity)**

$$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$$

using double-angle identity ***(A1)***

$$\frac{b}{\sin\theta} = \frac{a}{2\sin\theta\cos\theta} \text{ OR } b = \frac{a\sin\theta}{2\sin\theta\cos\theta} \text{ OR } b = \frac{a}{2\cos\theta}$$

correct working (involving substituting $\cos\theta = \frac{2}{3}$) that leads to ***AG*** ***A1***

$$b = \frac{a\sin\theta}{2\sin\theta\left(\frac{2}{3}\right)} \text{ OR } b = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)} \text{ OR } b = \frac{a}{2\left(\frac{2}{3}\right)} \text{ (or equivalent)}$$

$$b = \frac{3a}{4}$$ ***AG***

***[2 marks]***

(c) **METHOD 1 (using supplementary angles)**

recognizing $\mathrm{C\hat{A}D}$ and $\mathrm{B\hat{A}C}$ are supplementary ***(M1)***

recognizing supplementary angles have the same sine value ***(A1)***

$$\sin \mathrm{C\hat{A}D} = \sin 2\theta$$

$$\sin \mathrm{C\hat{A}D} = \frac{4\sqrt{5}}{9}$$ ***A1***

*continued…*

*Question 8 continued*

**METHOD 2 (using sine rule)**

recognizing $\mathrm{CD} = a$ ***(M1)***

$$\frac{a}{\sin \mathrm{C\hat{A}D}} = \frac{b}{\sin\theta}$$

correct substitution of $\sin\theta = \frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule ***(A1)***

$$\frac{a}{\sin \mathrm{C\hat{A}D}} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)} \quad \textbf{OR} \quad \sin \mathrm{C\hat{A}D} = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)} \quad \text{(or equivalent)}$$

$$\sin \mathrm{C\hat{A}D} = \frac{4\sqrt{5}}{9}$$ ***A1***

***[3 marks]***

(d) **METHOD 1 (using $\mathrm{C\hat{A}D}$ in area formula)**

recognizing $\mathrm{D\hat{C}A} = \theta$ ***(A1)***

recognizing $\mathrm{AD} = b \left(= \frac{3a}{4}\right)$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \mathrm{C\hat{A}D}$) ***(M1)***

$$\text{area} = \frac{1}{2}(b)(b)\left(\frac{4\sqrt{5}}{9}\right) \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin 2\theta \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin \mathrm{C\hat{A}D}$$

correct substitution in terms of $a$ ***(A1)***

$$\text{area} = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$$

$$\text{area} = \frac{\sqrt{5}\,a^2}{8}$$ ***A1***

*continued…*

*Question 8 continued*

**METHOD 2 (using $\hat{ACD}$ or $\hat{ADC}$ in area formula)**

recognizing $CD = a$ ***(A1)***

recognizing $AD = b \left( = \frac{3a}{4} \right)$ and/or $D\hat{C}A = \theta$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin A\hat{D}C$ or $\sin A\hat{C}D$ ) ***(M1)***

$\text{area} = \frac{1}{2}(a)(b)\left(\frac{\sqrt{5}}{3}\right)$ OR $\text{area} = \frac{1}{2}(a)(b)\sin\theta$ OR $\text{area} = \frac{1}{2}(a)(b)\sin A\hat{D}C$

OR $\text{area} = \frac{1}{2}(a)(b)\sin A\hat{C}D$

correct substitution in terms of $a$ ***(A1)***

$\text{area} = \frac{1}{2}(a)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\text{area} = \frac{\sqrt{5}\,a^2}{8}$ ***A1***

***[5 marks]***

***Total [14 marks]***

**1.** (a) Let $\mathrm{N}$ be North

$\mathrm{N\hat{J}D} = 34°$ OR $\mathrm{D\hat{J}L} = 56°$ (must be labelled or indicated in diagram): ***(A1)***

$\mathrm{J\hat{D}L} = 99(°)$ ***A1***

**Note:** Accept $\frac{11\pi}{20}, 1.73$ (radians).

***[2 marks]***

(b) attempt to apply the sine rule ***(M1)***

$\frac{\mathrm{DL}}{\sin 56°} = \frac{500}{\sin 99°}$ OR $\frac{\mathrm{DL}}{\sin 0.977384...} = \frac{500}{\sin 1.72787...}$ ***(A1)***

$419.685...$

$\mathrm{DL} = 420 \ (\mathrm{km})$ ***A1***

**Note:** Award ***M1A1A0*** for $261 \ (\mathrm{km})$ from use of degrees with GDC set in radians (with or without working).

***[3 marks]***

***Total [5 marks]***

**9.** (a) recognition that $45 = 10 + 10 + \text{arc length}$ ***(M1)***

arc length $= 25$ (cm) ***(A1)***

$25 = 12\theta$ ***A1***

$\theta = 2.08$ correct to 3 significant figures ***AG***

***[3 marks]***

*continued…*

*Question 9 continued*

(b)

**Note:** There are many different ways to dissect the cross-section to determine its area. In all approaches, candidates will need to find $w$ or $\frac{w}{2}$. Award the first three marks for work seen anywhere.

**EITHER**

evidence of using the cosine rule OR sine rule ***(M1)***

$w^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cos(2.08)$ OR $\frac{w}{\sin(2.08)} = \frac{12}{\sin(0.530796...)}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

**OR**

using trig ratios in a right triangle with angle $\frac{2.08}{2}$ and side length $\frac{w}{2}$ ***(M1)***

$\sin\left(\frac{2.08}{2}\right) = \frac{\frac{w}{2}}{12}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

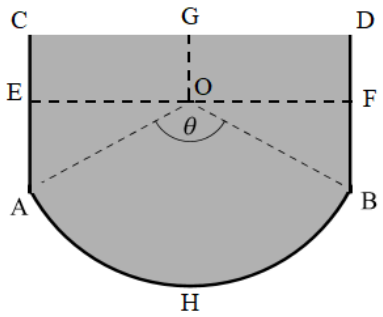
**Note:** Accept $w = 20.7179\ldots$ from use of $\frac{\theta}{2} = \frac{25}{24}$.

*continued…*

*Question 9 continued*

**THEN**

Let the points A, B, C, D, E, F, G, H lie on the figure as follows:




**EITHER**

(segment AHB =) sector OAB – triangle OAB ***(M1)***

$= \frac{1}{2}\times 12^2 \times 2.08 - \frac{1}{2}\times 12^2 \times \sin 2.08 \left(= 149.76 - 62.8655... = 86.8944...\right)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

sector OAB – triangle OAB + rectangle CDBA

$= 86.8944... + 10w \left(= 86.8944... + 206.977...\right)$

**Note:** Use of $\theta = \frac{25}{12}$ throughout leads to segment OAB $= 87.2517...$ and cross-sectional area $= 87.2517... + 207.179...$.

*continued…*

*Question 9 continued*

**OR**

trapezium $\mathrm{CGOA}$ (= rectangle $\mathrm{CGOE}$ + triangle $\mathrm{EOA}$) ***(M1)***

$= \frac{1}{2} \times \left(10 + \left(10 - 12\cos(1.04)\right)\right) \times \frac{20.6977...}{2}$ $(= 72.0557)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 × trapezium $\mathrm{CGOA}$ + sector $\mathrm{OAB}$

$= 2(72.0557...) + \frac{1}{2} \times 12^2 \times 2.08 \left(= 144.111... + 149.76\right)$

**Note:** Use of $\theta = \frac{25}{12}$ leads to area of trapezium $\mathrm{CGOA} = 72.2154...$ and cross-sectional area $= 144.430... + 150$.

**OR**

2 x area of trapezium $\mathrm{CGOA}$ (= area of rectangle $\mathrm{CDFE}$ + 2 x triangle EOA) ***(M1)***

$20.6977... \times \left(10 - 12\cos(1.04)\right) + 2 \times \frac{1}{2} \times 12\cos(1.04) \times 12\sin(1.04)$ ***(A1)***

$\left(= 81.2458... + 62.8655...\right)$

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 x trapezium $\mathrm{CGOA}$ + sector $\mathrm{OAB}$

$= 144.111... + \frac{1}{2} \times 12^2 \times 2.08 \ \left(= 144.111... + 149.76\right)$

**Note:** Use of $\theta = \frac{25}{12}$ leads to 2 x area of trapezium $\mathrm{CGOA} = 144.430...$ and cross-sectional area $= 144.430... + 150$.

*continued…*

*Question 9 continued*

**THEN**

area of cross-section $= 293.871...$ ($294.430...$ from exact answer)

$= 294$ (cm$^2$) ***A1***

***[7 marks]***

*continued…*

*Question 9 continued*

(c) **METHOD 1**

volume of gutter $=176323$ OR $176658$ (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

$\int_0^{60} R'(t)\,\mathrm{d}t \left(=\frac{250}{2\pi}\sin\left(\frac{2\pi\times 60}{5}\right)+3000\times 60\right)$ ***(A1)***

**Note:** Accept any 60 second interval or any interval which is a multiple of 5 seconds (one period) scaled up to 60 seconds e.g. $12\int_0^5 R'(t)\,\mathrm{d}t$.

rainfall over 60 seconds $=180000$ ($\mathrm{cm}^3$) ***A1***

the gutter will overflow because the rainfall > gutter volume ***A1***

**METHOD 2**

volume of gutter $=176323$ OR $176658$ (OR $600\times$ their area) (seen anywhere) ***A1***

recognition that cosine has a minimum value of -1 ***(M1)***

$R'(t)\geq -1\times 50+3000\left(\mathrm{cm}^3\mathrm{s}^{-1}\right)$ ***(A1)***

rainfall over 60 seconds $\geq 177000$ ***(A1)***

the gutter will overflow because the rainfall > gutter volume ***A1***

*continued…*

*Question 9 continued*

**METHOD 3**

volume of gutter $=176323$ OR $176658$ (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

attempt to solve $60 > 58.8$ OR $\int_0^T R'(t)\,\mathrm{d}t = 176658$ ***(M1)***

time to reach overflow point = $58.7875...$ OR $58.8990...$ ***A1***

the gutter will overflow because $60 > 58.8$ OR $60 > 58.9$ ***A1***

***[5 marks]***

***Total [15 marks]***

**2.** (a) attempt to substitute into cosine rule ***(M1)***

$154^2 = 150^2 + 90^2 - 2(150)(90)\cos \text{A}\hat{\text{P}}\text{B}$ OR $\cos \text{A}\hat{\text{P}}\text{B} = \dfrac{150^2 + 90^2 - 154^2}{2(150)(90)}$ ***(A1)***

$\text{A}\hat{\text{P}}\text{B} = 75.2286...°$ OR $1.31298...$ radians

$\text{A}\hat{\text{P}}\text{B} = 75.2°$ OR $1.31$ radians ***A1***

***[3 marks]***

(b) valid approach to find $\theta$ ***(M1)***

$\theta = \dfrac{180° - \text{A}\hat{\text{P}}\text{B}}{2}$ OR $\theta = \dfrac{180° - 75.2286...°}{2}$ $(= 52.3856...)$ OR

$\theta = \dfrac{\pi - 1.31298...}{2} (= 0.914302...)$

valid approach to express $h$ in terms of $\theta$ ***(M1)***

$\sin\theta = \dfrac{h}{150}$ OR $h = 150\sin 52.3856...°$

$h = 118.820...$

$h = 119$ (m) ***A1***

***[3 marks]***

***Total [6 marks]***

**8.** (a) $7.8 = \frac{2\pi}{\text{period}}$ ***(M1)***

$\frac{2\pi}{7.8} = 0.805536...$

$\text{period} = 0.806\left(= \frac{10\pi}{39}\right)$ ***A1***

***[2 marks]***

(b) **METHOD 1**

(i) $\text{amplitude} = \frac{\max - \min}{2}$ ***(M1)***

$\frac{1.8-1}{2}$

$a = -0.4$ ***A1***

(ii) $b = 1.4$ ***A1***

**METHOD 2**

attempt to form two simultaneous equations in $a$ and $b$ ***(M1)***

$H(0) = 1 \Rightarrow a + b = 1$, $H\left(\frac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$

$a = -0.4, b = 1.4$ ***A1A1***

***[3 marks]***

*continued…*

*Question 8 continued*

(c) **EITHER**

$\frac{5}{\text{period}} = 6.207... < 6\frac{1}{2}$ ***(A1)***

**OR**

consideration of number of maximums on graph in first 5 seconds ***(A1)***

**OR**

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ ***(A1)***

**THEN**

6 times ***A1***

***[2 marks]***

(d) recognizing that $H(t) = 1.5$ ***(M1)***

$-0.4\cos(7.8t) + 1.4 = 1.5$

$0.233779...$

$t = 0.234$ (seconds) ***A1***

***[2 marks]***

*continued…*

*Question 8 continued*

(e) finding second time height is 1.5 metres ***(M1)***

$t = 0.571757...$

in each period, height is greater than 1.5 metres for 0.337978… seconds ***(A1)***

**Note:** Award ***(M1)(A1)*** for total time $2.02787...$ seen.

multiplying their value by 6 and dividing by 5 ***(M1)***

$\frac{0.337978... \times 6}{5}$ OR $\frac{2.02787...}{5}$

$= 0.405574...$

$\mathrm{P}(\text{height is greater than } 1.5 \text{ m}) = 0.406$ ***A1***

***[4 marks]***

***Total [13 marks]***

2. (a) $BV = \sqrt{(8-4)^2+(6-3)^2+(0-10)^2}$ ***(A1)***

$=11.1803...$

$=11.2\ (=\sqrt{125}=5\sqrt{5})$ ***A1***

***[2 marks]***

(b) **METHOD 1**

$BV = VC$ AND $BC = 6$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle $BVC$ for any angle ***(M1)***

**Note:** Recognition must be shown in context either in terms of labelled sides or side lengths.

$\cos B\hat{V}C = \frac{11.1...^2+11.1...^2-6^2}{2\times 11.1...\times 11.1...}$ OR

$6^2 = 11.1....^2+11.1...^2-2\times 11.1...\times 11.1...\cos B\hat{V}C$ ***(A1)***

$B\hat{V}C = 0.543314...$

$B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept $31.1^\circ$) ***A1***

*continued…*

*Question 2 continued*

**METHOD 2**

let M be the midpoint of BC

$\mathrm{BM} = 3$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle $\mathrm{BMV}$ or $\mathrm{CMV}$ ***(M1)***

$\arcsin\frac{3}{\sqrt{125}}$ OR $\frac{\pi}{2} - \arccos\frac{3}{\sqrt{125}}$ OR $0.271657\ldots$ ***(A1)***

$\mathrm{B\hat{V}C} = 0.543314...$

$\mathrm{B\hat{V}C} = 0.543$ (0.542 from 3 sf) (accept $31.1^\circ$) ***A1***

***[4 marks]***

***Total [6 marks]***

**4.** (a) use of sector area formula to find area of at least one sector ***(M1)***

$\frac{1}{2}\times 4.8\times 100-\frac{1}{2}\times 4.8\times r^2$ OR $10^2\pi-\pi r^2-\left(\frac{1}{2}10^2\times(2\pi-4.8)-\frac{1}{2}\times(2\pi-4.8)r^2\right)$ ***A1***

$(\text{area})=240-2.4r^2$ ***AG***

***[2 marks]***

(b) (i) $240-2.4r^2=176$ ***(A1)***

$r=5.16397...$

$=5.16$ (cm) $\left(\frac{4\sqrt{15}}{3}\text{ exact}\right)$ ***A1***

(ii) $10\times 4.8$ OR $5.16...\times 4.8$ ***(A1)***

substituting their value of $r$ into $10\times 4.8+r\times 4.8+2(10-r)$ (or equivalent) ***(M1)***

Perimeter $=10\times 4.8+5.16...\times 4.8+2(10-5.16...)$ $(=48+24.7870...+9.67204...)$

$=82.4591\ldots$

$=82.5$ (cm) (82.4 from 3 sf) ***A1***

***[5 marks]***

***Total [7 marks]***

**2.** (a) $\mathrm{BV} = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ ***(A1)***

$= 10.2956...$

$= 10.3\ (=\sqrt{106})$ ***A1***

***[2 marks]***

(b) **METHOD 1**

$\mathrm{BV} = \mathrm{VC}$ AND $\mathrm{BC} = 8$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle $\mathrm{BVC}$ for any angle ***(M1)***

**Note:** Recognition must be shown in context either in terms of labelled sides or in side lengths.

$\cos \mathrm{B\hat{V}C} = \dfrac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...}$ OR

$8^2 = 10.2....^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \mathrm{B\hat{V}C}$ ***(A1)***

$\mathrm{B\hat{V}C} = 0.798037...$

$\mathrm{B\hat{V}C} = 0.798$ (accept $45.7^\circ$) ***A1***

**METHOD 2**

let M be the midpoint of BC

$\mathrm{BM} = 4$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle $\mathrm{BMV}$ or $\mathrm{CMV}$ ***(M1)***

$\arcsin \dfrac{4}{\sqrt{106}}$ OR $\dfrac{\pi}{2} - \arccos \dfrac{4}{\sqrt{106}}$ OR $0.399018$ ***(A1)***

$\mathrm{B\hat{V}C} = 0.798037...$

$\mathrm{B\hat{V}C} = 0.798$ (accept $45.7^\circ$) ***A1***

***[4 marks]***

***Total [6 marks]***

**4.** (a) use of sector area formula to find area of at least one sector ***(M1)***

$\frac{1}{2}\times 5.2\times 100-\frac{1}{2}\times 5.2\times r^2$ OR $10^2\pi-\frac{1}{2}10^2\times(2\pi-5.2)-\left(\pi r^2-\frac{1}{2}\times(2\pi-5.2)\times r^2\right)$ ***A1***

$(\text{area})=260-2.6r^2$ ***AG***

**Note:** There are many different ways to find the area of the "C". In all methods, the ***A*** mark is awarded for working which leads directly to the ***AG***.

***[2 marks]***

(b) (i) $260-2.6r^2=64$ ***(A1)***

$r=8.68243...$

$=8.68\,(\text{cm})\left(\frac{14\sqrt{65}}{13}\text{ exact}\right)$ ***A1***

(ii) $10\times 5.2$ OR $8.68...\times 5.2$ ***(A1)***

substituting their value of $r$ into $10\times 5.2+r\times 5.2+2(10-r)$ (or equivalent) ***(M1)***

Perimeter = $10\times 5.2+8.68...\times 5.2+2(10-8.68...)$ $(=52+45.1486...+2.63513...)$

= $99.7837\ldots$

= $99.8$ (cm) ***A1***

***[5 marks]***

***Total [7 marks]***

**4.** (a) $2r + r\theta = 10$ ***A1***

$\frac{1}{2}r^2\theta = 6.25$ ***A1***

attempt to eliminate $\theta$ to obtain an equation in $r$ ***M1***

correct intermediate equation in $r$ ***A1***

$10 - 2r = \frac{25}{2r}$ OR $\frac{10}{r} - 2 = \frac{25}{2r^2}$ OR $\frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25$ OR $12.5 + 2r^2 = 10r$

$4r^2 - 20r + 25 = 0$ ***AG***

***[4 marks]***

(b) attempt to solve quadratic by factorizing or use of formula or completing the square ***(M1)***

$(2r-5)^2 = 0$ OR $r = \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)}\left(= \frac{20 \pm \sqrt{400-400}}{8}\right)$

$r = \frac{5}{2}$ ***A1***

attempt to substitute their value of $r$ into their perimeter or area equation ***(M1)***

$\theta = \frac{10 - 2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)}$ or $\theta = \frac{25}{2\left(\frac{5}{2}\right)^2}$

$\theta = 2$ ***A1***

***[4 marks]***

***Total [8 marks]***

**2.** $\tan^{-1}1 = 45^\circ$ or equivalent ***(A1)***

attempt to equate $2x - 5^\circ$ to their reference angle ***(M1)***

Note: Do not accept $2x - 5° = 1$.

$2x - 5^\circ = 45^\circ, \left(225^\circ\right)$

$x = 25^\circ, 115^\circ$ ***A1A1***

**Note:** Do not award the final ***A1*** if any additional solutions are seen.

***[4 marks]***

**7.** (a) $12b = 2\pi$ OR $(b =)\frac{2\pi}{12}$ OR $12 = \frac{2\pi}{b}$ ***A1***

$b = \frac{\pi}{6}$ ***AG***

***[1 mark]***

(b) (i) $a = 5$ ***A1***

(ii) $c = 15$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued.*

(c) (i) attempt to substitute $x = 5$ into $g(x)$ ***(M1)***

$g(5) = 3.5\sin\frac{5\pi}{6} + 11$

$\sin\frac{5\pi}{6} = \frac{1}{2}$ ***(A1)***

$g(5) = 3.5 \times \frac{1}{2} + 11$

$g(5) = 12.75\left(= \frac{51}{4}\right)$ ***A1***

(ii) **METHOD 1 (finding maximum temperature)**

considering the maximum value of $\sin\frac{\pi}{6}x(=1)$ OR $g'(x) = 0$ at maximum

OR maximum = vertical shift + amplitude (may be seen on a graph) (***M1)***

$g_{\max} = 3.5 + 11$ OR $\frac{\pi}{6} \cdot 3.5\cos\left(\frac{\pi}{6}x\right) = 0$ OR $x = 3$

$g_{\max} = 14.5$ ***A1***

$14.5 < 15$ (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

***Note:*** Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** *or* ***M0A0R1*** ) .

Worded conclusions are acceptable for the ***R1***, as long as the reasoning is clear that the water does not reach $15°$, so not warm enough for Alex.

*continued…*

*Question 7 continued.*

**METHOD 2 (working with inequality)**

$3.5\sin\left(\frac{\pi}{6}x\right)+11\geq 15$ (***M1)***

$\sin\left(\frac{\pi}{6}x\right)\geq\frac{8}{7}$ ***A1***

sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

| | |
|---|---|
| ***Note:*** | Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** *or* ***M0A0R1*** ) .<br><br>If candidate works with an equation throughout, the ***M1*** and ***A1*** may be awarded, if appropriate. A correct inequality is required for the ***R1*** to be awarded. |

***[6 marks]***

*continued…*

*Question 7 continued.*

(d) **EITHER**

attempt to find $0.7f(x)$ OR $0.7f(x)+q$ **(*M1*)**

$0.7f(x)=3.5\sin\frac{\pi}{6}x+10.5$ OR $0.7f(x)+q=3.5\sin\frac{\pi}{6}x+10.5+q$ OR

$10.5+q=11$ ***(A1)***

**OR**

attempt to find $0.7f(x)$ for a particular value of $x$ **(*M1*)**

eg maximum $20\times0.7=14$ ***(A1)***

**THEN**

$q=0.5$ ***A1***

***[3 marks]***

***Total [12 marks]***

**3.** (a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r+5)$ ***(M1)***

$\tan 58^\circ = \frac{18.2}{r+5}$ OR $\frac{r+5}{\sin 32^\circ} = \frac{18.2}{\sin 58^\circ}$ OR $(r+5=)\,11.3726...$ ***(A1)***

$r = 6.37262...$ (m)

$(r=)\,6.37$ (m) ***A1***

***[3 marks]***

(b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$$V = \frac{\pi(6.37262...)^2(20)}{3}$$

$$= 850.540\ldots$$

$$= 851\ (\text{m}^3)$$ ***A1***

**Note**: Accept $849.840\ldots\ (850)$ obtained from using $r = 6.37$.

***[2 marks]***

***Total [5 marks]***

**8.** (a) **EITHER**

attempt to find value of $t$ for the first low tide OR the first high tide ***(M1)***

$11.2619\ldots - 5.13801\ldots$

$= 6.12396\ldots$ ***(A1)***

**OR**

attempt to find half of the period ***(M1)***

$\frac{1}{2} \times \frac{2\pi}{0.513}$

$= 6.12396\ldots$ ***(A1)***

**THEN**

$m = (6.12396\ldots - 6) \times 60 = 7.43773...$

$m = 7$ ***A1***

***[3 marks]***

(b) attempt to solve $H(t) = 1$ ***(M1)***

$3.56919\ldots$ OR $6.70684\ldots$ OR $15.8171\ldots$ OR $18.9547\ldots$

$(6.70684\ldots - 3.56919\ldots =)\ 3.13764...$

$= 3.14$ (hours) ***A1***

***[2 marks]***

(c) recognition that $H'(13)$ is required ***(M1)***

$= -0.650622\ldots$

$= -0.651$ (m/h) ***A1***

***[2 marks]***

*continued…*

*Question 8 continued*

(d)

**Note**: In part (d), award the marks for $a$, $b$, $c$ and $d$ independent of each other.

**METHOD 1**

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

attempt to find time between low and high tide in hours ***(M1)***

$6$ hours and $21$ minutes = $6.35$ hours

(period =) $12.7$ ***(A1)***

$b = \frac{2\pi}{12.7} = 0.494739\ldots$

$b = 0.495 \left( = \frac{60\pi}{381} \right)$ ***A1***

attempt to find mean of low and high tide times OR substitute values of a known point ***(M1)***

$c = \frac{1}{2}\left( 2\frac{41}{60} + 9\frac{2}{60} \right)$ OR eg $0.40 = 1.17\sin\left(0.495\left(2.68333... - c\right)\right) + 1.57$

$c = 5.85833...$

$c = 5.86$ ***A1***

**Note:** Award ***(M1)A1*** for $c = 18.6$.
Award ***(M1)A0*** for $c = -6.84$.

***[7 marks]***

*continued…*

*Question 8 continued*

**METHOD 2**

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

substituting at least one point into $h(t)$ ***(M1)***

$1.17\sin\left(b\left(2\frac{41}{60} - c\right)\right) + 1.57 = 0.4$ OR $1.17\sin\left(b\left(9\frac{2}{60} - c\right)\right) + 1.57 = 2.74$

$b\left(2\frac{41}{60} - c\right) = -\frac{\pi}{2}(= -1.57)$ **AND** $b\left(9\frac{2}{60} - c\right) = \frac{\pi}{2}(= 1.57)$ ***(A1)***

**Note:** accept any angles of the form $-\frac{\pi}{2} + 2\pi k$ and $\frac{\pi}{2} + 2\pi k$.

**EITHER**

use of graph or table to find their intersection ***(M1)***

**OR**

attempt to solve their equations simultaneously ***(M1)***

$$\frac{2\frac{41}{60} - c}{9\frac{2}{60} - c} = -1$$

**THEN**

$c = 5.85833...$

$c = 5.86$ ***A1***

$b = 0.494739...$

$b = 0.495$ ***A1***

***[7 marks]***

(e) attempt to find point of intersection of two graphs ***(M1)***

$T = 4.16292\ldots$ OR $T = 4.16417\ldots$ ( using 3 sf)

$T = 4.16$ ***A1***

***[2 marks]***

***Total [16 marks]***

**9.** (a) (i) **METHOD 1**

attempt to use the sine rule to find $\text{A}\hat{\text{B}}\text{O}$ ***(M1)***

$$\frac{25}{\sin 28°} = \frac{50}{\sin \text{A}\hat{\text{B}}\text{O}}$$

$\left(\text{A}\hat{\text{B}}\text{O} =\right) 69.8748\ldots^\circ$ or $\left(\text{A}\hat{\text{B}}\text{O} =\right) 110.125\ldots^\circ$ ***(A1)(A1)***

**Note:** Award ***A1*** for each value.

attempt to find at least one possible angle for $\text{O}\hat{\text{A}}\text{B}$ ***(M1)***

$\left(\text{O}\hat{\text{A}}\text{B} =\right) 180^\circ - 28^\circ - 69.8748\ldots^\circ$ OR $\left(\text{O}\hat{\text{A}}\text{B} =\right) 180^\circ - 28^\circ - 110.125\ldots^\circ$

$\text{O}\hat{\text{A}}\text{B} = 82.1251...^\circ, 41.8748...^\circ$

$\text{O}\hat{\text{A}}\text{B} = 82.1^\circ, 41.9^\circ$ ***A1***

**METHOD 2**

attempt to use the cosine rule to find OB ***(M1)***

$25^2 = 50^2 + \text{OB}^2 - 2(50)(\text{OB})\cos 28^\circ$

$\text{OB} = 52.7491...$ or $35.5455...$ ***(A1) (A1)***

attempt to use the sine rule to find $\text{O}\hat{\text{A}}\text{B}$ ***(M1)***

$$\frac{25}{\sin 28^\circ} = \frac{\text{OB}}{\sin \text{O}\hat{\text{A}}\text{B}}$$

$\text{O}\hat{\text{A}}\text{B} = 82.1251...^\circ, 41.8748...^\circ$

$\text{O}\hat{\text{A}}\text{B} = 82.1^\circ, 41.9^\circ$ ***A1***

*continued…*

*Question 9 continued*

(ii) attempt to substitute two sides of triangle OAB and one of their angles into the area formula $\frac{1}{2}ab\sin C$ ***(M1)***

$\frac{1}{2}(\text{OA})(\text{AB})\sin 82.1251\ldots^\circ$ OR $\frac{1}{2}(50)(25)\sin 41.8748\ldots^\circ$ OR

$\frac{1}{2}(50)(52.7491...)\sin 28^\circ$ OR $\frac{1}{2}(50)(35.5455)\sin 28^\circ$

Area $= 619.106\ldots$ OR $= 417.190\ldots$

$= 619\ (\text{m}^2)$ OR $= 417\ (\text{m}^2)$ ***A1A1***

***[8 marks]***

(b) attempt to use the cosine rule in triangle OCD ***(M1)***

$10^2 = x^2 + y^2 - 2xy\cos 28^\circ$ ***A1***

attempt to use the area formula in triangle OCD ***(M1)***

$\frac{1}{2}xy\sin 28^\circ = 60$ ***A1***

**Note:** Award ***(M1)A1*** for use of the area formula independently of the ***(M1)A1*** for use of the cosine rule.

$xy = \frac{120}{\sin 28^\circ}$

$100 = x^2 + y^2 - \frac{240\cos 28^\circ}{\sin 28^\circ}$ ***A1***

$x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$ ***AG***

***[5 marks]***

*continued…*

*Question 9 continued*

(c) **EITHER**

attempt to eliminate $y$ or $x$ ***(M1)***

$$100 = x^2 + \left(\frac{120}{x\sin 28^\circ}\right)^2 - \frac{240}{\tan 28^\circ}$$

**OR**

attempt to find the intersection of the graph of their cosine rule equation and the graph of their area formula ***(M1)***

**Note**: Award ***(M1)*** only if their graphs are of functions with the same subject e.g. both "$y = ...$" or both "$y^2 = ...$".

**THEN**

$x = 13.1300...$ or $x = 19.4673...$

$x = 13.1$ or $x = 19.5$ ***A1A1***

$\text{OC} = 13.1(\text{m})$ or $19.5(\text{m})$

***[3 marks]***

***Total [16 marks]***

**9.** (a) **METHOD 1**

let $\mathrm{M}$ be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

attempts to use Pythagoras' theorem to find $\mathrm{AM}^2$ OR $\mathrm{AM}$ ***(M1)***

$\mathrm{AM}^2 = 20^2 - 14^2 \ (= 204)$ OR $\mathrm{AM} = \sqrt{20^2 - 14^2} \ \left(= 14.2828... = \sqrt{204} = 2\sqrt{51}\right)$

recognizes that $\mathrm{AB} = 2\mathrm{AM}$ ***(A1)***

$\mathrm{AB} = 2 \times 14.2828... \ (= 28.5657...) \ \left(= 2\sqrt{204} = 4\sqrt{51}\right)$ ***A1***

$\mathrm{AB} = 28.5657...$

$\mathrm{AB} = 28.57$ (m) ***AG***

**METHOD 2**

let $\mathrm{M}$ be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

let $\theta = \mathrm{A\hat{S}M}$

$\theta = 0.795398... \left(= \cos^{-1}\frac{14}{20}\right)$ ***(A1)***

attempts to use a valid trigonometric ratio ***(M1)***

**EITHER**

$\mathrm{AM} = 14\tan(0.795398...) \left(= 14.2828... = 14\tan\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

**OR**

$\mathrm{AM} = 20\sin(0.795398...) \left(= 14.2828... = 20\sin\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

**THEN**

$\mathrm{AB} = 28.5657...$

$\mathrm{AB} = 28.57$ (m) ***AG***

***[3 marks]***

*continued…*

*Question 9 continued.*

(b) **EITHER**

the sprinkler rotates through (an angle of) $2\pi$ (radians) every $16$ seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in $1$ second ***A1***

**OR**

$\left(\frac{2\pi}{n}=16 \Rightarrow n=\right)\frac{2\pi}{16}\ \left(=\frac{\pi}{8}\right)$ ***A1***

**THEN**

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***

***[1 mark]***

*continued…*

*Question 9 continued.*

(c)

| Note: For candidates that used Method 2 in part (a) apply full FT from their value of $\theta$. |
|---|

attempts to find $2\theta$ where $\theta = \text{A}\hat{\text{S}}\text{M}$ ***(M1)***

$= 2(0.795398...)$ $\left(1.59079... = 2\cos^{-1}\frac{14}{20}\right)$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving $T$ ***(M1)***

$\frac{2\pi}{16} = \frac{1.59079...}{T}$ $\left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T}\right)$ ***(A1)***

$T = 4.05093...$ $\left(= \frac{1.59079...}{\frac{2\pi}{16}}\right)$ $\left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$

$T = 4.05$ (s) ***A1***

***[4 marks]***

*continued…*

*Question 9 continued.*

(d) $\alpha = \frac{\pi t}{8}$ ***A1***

***[1 mark]***

(e) applies sine rule in $\Delta ASD$ ***A1***

$\frac{d}{\sin\alpha} = \frac{20}{\sin A\hat{D}S}$

attempts to find $A\hat{D}S$ in terms of $\alpha$ ***M1***

$A\hat{D}S = \pi - \beta - \alpha \ \left(= \pi - 0.7754 - \alpha\right)\left(= 2.366... - \alpha\right) \ \left(= 2.37 - \alpha\right)$

$d = \frac{20\sin\alpha}{\sin\left(2.366... - \alpha\right)} \left(= \frac{20\sin\alpha}{\sin\left(2.37 - \alpha\right)}\right)$ (accept $d = \frac{20\sin\alpha}{\sin\left(\pi - \beta - \alpha\right)}$ ) ***A1***

$d = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$ ***AG***

***[3 marks]***

(f) $18$ (m) ***A1***

***[1 mark]***

*continued…*

*Question 9 continued.*

(g) (i) $w = \left| 0.05t^2 + 1.1t + 18 - \dfrac{20\sin\left(\dfrac{\pi t}{8}\right)}{\sin\left(2.37 - \dfrac{\pi t}{8}\right)} \right|$ ***A1***

(ii) attempts to solve $w = 0$ for $t$ ***(M1)***

$t = 3.34880...\left(12.7765...\right)$

$t = 3.35$ (s) ***A1***

$22.2444...$

$22.2$ (m) (south of $\mathrm{A}$) ***A1***

***[4 marks]***

***Total [17 marks]***

---

**2.** (a) $\frac{1}{2}r^2(\theta)=6$ OR $\frac{1}{2}r^2(0.75)=6$ ***(A1)***

attempt to solve their equation to find $r$ or $r^2$ ***(M1)***

**Note:** To award the ***M1***, candidate's equation must include $r^2$ and $\theta=1.5$, and they must isolate $r^2$ or $r$.

$r^2=16$

$r=4$ (cm) ***A1***

***[3 marks]***

(b) evidence of summing the two radii and the arc length ***(M1)***

perimeter $=2r+r\theta$

$=8+4(0.75)$

$=11$ (cm) ***A1***

***[2 marks]***

***Total [5 marks]***

**5.** attempt to substitute into cosine rule ***(M1)***

$\left(\cos 2\theta =\right)\dfrac{3^2+10^2-8^2}{2\times 3\times 10}$ OR $8^2=3^2+10^2-2\times 3\times 10\times\cos 2\theta$

$\left(\cos 2\theta =\right)\dfrac{45}{60}\left(=\dfrac{3}{4}\right)$ ***(A1)***

attempt to use $\cos 2\theta = 2\cos^2\theta - 1$ ***(M1)***

$\cos^2\theta = \dfrac{1+\dfrac{45}{60}}{2}\left(=\dfrac{1+\dfrac{3}{4}}{2}\right)$

$\cos^2\theta = \dfrac{105}{120}\left(=\dfrac{7}{8}\right)$ ***A1***

$\cos\theta = (\pm)\sqrt{\dfrac{105}{120}}\left(=\sqrt{\dfrac{7}{8}}\right)$ ***(A1)***

$=\dfrac{\sqrt{7}}{2\sqrt{2}}$

$=\dfrac{\sqrt{14}}{4}\ (p=14, q=4)$ ***A1***

**Note:** The final answer must be positive.

***[6 marks]***

**2.** (a) $\frac{1}{2}r^2\theta = 48$ OR $\frac{1}{2}r^2(1.5) = 48$ ***(A1)***

attempt to solve their equation to find $r$ or $r^2$ ***(M1)***

**Note:** To award the ***M1***, candidate's equation must include $r^2$ and $\theta = 1.5$, and they must attempt to isolate $r^2$ or $r$.

$r^2 = 64$

$r = 8$ (cm) ***A1***

***[3 marks]***

(b) evidence of summing the two radii and the arc length ***(M1)***

perimeter $= 2r + r\theta$

$= 16 + 8(1.5)$

$= 28$ (cm) ***A1***

***[2 marks]***

***Total [5 marks]***

**5.** attempt to substitute into cosine rule ***(M1)***

$\left(\cos 2\theta =\right)\dfrac{4^2+6^2-5^2}{2\times 4\times 6}$ OR $5^2=4^2+6^2-2\times 4\times 6\times\cos 2\theta$

$\left(\cos 2\theta =\right)\ \dfrac{27}{48}\left(=\dfrac{9}{16}\right)$ ***(A1)***

attempt to use $\cos 2\theta = 2\cos^2\theta - 1$ ***(M1)***

$\cos^2\theta = \dfrac{1+\dfrac{27}{48}}{2}\ \left(=\dfrac{1+\dfrac{9}{16}}{2}\right)$

$\cos^2\theta = \dfrac{75}{96}\ \left(=\dfrac{25}{32}\right)$ ***A1***

$\cos\theta = (\pm)\sqrt{\dfrac{75}{96}}\ \left(=\sqrt{\dfrac{25}{32}}=\dfrac{5}{\sqrt{32}}\right)$ ***(A1)***

$=\dfrac{5}{4\sqrt{2}}$

$=\dfrac{5\sqrt{2}}{8}\ \ (p=5, q=8)$ ***A1***

**Note:** The final answer must be positive.

***[6 marks]***

**5.** (a) **METHOD 1**

attempt to use right triangle trigonometry ***(M1)***

$\tan \hat{BAE} = \frac{15}{4}$ OR $\tan\left(90° - \hat{BAE}\right) = \frac{4}{15}$ ***(A1)***

$75.0685...$

$\hat{BAE} = 75.1°$ ***A1***

**Note:** Award ***(M1)(A1)A0*** for the equivalent radian value of $1.31$.

**METHOD 2**

attempt to find $\hat{BAE}$ using sine rule OR cosine rule (***M1)***

$\frac{\sin \hat{BAE}}{15} = \frac{\sin 90}{\sqrt{15^2 + 4^2}}$ OR $15^2 = 4^2 + 241 - 2 \times 4 \times \sqrt{15^2 + 4^2} \times \cos \hat{BAE}$ ***(A1)***

$75.0685...$

$\hat{BAE} = 75.1°$ ***A1***

**Note:** Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

***[3 marks]***

(b) (i) **METHOD 1**

attempt to find $DE$ using right angle trigonometry ***(M1)***

$\sin 75.0685...° = \frac{350}{DE}$ OR equivalent ***(A1)***

$DE = 362.230...$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

**METHOD 2**

**Let** $DE = EF = x$

attempt to find $DE$ using their $\hat{DEF}$ and the sine rule OR cosine rule (***M1)***

$\frac{700}{\sin(150.137...)} = \frac{DE}{\sin(14.9314...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 14.9314...$ ***(A1)***

$DE = 362.230...$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

*continued…*

*Question 5 continued*

**METHOD 3**

Let $G$ be the midpoint of $DF$

$EG = \frac{4}{15} \times 350 \left( = \frac{280}{3} = 93.3333... \right)$ ***(A1)***

use of Pythagoras' with their $EG$ to find $DE$ ***(M1)***

$(DE =) \sqrt{93.3333...^2 + 350^2} \ (= 362.230...)$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

(ii) $\tan(75.0685...°) = \frac{20}{x}$ OR $\frac{15}{4} = \frac{20}{x}$ ***(A1)***

$x = 5.33$

$BA = 402.230... + 5.33333...$

$= 407.564...$

$= 408$ (cm) (accept $407$ from previous 3sf answer) ***A1***

***[5 marks]***

***Total [8 marks]***

**9.** (a) (i) 3 ***A1***

(ii) attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{4\pi}$

period = $\frac{1}{2}$ ***A1***

***[3 marks]***

(b) (i) evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii,or iii) ***(M1)***

$a = 5$ ***A1***

(ii) $b = 4\pi$ ***A1***

(iii) $0.198792\ldots$

$c = 0.199$ ***A1***

***[4 marks]***

(c) $19$ (seconds) ***A1***

***[1 mark]***

(d) 5 (speed bumps) ***A1***

***[1 mark]***

(e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{14}$) ***(M1)***

$(3.5)\left(\frac{\pi}{14}\right)\sin\left(\frac{\pi}{14}(t-5)\right)$ OR $0.785398\ldots$

$v'(t) = 0.785\sin\left(\frac{\pi}{14}(t-5)\right)$ $\left(= \frac{\pi}{4}\sin\left(\frac{\pi}{14}(t-5)\right)\right)$ ***A1***

(ii) recognition that $v' = a$ ***(M1)***

$0.785\left(\text{ms}^{-2}\right)$ $\left(= \frac{\pi}{4}\right)$ ***A1***

***[4 marks]***

*continued…*

*Question 9 continued*

(f) recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by $28$ seconds) ***(A1)***

$\int_{5}^{33} v(t)dt \left( = \int_{k}^{k+28} |v(t)|\,dt \right)$

$252(\text{m})$ (exact) ***A1***

***[3 marks]***

***Total [16 marks]***

---

**5.** (a) **METHOD 1**

attempt to use right triangle trigonometry ***(M1)***

$\tan \hat{BAE} = \frac{12}{7}$ OR $\tan\left(90° - \hat{BAE}\right) = \frac{7}{12}$ ***(A1)***

$59.7435...$

$\hat{BAE} = 59.7°$ ***A1***

**Note:** Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

**METHOD 2**

attempt to find $\hat{BAE}$ using sine rule OR cosine rule (***M1)***

$\frac{\sin \hat{BAE}}{12} = \frac{\sin 90}{\sqrt{12^2 + 7^2}}$ OR $12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \hat{BAE}$ ***(A1)***

$\hat{BAE} = 59.7435...$

$\hat{BAE}=59.7°$ ***A1***

**Note:** Award ***(M1)(A1)A0*** for the equivalent radian value of $1.04$.

***[3 marks]***

(b) (i) **METHOD 1**

attempt to find $DE$ using right angle trigonometry ***(M1)***

$\sin 59.7435...° = \frac{350}{DE}$ OR equivalent ***(A1)***

$DE = 405.196...$

$CE = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

**METHOD 2**

Let $DE = EF = x$

attempt to find $DE$ using their $D\hat{E}F$ and the sine rule OR cosine rule (***M1)***

$\frac{700}{\sin(119.487...)} = \frac{DE}{\sin(30.2564...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564...$ ***(A1)***

$DE = 405.196...$

$CE = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

*continued…*

*Question 5 continued.*

**METHOD 3**

Let G be the midpoint of DF

$\text{EG} = \frac{7}{12} \times 350 \left( = \frac{1225}{6} = 204.166... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\text{DE} = \sqrt{204.166...^2 + 350^2} \ \left( = 405.196... \right)$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455 \text{ (cm)}$ ***A1***

(ii) $\tan(59.7435...°) = \frac{30}{x}$ OR $\frac{12}{7} = \frac{30}{x}$ ***(A1)***

$x = 17.5$

$\text{BA} = 455.196... + 17.5$

$= 472.696...$

$= 473 \text{ (cm)}$ ***A1***

***[5 marks]***

***Total [8 marks]***

**9.** (a) (i) 6 ***A1***

(ii) attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{\pi}$

period = $2$ ***A1***

***[3 marks]***

(b) (i) evidence of considering the graph of $6\cos(\pi x) - 8\sin(\pi x)$ (seen in i, ii, or iii) ***(M1)***

$a = 10$ ***A1***

(ii) $b = \pi$ ***A1***

(iii) $1.70483\ldots$

$c = 1.70$ ***A1***

***[4 marks]***

(c) 23 (seconds) ***A1***

***[1 mark]***

(d) 4 (speedbumps) ***A1***

***[1 mark]***

*continued…*

*Question 9 continued.*

(e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{19}$) ***(M1)***

$\left(4.5\right)\left(\frac{\pi}{19}\right)\sin\left(\frac{\pi}{19}\left(t-4\right)\right)$ OR $0.744061\ldots$

$v'(t)=0.744\sin\left(\frac{\pi}{19}\left(t-4\right)\right)\left(=\left(\frac{9\pi}{38}\right)\sin\left(\frac{\pi}{19}\left(t-4\right)\right)\right)$ ***A1***

(ii) recognition that $v'=a$ ***(M1)***

$0.744\left(\text{ms}^{-2}\right)$ ***A1***

***[4 marks]***

(f) recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by $38$ seconds) ***(A1)***

$\int_{4}^{42}v(t)dt\left(=\int_{k}^{k+38}\left|v(t)\right|dt\right)$

$399(\text{m})$ (exact) ***A1***

***[3 marks]***

***Total [16 marks]***

**9.** (a) (i) recognition that $\mathrm{AB} = 2r$ and $\mathrm{BC} = 2R$ (seen anywhere) ***(M1)***

$\cos\theta = \frac{2R}{2r}$ OR $\frac{2r}{\sin 90^\circ} = \frac{2R}{\sin\left(90^\circ - \theta\right)}\left(= \frac{2R}{\cos\theta}\right)$ ***A1***

$R = r\cos\theta$ ***AG***

(ii) attempt to use Pythagoras, sine or the sine rule or cosine rule in triangle $\mathrm{A\hat{B}C}$ or similar ***(M1)***

$\sin\theta = \frac{h}{2r}$ OR $h^2 + (2R)^2 = (2r)^2$ OR $\frac{2r}{\sin 90^\circ} = \frac{h}{\sin\theta}$ OR

$h^2 = (2R)^2 + (2r)^2 - 2\times 2R\times 2r\cos(\theta)$

$h = 2r\sin\theta$ OR $h = \sqrt{4r^2 - 4r^2\cos^2\theta}$ ***A1***

***[4 marks]***

(b) area of one circle $= \pi R^2$

$= \pi(r\cos\theta)^2$ ***A1***

curved surface area $= 2\pi Rh$

$= 2\pi(r\cos\theta)(2r\sin\theta)$ OR $= 2\pi(r\cos\theta)\sqrt{4r^2 - 4r^2\cos^2\theta}$ ***A1***

**Note:** these ***A1*** marks can be awarded independently.

recognition that total surface area = area of two circles + curved surface area

$= 2\pi R^2 + 2\pi Rh$ (seen anywhere) ***M1***

$= 2\pi r^2\cos^2\theta + 4\pi r^2\sin\theta\cos\theta$ OR $= 2\pi r^2\cos^2\theta + 2\pi r\cos\theta\sqrt{4r^2 - 4r^2\cos^2\theta}$

$= 2\pi r^2(1 - \sin^2\theta) + 4\pi r^2\sin\theta\cos\theta$ ***A1***

total surface area of the cylinder $= 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$ ***AG***

***[4 marks]***

(c) attempt to equate external surface area of the sphere to $2S$

$4\pi r^2 = 4\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$ or equivalent ***M1***

$2\sin\theta\cos\theta - \sin^2\theta = 0$ ***A1***

$\sin\theta \neq 0$ ***R1***

$2\cos\theta - \sin\theta = 0$ ***A1***

$\tan\theta = 2$ ***AG***

***[4 marks]***

*continued…*

*Question 9 continued.*

(d) attempt to find volume of the cylinder in terms of $r$ and $\theta$ ***(M1)***

Volume of cylinder $= \pi R^2 h = \pi\left(r\cos\theta\right)^2\left(2r\sin\theta\right)$ OR $\pi r^2\cos^2\theta\sqrt{4r^2-4r^2\cos^2\theta}$

OR $\pi\left(r\cos\theta\right)^2\times 4r\cos\theta$ ***A1***

attempt to use right angled triangle to find $\cos\theta$ and $\sin\theta$ ***(M1)***

$\sqrt{1^2+2^2}=\sqrt{5}$, $\cos\theta=\frac{1}{\sqrt{5}}$, $\sin\theta=\frac{2}{\sqrt{5}}$

volume $= 2\pi r^3\left(\frac{1}{\sqrt{5}}\right)^2\left(\frac{2}{\sqrt{5}}\right)$ OR $4\pi r^3\left(\frac{1}{\sqrt{5}}\right)^3$ ***A1***

$=\frac{4}{25}\pi r^3\sqrt{5}$ ***A1***

$p=\frac{4}{25}$

***[5 marks]***

***Total [17 marks]***

---

**4.**

(a) correct substitution in sine rule ***(A1)***

$\frac{\sin\theta}{5} = \frac{\sin 2\theta}{6\sqrt{2}}$ (or equivalent)

attempt to use double angle rule for $\sin 2\theta$ ***(M1)***

$\frac{\sin\theta}{5} = \frac{2\sin\theta\cos\theta}{6\sqrt{2}}$

$6\sqrt{2}\sin\theta = 10\sin\theta\cos\theta$ OR $\frac{1}{5} = \frac{2\cos\theta}{6\sqrt{2}}$ OR equivalent ***A1***

$\cos\theta = \frac{3\sqrt{2}}{5}$ ***AG***

***[3 marks]***

(b) valid attempt to find $\sin\theta$ ***(M1)***

$\sin^2\theta + \left(\frac{3\sqrt{2}}{5}\right)^2 = 1$ OR right triangle with adjacent side and hypotenuse labelled

$\sin\theta = \frac{\sqrt{7}}{5}$ ***A1***

***[2 marks]***

(c) $\frac{1}{2} \times 6\sqrt{2} \times \mathrm{DC} \times \frac{\sqrt{7}}{5} = 2\sqrt{14}$ ***(A1)***

$\mathrm{DC} = \frac{10}{3}$ ***A1***

***[2 marks]***

***Total [7 marks]***

**8.** (a) (i) amplitude is $\frac{72}{2} = 36$ OR $8 = a\cos(b \times 0) + 44$ ***(A1)***

$a = -36$ ***A1***

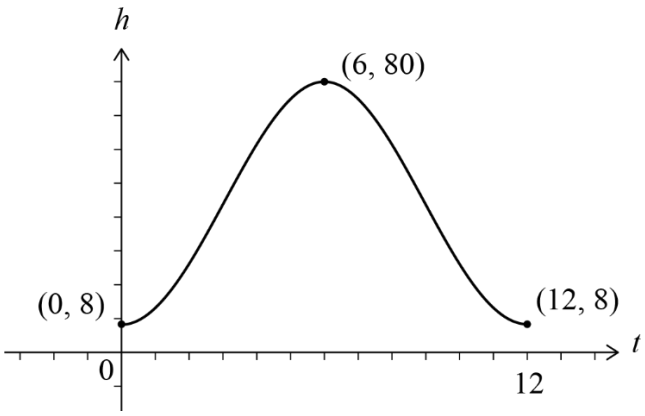
**Note**: If no working shown, award ***A1A0*** for $a = 36$.

(ii) $\frac{2\pi}{b} = 12$ (accept $\frac{360}{b} = 12$) OR $80 = -36\cos(6b) + 44$ ***(M1)***

$b = \frac{\pi}{6}$ (do not accept 30) ***A1***

***[4 marks]***

(b)




***A1A1***

**Note**: Award ***A1*** for maximum at $(6, 80)$.

For an approximately correct sinusoidal shape,

award ***A1*** for correct end points (accept $y$-intercept labelled as $8$).

Accept coordinate values clearly aligned and labelled on the axes.

***[2 marks]***

*continued…*

*Question 8 continued.*

(c) equating their $-36\cos\left(\frac{\pi}{6}t\right)+44\geq 26$ ***(M1)***

**Note**: Accept working with = or > sign.

$\cos\left(\frac{\pi}{6}t\right)\leq\frac{1}{2}$

$\frac{\pi}{6}t=\frac{\pi}{3}$ OR $\frac{\pi}{6}t=\frac{5\pi}{3}$ (accept $30t=60$ or $30t=300$ ) ***(A1)***

$t=2$ OR $t=10$ ***A1***

valid attempt to find interval ***(M1)***

$12-2\times 2$ OR $10-2$

$T=8$ (minutes) ***A1***

**Note**: If their values from part (a) result in a single solution for $t$ only, award ***M1A1FTA1FTM0A0*** as appropriate.

***[5 marks]***

(d) (i) attempt to differentiate using chain rule ***(M1)***

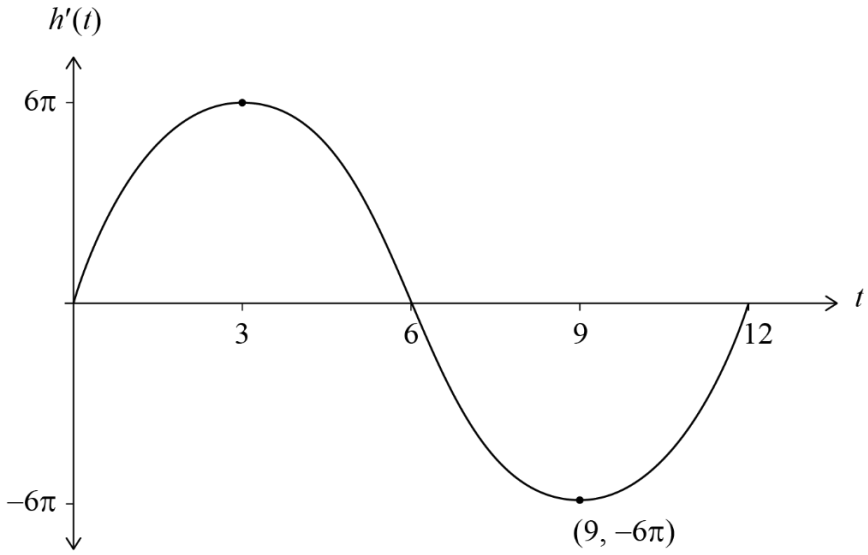
$h'(t)=-36\times-\sin\left(\frac{\pi}{6}t\right)\times\frac{\pi}{6}\left(=6\pi\sin\left(\frac{\pi}{6}t\right)\right)$ ***A1***

*continued…*

*Question 8 continued.*

(ii) **METHOD 1**

sketch of their $h'$ (with their minimum point labelled) ***(M1)***




$t = 9$ ***A1***

**METHOD 2**

attempt to find and solve their $h''(t) = 0$ ***(M1)***

$h''(t) = -36 \times -\cos\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} \times \frac{\pi}{6} \left( = \pi^2 \cos\left(\frac{\pi}{6}t\right)\right) = 0$

$t = 3, t = 9$

$t = 9$ ***A1***

**METHOD 3**

recognising (minimum value of sine $-1$) occurs at $\frac{3}{4}$ of revolution ***(M1)***

$\frac{\pi}{6}t = \frac{3\pi}{2}$ (accept $30t = 270$)

$t = 9$ ***A1***

***[4 marks]***

***Total [15 marks]***

**2.** (a) **EITHER**

$210^\circ = \frac{210\pi}{180}\left(= \frac{7\pi}{6} = 3.66519...\right)$ radians ***(A1)***

attempt to use radian formula for area of sector ***(M1)***

area $= \frac{1}{2}(19.5)^2\left(\frac{7\pi}{6}\right)$

**OR**

attempt to use degree formula for area of sector ***(M1)***

area $= \frac{210}{360}\pi(19.5)^2$ ***(A1)***

**THEN**

area $= \frac{3549\pi}{16} = 696.844...$

$= 697\left(= \frac{3549\pi}{16}\right)$ (cm$^2$) ***A1***

***[3 marks]***

(b) **EITHER**

arc length $= 19.5\left(\frac{7\pi}{6}\right)$ OR $= \frac{210}{360}(2\pi(19.5))\left(= \frac{91\pi}{4} = 71.4712...\right)$ ***(A1)***

attempt to set $2\pi r$ equal to arc length ***(M1)***

$2\pi r = 71.4712...$

**OR**

attempt to set $\pi rl$ equal to their area from (a) ***(M1)***

$19.5\pi r = 696.844...$ ***(A1)***

**THEN**

$r = 11.4\left(= \frac{91}{8} = 11.375\right)$ (cm) ***A1***

***[3 marks]***

***Total[6 marks]***

**3.** (a) period is $\frac{\pi}{2}\left(=1.57079...=1.57\right)$ ***A1***

***[1 mark]***

(b) attempt to substitute $x=\frac{\pi}{12}, f(x)=5$ and $x=\frac{\pi}{3}, f(x)=7$ to obtain two equations ***(M1)***

**Note:** accept work where $x$ values have been converted into degrees

$a\tan\left(\frac{\pi}{6}\right)+b=5$ and $a\tan\left(\frac{2\pi}{3}\right)+b=7\left(\Rightarrow\frac{a}{\sqrt{3}}+b=5 \text{ and } -a\sqrt{3}+b=7\right)$

$a=-\frac{\sqrt{3}}{2}\left(=-0.866025...=-0.866\right)$ ***A1***

$b=\frac{11}{2}\left(=5.5\right)$ ***A1***

**Note:** These ***A1*** marks may be awarded independently.

***[3 marks]***

***Total [4 marks]***

**2.**

(a) attempt to find distance between two points ***(M1)***

$\sqrt{(-3-1)^2+(4-7)^2+(2-0)^2}\ \ (=5.38516...)$

$\mathrm{VX}=5.39\ \left(=\sqrt{29}\right)$ ***A1***

***[2 marks]***

(b) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{5^2+5^2}\ \ (=7.07106...)$

$\mathrm{AC}=7.07\ \left(=5\sqrt{2}\right)$ ***A1***

***[2 marks]***

(c) valid attempt to use trig ratio in triangle $\mathrm{VXC}$ ***(M1)***

$\tan\theta=\dfrac{\sqrt{29}}{\frac{5\sqrt{2}}{2}}\left(=\dfrac{5.38516...}{3.53553...}\right)$

$56.7138...^\circ$ OR $0.989842...$ rad

$\theta=56.7^\circ$ OR $\theta=0.990$ rad (accept $\theta=0.99$) ***A1***

***[2 marks]***

***Total [6 marks]***

**6.**

**METHOD 1**

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)***

$\text{area}_{\text{triangle}} = 21.3333...$ OR $\text{area}_{\text{sec tor}} = 34.1333...$ ***A1***

correct equation in $r$ and $\theta$ ***(A1)***

$\frac{1}{2}r^2\theta = 34.1333...$ OR $\frac{1}{2}r^2\sin\theta = 21.3333...$ OR $\frac{1}{2}r^2(\theta - \sin\theta) = 12.8$ (seen anywhere)

correct equation in one variable ***A1***

$\frac{1}{2}r^2\left(\frac{68.2666...}{r^2} - \sin\left(\frac{68.2666...}{r^2}\right)\right) = 12.8$ OR $\frac{1}{2}\left(\frac{68.2666...}{\theta}\right)\sin\theta = 21.3333...$ OR

$\frac{1}{2}\left(\frac{68.2666...}{\theta}\right)(\theta - \sin\theta) = 12.8$

attempt to solve their equation or use of graph ***(M1)***

$\theta = 1.59934...$

$6.53330...$

$r = 6.53$ ***A1***

**METHOD 2**

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)***

$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$ (seen anywhere) ***(A1)***

$$\frac{\frac{1}{2}r^2(\theta - \sin\theta)}{\frac{1}{2}r^2\sin\theta} = \frac{3}{5}$$

correct equation without $r$ ***A1***

$$\frac{(\theta - \sin\theta)}{\sin\theta} = \frac{3}{5}$$

$\theta = 1.59934...$ ***(A1)***

*continued…*

*Question 6 continued*

attempt to solve for $r$ using their $\theta$ ***(M1)***

$\frac{1}{2}r^2\left(1.59... - \sin 1.59...\right) = 12.8$

$6.53330...$

$r = 6.53$ ***A1***

**METHOD 3**

recognising $\text{area}_{\text{segment}} = \frac{3}{8} \times \text{area}_{\text{sector}}$ (seen anywhere) ***(M1)***

$\text{area}_{\text{segment}} = \frac{1}{2}r^2\left(\theta - \sin\theta\right)$ (seen anywhere) ***(A1)***

$\frac{1}{2}r^2\left(\theta - \sin\theta\right) = \frac{3}{8} \times \frac{1}{2}r^2\theta$

correct equation without $r$ ***A1***

$\frac{1}{2}\left(\theta - \sin\theta\right) = \frac{3}{8} \times \frac{1}{2}\theta$

$\theta = 1.59934...$ ***(A1)***

attempt to solve for $r$ using their $\theta$ ***(M1)***

$\frac{1}{2}r^2\left(1.59... - \sin 1.59...\right) = 12.8$

$6.53330...$

$r = 6.53$ ***A1***

***[6 marks]***

**SECTION B**

**7.** (a) $\hat{JCL} = 28°$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$JL^2 = (5.5)^2 + 8^2 - 2(5.5)(8)\cos 28°$ ***(A1)***

$4.068244\ldots$

$JL = 4.07$ (km) ***A1***

**Note**: Award ***(A1)(M1)(A1)A0*** for an answer of $13.4\,(13.3775...)$ (km) for working in radians.

***[4 marks]***

(b) valid attempt to solve ***(M1)***

use a trigonometric ratio or sine rule in triangle $BLX$

$\tan 0.94^\circ = \frac{60}{BL}$ OR $\tan 89.06^\circ = \frac{BL}{60}$ OR $\frac{BL}{\sin 89.06^\circ} = \frac{60}{\sin 0.94^\circ}$ ***(A1)***

$3656.849288\ldots$ OR $3.656849\ldots$

$BL = 3660$ (m) OR $BL = 3.66$ (km) ***A1***

***[3 marks]***

(c) valid approach ***(M1)***

clear diagram OR $180 + 121$ OR $360 - (180 - 121)$ OR $270 + (121 - 90)$

$301°$ (accept $N59°W$) ***A1***

***[2 marks]***

(d) $\frac{t}{3.656\ldots} = \frac{60}{48}$ OR $t = \frac{3.65684\ldots}{48} (= 0.0761843\ldots)$ ***(A1)***

$4.57106\ldots$

$t = 4.57$ (min) ***A1***

***[2 marks]***

*continued…*

*Question 7 continued*

(e) attempt to use cosine rule to find distance $\mathrm{BC}$ ***(M1)***

$(3.656\ldots)^2 + 8^2 - 2(3.656\ldots)(8)\cos 31^\circ \quad (= 27.2200...)$

$\mathrm{BC} = 5.21728...$ ***(A1)***

$\frac{t}{5.21728...} = \frac{60}{55}$ OR $t = \frac{5.21728...}{60} (= 0.094859\ldots)$

time for coast guard rescue boat $= 5.69158...$ (min) OR $0.094859\ldots$ (hour) ***A1***

$5.69158... > 4.57106....$ (min) OR $0.094859\ldots > 0.076184...$ (hour)

the lighthouse rescue boat will reach the jet-ski first. ***A1***

***[4 marks]***

***Total [15 marks]***

**1.** (a) substituting into distance between two points formula in 3D ***(M1)***

$\sqrt{(6-4)^2+(-1-(-5))^2+(-3-(-9))^2}$

$=7.48331...$

(radius) $=7.48$ $\left(=\sqrt{56} \text{ or } 2\sqrt{14} \text{ (exact)}\right)$ ***A1***

***[2 marks]***

(b) valid approach to find surface area of solid ***(M1)***

$\frac{1}{2}\times 4\pi r^2+\pi r^2$

$\frac{1}{2}\times 4\pi\times(7.48331...)^2+\pi\times\left(7.48331...\right)^2$ ***(A1)***

$=527.787...$

(surface area =) $528$ $(=168\pi)$ (Accept $527$ from 3sf answer to (a)) ***A1***

**Note:** Award ***(M1)A0A0*** for $351.858...(=112\pi)$ from only considering curved surface.

***[3 marks]***

***Total [5 marks]***



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**2.** (a) substituting into cosine rule ***(M1)***

$\left(\text{BC}^2 =\right) 12^2 + 7^2 - 2\times 12\times 7\times \cos\left(116^\circ\right)$ ***(A1)***

$\sqrt{266.646...} = 16.3293...$

$\left(\text{BC} =\right) 16.3$ ***A1***

***[3 marks]***

(b) substituting into sine rule or cosine rule for angle $\text{A}\hat{\text{C}}\text{B}$ or angle $\text{A}\hat{\text{B}}\text{C}$ ***(M1)***

$\frac{\sin\left(\text{A}\hat{\text{C}}\text{B}\right)}{7} = \frac{\sin\left(116^\circ\right)}{16.3293...}$ **OR** $7^2 = 12^2 + 16.3293...^2 - 2\times 12\times 16.3293...\times\cos(\text{A}\hat{\text{C}}\text{B})$

**OR** $180° - 116° - 41.3°$ from finding angle $\text{A}\hat{\text{B}}\text{C}$ ***(A1)***

$22.6618\ldots$

$\left(\text{A}\hat{\text{C}}\text{B} =\right) 22.7\ (^\circ)$ (accept $22.8(^\circ)$ if using previous 3sf AND the cosine rule) ***A1***

***[3 marks]***

***Total [6 marks]***

**5.** Attempts to use the area formula for triangle $\mathrm{AOB}$ **AND** equate it to 26 ***(M1)***

$\frac{1}{2}a \times b \times \sin(2.51) = 26$ **OR** $\frac{1}{2}\mathrm{OA} \times \mathrm{OB} \times \sin \mathrm{AOB} = 26$

$\frac{1}{2} \times r^2 \times \sin(2.51) = 26$ or equivalent ***(A1)***

$r = 9.38462...$ ***(A1)***

use of arc length formula to find the major arc $\mathrm{ACB}$ ***(M1)***

$9.38462... \times (2\pi - 2.51)$ **OR** $2\pi \times 9.38462... - 9.38462... \times 2.51$ **OR** $9.38462... \times 3.773185...$

$35.4099\ldots$

(arc length =) $35.4$ $(\mathrm{cm})$ ***A1***

***[5 marks]***

**9.** (a) attempt to substitute $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$ (in numerator or denominator) ***(M1)***

uses $\cos^2\theta+\sin^2\theta=1$ or $2\sin\theta\cos\theta=\sin 2\theta$ in their expression (seen anywhere) ***(M1)***

correct working leading to given answer ***A1***

$\sin 2\theta$ ***AG***

**Note:** Candidates perform substitutions in different orders. Use the above as a guide.
Award ***(M1)(M1)A0AG*** if a candidate substitutes $\frac{\sin\theta}{\cos\theta}$ and with no other interim working writes $2\sin\theta\cos\theta$ leading to the ***AG***.

***[3 marks]***

(b) $\sin 2\theta=\frac{1}{2}$ ***(A1)***

$(2\theta=)\ 30,150$ (accept $\frac{\pi}{6},\frac{5\pi}{6}$ and condone extra values) ***(A1)(A1)***

attempt to halve their angle(s) ***(M1)***

$\theta=15^\circ,75^\circ$ ***A1***

**Note:** Award final ***A0*** for extra or missing values of $\theta$.
Award ***(A1)(A1)(A1)(M1)A0*** for $\theta=\frac{\pi}{12},\frac{5\pi}{12}$.
Award ***(A1)(A1)A0(M1)A0*** for only one of $\theta=15^\circ$ OR $\theta=75^\circ$ with no working.

***[5 marks]***

*continued…*

*Question 9 continued.*

(c) $\dfrac{2\tan\left(22\frac{1}{2}^\circ\right)}{1+\tan^2\left(22\frac{1}{2}^\circ\right)} = \sin 45^\circ$ ***(A1)***

$\sin 45^\circ = \dfrac{1}{\sqrt{2}}$ (seen anywhere) ***(A1)***

attempt to form an equation in terms of $x$ or $\tan(22.5)$ ***M1***

$\dfrac{2x}{1+x^2} = \dfrac{1}{\sqrt{2}} \quad \left(= \dfrac{\sqrt{2}}{2}\right)$ OR $2\sqrt{2}\tan(22.5) = 1+\left(\tan(22.5)\right)^2$

**Note:** Accept working in radians.

$2\sqrt{2}x = 1+x^2$ **OR** $\sqrt{2}x^2 - 4x + \sqrt{2} = 0$ **OR** $\tan^2(22.5) - 2\sqrt{2}\tan(22.5) + 1 = 0$ ***A1***

$x^2 - 2\sqrt{2}x + 1 = 0$ ***AG***

**Note:** Substituting $\tan(22.5)$ directly into the ***AG*** is working backwards; award no marks.

**Exception:** If a candidate rearranges the ***AG*** to $\dfrac{\sqrt{2}\times 2x}{1+x^2} = 1$ and then substitutes $\sin 45^\circ$ for $\dfrac{2x}{1+x^2}$, award the first ***A1***; then a maximum of ***A1M0A0*** may be awarded after that.

***[4 marks]***

(d) attempt to use the quadratic formula ***(M1)***

$x = \dfrac{2\sqrt{2} \pm \sqrt{8-4}}{2}$

$x = \sqrt{2} \pm 1$ ***(A1)***

$\tan\left(22\frac{1}{2}^\circ\right) = \sqrt{2}-1$, (since $\tan\left(22\frac{1}{2}^\circ\right) < 1$) ***A1***

***[3 marks]***

***Total [15 marks]***

**7.** (a) **METHOD 1** (cosine rule)

attempt to use cosine rule ***(M1)***

$\mathrm{PQ}^2 = 2^2 + 3^2 - 2(2)(3)\cos 60^\circ$ OR $\cos 60^\circ = \dfrac{2^2 + 3^2 - \mathrm{PQ}^2}{2(2)(3)}$ ***(A1)***

$\mathrm{PQ}^2 = 7$

$\mathrm{PQ} = \sqrt{7}$ ***A1***

**METHOD 2** (drawing altitude from Q perpendicular to PR to create two triangles)

attempt to find length of altitude segment $\left(= \sqrt{3}\right)$ ***(M1)***

$\mathrm{PQ}^2 = 2^2 + \left(\sqrt{3}\right)^2$ ***(A1)***

$= 7$

$\mathrm{PQ} = \sqrt{7}$ ***A1***

***[3 marks]***

(b) attempt to use cosine rule ***(M1)***

$\mathrm{AB}^2 = (x+2)^2 + (3x-1)^2 - 2(x+2)(3x-1)\cos 60^\circ$ OR $\cos 60^\circ = \dfrac{(x+2)^2 + (3x-1)^2 - \mathrm{AB}^2}{2(x+2)(3x-1)}$ ***(A1)***

$\left(\mathrm{AB}^2 =\right) \left(x^2 + 4x + 4\right) + \left(9x^2 - 6x + 1\right) - \left(3x^2 + 5x - 2\right)$ ***A1A1***

**Note:** Correct expansions may be seen anywhere in the candidate's working.

Award ***A1*** for $\left(x^2 + 4x + 4\right)$ AND $\left(9x^2 - 6x + 1\right)$

Award ***A1*** for $-2(x+2)(3x-1)\left(\dfrac{1}{2}\right)$ OR $-\left(3x^2 + 5x - 2\right)$.

$\left(\mathrm{AB}^2 =\right) 7x^2 - 7x + 7$ ***A1***

$\left(\mathrm{AB} =\right) \sqrt{7x^2 - 7x + 7}$ ***AG***

***[5 marks]***

*continued…*

*Question 7 continued*

(c) (i) $\sqrt{7x^2 - 7x + 7} = 7$

$7x^2 - 7x + 7 = 49$ OR $x^2 - x - 6 = 0$ (or equivalent) ***(A1)***

valid attempt to solve their quadratic ***(M1)***

$x = 3$ ***A1***

**Note:** Award ***A0*** if $x = -2$ is included in the final answer.

(ii) attempt to use $A = \frac{1}{2}(x+2)(3x-1)\sin 60$ with their value of $x$ ***(M1)***

$\frac{1}{2} \cdot 5 \cdot 8 \cdot \sin 60$ OR $\frac{1}{2} \cdot 5 \cdot 8 \cdot \frac{\sqrt{3}}{2}$ (or equivalent) ***(A1)***

$= 10\sqrt{3}$ ***A1***

**Note**: Do not award FT for $x \leq \frac{1}{3}$.

***[6 marks]***

***Total [14 marks]***

**1.** (a) (i) $1.25663\ldots$

$\theta = 1.26 \left(= \frac{2\pi}{5}\right)$ ***A1***

(ii) $\frac{2\pi}{5} \times r = 12$ or equivalent ***(A1)***

$9.54929...$

$r = 9.55$ (cm) $\left(= \frac{60}{2\pi} = \frac{30}{\pi}\right)$ ***A1***

***[3 marks]***

(b) **METHOD 1**

attempt to find area of at least one shaded segment ***(M1)***

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$ OR $\frac{1}{2}r^2(\theta - \sin\theta)$

$\frac{1}{2}(9.54929...)^2 \times 1.25663\ldots - \frac{1}{2}(9.54929...)^2 \sin 1.25663\ldots$ (or equivalent) ***(A1)***

shaded area $= 5 \times (57.2957\ldots - 43.3629\ldots) (= 5 \times 13.9328...)$

$69.6640...$

$= 69.7\ (\text{cm}^2)$ ***A1***

***[3 marks]***

**METHOD 2**

attempt to find area of one triangle ***(M1)***

$\frac{1}{2} \times (9.54929...)^2 \times \sin 1.25663...$

attempt to subtract five times their triangle area from their circle area ***(M1)***

$\pi \times (9.54929...)^2 - 5 \times \frac{1}{2} \times (9.54929...)^2 \times \sin 1.25663...$

$69.6640...$

$= 69.7\ (\text{cm}^2)$ ***A1***

***[3 marks]***

***Total [6 marks]***



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**5.** (a) (i) $a = -5$ ***A1***

(ii) **EITHER**

period $= 32$ ***(A1)***

**OR**

correct substitution of a point on the graph of $g$ and their $a$ ***(A1)***

$2 = -5\cos(b \times 16) - 3$ OR $-3 = -5\cos(b \times 8) - 3$

**Note:** Award ***A0*** if candidate substitutes $x = 0, y = -8$.

**THEN**

$0.196349...$

$b = 0.196 \left(= \frac{2\pi}{32} = \frac{\pi}{16}\right)$ ***A1***

***[3 marks]***

(b) $k = 32$ ***A1***

***[1 mark]***

(c) (i) **EITHER**

attempt to use a horizontal translation ***(M1)***

$24 - q = 16$ OR $16 - q = 8$

**OR**

substituting a point on the graph of $g$ into $f(x) = p\sin(b(x-q)) - 3$ ***(M1)***

$-8 = p\sin\left(\frac{\pi}{16}(0-q)\right) - 3$ OR $-3 = p\sin\left(\frac{\pi}{16}(8-q)\right) - 3$ OR $2 = p\sin\left(\frac{\pi}{16}(16-q)\right) - 3$

**THEN**

$q = 8$ ***A1***

(ii) $p = 5$ ***A1***

***[3 marks]***

***Total [7 marks]***

**2.** (a) $34^\circ$ ***A1***

***[1 mark]***

(b) $\hat{\text{LBP}} = 20°$ (seen anywhere) (***A1)***

attempt to use the sine rule ***(M1)***

$$\frac{\text{LB}}{\sin 34^\circ} = \frac{5}{\sin 20^\circ}$$ ***(A1)***

$$\text{LB} = 8.17485...$$

$\text{LB} = 8.17\,(\text{km})$ ***A1***

***[4 marks]***

***Total [5 marks]***

**3.** (a) attempt to use double angle formula for $\cos 2x$ (seen anywhere) ***M1***

**Note:** Only award the ***M1*** if a double angle formula for $\cos 2x$ is used, and not only for stating it.

$3\left(1-2\sin^2 x\right)=3-6\sin^2 x$ (seen anywhere) ***A1***

$3\cos 2x+11\sin x=3+11\sin x-6\sin^2 x$ ***AG***

***[2 marks]***

(b) **METHOD 1**

attempt to solve a quadratic equation in $\sin x$ (***M1)***

$\sin x=0.333333...\left(=\frac{1}{3}\right)$

$x=19.4712....,160.528...$

$x=19.5,161$ ***A1A1***

**METHOD 2**

attempt to solve $3\cos 2x+11\sin x-6=0$ (may be seen in a sketch) ***(M1)***

$x=19.4712....,160.528...$

$x=19.5,161$ ***A1A1***

***[3 marks]***

***Total [5 marks]***

**5.** (a) perimeter of sector is $1.5 \times \text{circumference of circle}$ (seen anywhere) ***A1***

$1.5 \times 16\pi$ OR $24\pi$ OR $75.3982...$

perimeter of sector $r\theta + 2r$ ***(M1)***

$24\pi = r\theta + 2r$ ***A1***

$r = \dfrac{24\pi}{2+\theta}$ ***AG***

***[3 marks]***

(b) correct substitution of $r = \dfrac{24\pi}{2+\theta}$ into $\dfrac{1}{2}r^2\theta$ ***(A1)***

$64\pi = \dfrac{1}{2} \times \theta \times \left(\dfrac{24\pi}{2+\theta}\right)^2$

attempt to solve their equation (must be quadratic) ***(M1)***

$\theta = 0.411273...$

$\theta = 0.411$ ***A1***

**Note:** Award ***(A1)(M1)A0*** if additional answers are given.

***[3 marks]***

***Total [6 marks]***

**8.** (a) (i) attempt to use volume of cone or sphere or hemisphere (***M1***)

$\frac{1}{3}\pi\times3^2\times8=75.3982...(24\pi)$ OR $\frac{4}{3}\pi\times3^3=113.097...(36\pi)$ OR $\frac{2}{3}\pi\times3^3=56.5486...(18\pi)$

recognize that total volume is half the volume of a sphere plus the volume of a cone (***M1***)

$\frac{2}{3}\pi\times r^3+\frac{1}{3}\pi\times r^2h$ OR $V=56.5486...+75.3982...(18\pi+24\pi)$

$V=131.946....$ OR $V=42\pi$ ***A1***

$V=132$ (cm$^3$) ***AG***

(ii) **EITHER**

attempt to use Pythagoras to find the slant height. (***M1***)

$l^2=3^2+8^2$ OR $l=\sqrt{3^2+8^2}$

**OR**

attempt to use trigonometry to find the angle **and** the slant height ***(M1)***

$\arctan\frac{8}{3}=1.21202...\text{rad or } 69.4439...°$ **and** $l=\frac{8}{\sin69.4439...}\text{or}\frac{3}{\cos69.4439...}$

**THEN**

$l=8.54400...(\sqrt{73})$ ***(A1)***

attempt to use formula for surface area with their $l$ ***(M1)***

$S=\pi\times3\times8.54400...$

**Note**: Do not award the ***(M1)*** if they use $l=8$.

$S=80.5(80.5253...=3\pi\sqrt{73})$ (cm$^2$) ***A1***

***[7 marks]***

*continued…*

*Question 8 continued*

(b) recognize that volume of cone + volume of hemisphere = $120$ ***(M1)***

$\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = 120$ ***A1***

$h = \left(120 - \frac{2}{3}\pi r^3\right) \div \left(\frac{1}{3}\pi r^2\right)$ OR $h = \left(360 - 2\pi r^3\right) \div \left(\pi r^2\right)$ OR $\pi r^2 h = 360 - 2\pi r^3$ ***A1***

$h = \frac{360 - 2\pi r^3}{\pi r^2}$ ***AG***

***[3 marks]***

(c) attempt to use Pythagoras to find $l$ in terms of $r$ and $h$ ***(M1)***

$l = \sqrt{r^2 + h^2}$

$l = \sqrt{r^2 + \left(\frac{360 - 2\pi r^3}{\pi r^2}\right)^2}$ ***A1***

$S = \pi r \sqrt{r^2 + \left(\frac{360 - 2\pi r^3}{\pi r^2}\right)^2}$ ***AG***

***[2 marks]***

(d) attempt to find the minimum value of $S$ ***(M1)***

$S = 44.4045\ldots$, $r = 3.67084\ldots$, sketch

$S = 44.4$ (cm$^2$), $r = 3.67$ (cm) ***A1A1***

***[3 marks]***

***Total [15 marks]***

APOLLO

# Statistics and probability

81 questions

**4.** (a) attempt to use definition of outlier
$1.5\times 20+Q_3$ ***(M1)***

$1.5\times 20+U\geq 75$ $\left(\Rightarrow U\geq 45\text{, accept } U>45\right)$ OR $1.5\times 20+Q_3=75$ ***A1***

minimum value of $U=45$ ***A1***

***[3 marks]***

(b) attempt to use interquartile range ***(M1)***
$U-L=20$ (may be seen in part (a)) OR $L\geq 25$ (accept $L>25$)
minimum value of $L=25$ ***A1***

***[2 marks]***
***Total [5 marks]***

**9.** (a) recognising probabilities sum to 1 ***(M1)***

$p + p + p + \frac{1}{2}p = 1$

$p = \frac{2}{7}$ ***A1***

***[2 marks]***

(b) valid attempt to find $\mathrm{E}(X)$ ***(M1)***

$1\times p + 2\times p + 3\times p + 4\times \frac{1}{2}p \left(= 8p\right)$

$\mathrm{E}(X) = \frac{16}{7}$ ***A1***

***[2 marks]***

(c) (i) $0 \le r \le 1$ ***A1***

(ii) attempt to find a value of $q$ ***(M1)***

$0 \le 1 - 3q \le 1$ OR $r = 0 \Rightarrow q = \frac{1}{3}$ OR $r = 1 \Rightarrow q = 0$

$0 \le q \le \frac{1}{3}$ ***A1***

***[3 marks]***

(d) $\mathrm{E}(Y) = 1\times q + 2\times q + 3\times q + 4\times r \left(= 2 + 2r \text{ OR } 4 - 6q\right)$ ***(A1)***

one correct boundary value ***A1***

$1\times\frac{1}{3} + 2\times\frac{1}{3} + 3\times\frac{1}{3} + 4\times 0 \left(= 2\right)$ OR

$1\times 0 + 2\times 0 + 3\times 0 + 4\times 1 \left(= 4\right)$ OR

$2 + 2(0) \left(= 2\right)$ OR

$2 + 2(1) \left(= 4\right)$ OR

$4 - 6(0) \left(= 4\right)$ OR $4 - 6\left(\frac{1}{3}\right) \left(= 2\right)$

$2 \le \mathrm{E}(Y) \le 4$ ***A1***

***[3 marks]***

*continued…*

*Question 9 continued*

(e) **METHOD 1**

evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4 ***(M1)***

$\frac{6}{7}q+\frac{6}{7}r$ OR $3pq+3pr$ OR $pq+pq+p(1-3q)+pq+p(1-3q)+p(1-3q)$ ***(A1)***

solving for either $q$ or $r$ ***M1***

$\frac{6}{7}(q+1-3q)=\frac{1}{2}$ OR $\frac{6}{7}\left(\frac{1-r}{3}+r\right)=\frac{1}{2}$ OR $3pq+3p(1-3q)=\frac{1}{2}$

OR $3p\left(\frac{1-r}{3}\right)+3pr=\frac{1}{2}$

***EITHER*** two correct values

$q=\frac{5}{24}$ and $r=\frac{3}{8}$ ***A1A1***

***OR*** one correct value

$q=\frac{5}{24}$ OR $r=\frac{3}{8}$ ***A1***

substituting their value for $q$ or $r$ ***A1***

$4-6\left(\frac{5}{24}\right)$ OR $2+2\left(\frac{3}{8}\right)$

***THEN***

$\mathrm{E}(Y)=\frac{11}{4}$ ***A1***

***[6 marks]***

*continued…*

*Question 9 continued*

**METHOD 2** (solving for $\mathrm{E}(Y)$)

evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4 ***(M1)***

$\frac{6}{7}q+\frac{6}{7}r$ OR $3pq+3pr$ OR $pq+pq+p(1-3q)+pq+p(1-3q)+p(1-3q)$ ***(A1)***

rearranging to make $q$ the subject ***M1***

$$q=\frac{4-\mathrm{E}(Y)}{6}$$

$3pq+3p(1-3q)=\frac{1}{2}$ ***M1***

$$\frac{6}{7}\times\left(\frac{4-\mathrm{E}(Y)}{6}\right)+\frac{6}{7}\left(1-3\left(\frac{4-\mathrm{E}(Y)}{6}\right)\right)=\frac{1}{2}$$ ***A1***

$$\frac{2(\mathrm{E}(Y)-1)}{7}=\frac{1}{2}$$

$\mathrm{E}(Y)=\frac{11}{4}$ ***A1***

***[6 marks]***
***Total [16 marks]***

**7.** (a) evidence of median position ***(M1)***

40 students

median $=14$ (hours) ***A1***

***[2 marks]***

(b) recognizing there are $8$ students in the top $10\%$ ***(M1)***

$72$ students spent less than $k$ hours ***(A1)***

$k=18$ (hours) ***A1***

***[3 marks]***

(c) 15 hours is 60 students OR $p=60-4$ ***(M1)***

$p=56$ ***A1***

21 hours is 76 students OR $q=80-76$ OR $q=80-4-56-16$ ***(A1)***

$q=4$ ***A1***

***[4 marks]***

(d) 20 of the 80 students OR $\frac{1}{4}$ spend more than 15 hours doing homework ***(A1)***

$\frac{20}{80}=\frac{x}{320}$ OR $\frac{1}{4}\times 320$ OR $4\times 20$ ***(A1)***

$80$ (students) ***A1***

***[3 marks]***

(e) (i) only year 12 students surveyed OR amount of homework might be different for different year levels ***R1***

(ii) stratified sampling OR survey students in all years ***R1***

***[2 marks]***

***Total [14 marks]***

**2.** (a) $a = 0.433156...,\ b = 4.50265...$

$a = 0.433,\ b = 4.50$ ***A1A1***

***[2 marks]***

(b) attempt to substitute $x = 18$ into their equation ***(M1)***

$y = 0.433 \times 18 + 4.50$

$= 12.2994...$

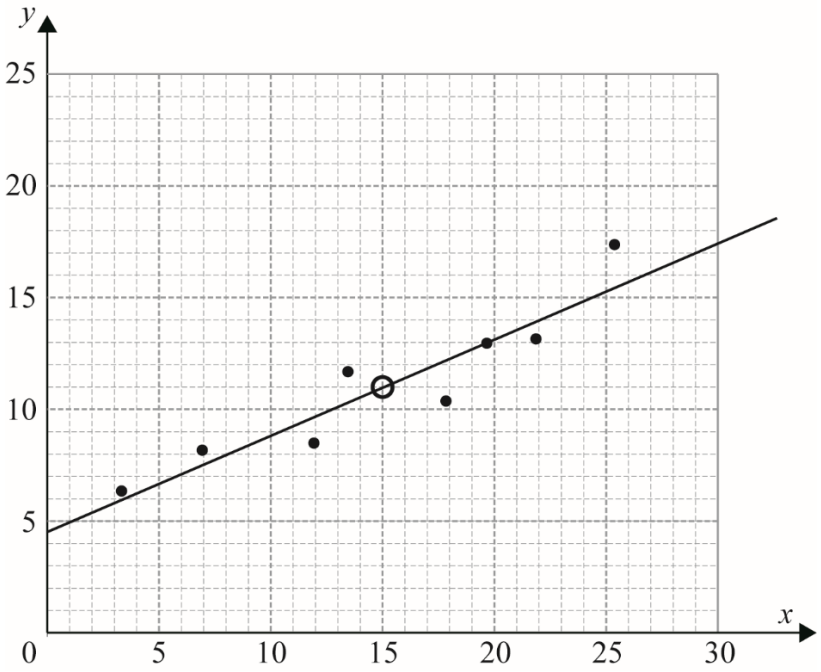
$= 12.3$ ***A1***

***[2 marks]***

(c) $\overline{x} = 15\ ,\ \overline{y} = 11$ ***A1***

***[1 mark]***

(d)




***A1A1***

**Note:** Award marks as follows:

***A1*** for a straight line going through $(15,11)$

***A1*** for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \leq x \leq 25.3$.

If the candidate does not use a ruler, award ***A0A1*** where appropriate.

***[2 marks]***

***Total [7 marks]***

**3.**

**Note:** In this question, do not penalise incorrect use of strict inequality signs.

Let $X$ = mass of a bag of sugar

(a) evidence of identifying the correct area ***(M1)***

$\mathrm{P}(X < 995) = 0.0765637...$

$= 0.0766$ ***A1***

***[2 marks]***

(b) $0.0766 \times 100$

$\approx 8$ ***A1***

***[1 mark]***

**Note:** Accept 7.66.

(c) recognition that $\mathrm{P}(X > 1005 | X \geq 995)$ is required ***(M1)***

$\dfrac{\mathrm{P}(X \geq 995 \cap X > 1005)}{\mathrm{P}(X \geq 995)}$

$\dfrac{\mathrm{P}(X > 1005)}{\mathrm{P}(X \geq 995)}$ ***(A1)***

$\dfrac{0.0765637\ldots}{1 - 0.0765637\ldots} \left( = \dfrac{0.0765637\ldots}{0.923436\ldots} \right)$

$= 0.0829$ ***A1***

***[3 marks]***
***Total [6 marks]***

**1.** (a) (i) $a = 0.805084...$ and $b = 2.88135...$

$a = 0.805$ and $b = 2.88$ ***A1A1***

(ii) $r = 0.97777...$

$r = 0.978$ ***A1***

***[3 marks]***

(b) $a$ represents the (average) increase in waiting time ($0.805$ mins) per additional customer (waiting to receive their coffee) ***R1***

***[1 mark]***

(c) attempt to substitute $x = 7$ into their equation ***(M1)***

$8.51693...$

$8.52$ (mins) ***A1***

***[2 marks]***

***Total [6 marks]***

**4.** (a) **EITHER**

$\mathrm{P}(S)+\mathrm{P}(T)+\mathrm{P}(S'\cap T')-\mathrm{P}(S\cap T)=1$ OR $\mathrm{P}(S\cup T)=\mathrm{P}\left((S'\cap T')'\right)$ ***(M1)***

$0.7+0.2+0.18-\mathrm{P}(S\cap T)=1$ OR $\mathrm{P}(S\cup T)=1-0.18$

**OR**

a clearly labelled Venn diagram ***(M1)***

**THEN**

$\mathrm{P}(S\cap T)=0.08$ (accept 8%) ***A1***

**Note:** To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S\cap T'$.

***[2 marks]***

(b) **EITHER**

$\mathrm{P}(T\cap S')=\mathrm{P}(T)-\mathrm{P}(T\cap S)(=0.2-0.08)$ OR

$\mathrm{P}(T\cap S')=\mathrm{P}(T\cup S)-\mathrm{P}(S)(=0.82-0.7)$

***(M1)***

**OR**

a clearly labelled Venn diagram including $\mathrm{P}(S)$, $\mathrm{P}(T)$ and $\mathrm{P}(S\cap T)$ ***(M1)***

**THEN**

$=0.12$ (accept 12%) ***A1***

***[2 marks]***

*continued…*

*Question 4 continued*

(c) $\mathrm{P}(G \cap T) = \mathrm{P}(T / G)\mathrm{P}(G)$ $(0.25 \times 0.48)$ ***(M1)***

$= 0.12$ ***A1***

***[2 marks]***

(d) **METHOD 1**

$\mathrm{P}(G) \times \mathrm{P}(T) (= 0.48 \times 0.2) = 0.096$ ***A1***

$\mathrm{P}(G) \times \mathrm{P}(T) \neq \mathrm{P}(G \cap T) \Rightarrow$ $G$ and $T$ are not independent ***R1***

**METHOD 2**

$\mathrm{P}(T \mid G) = 0.25$ ***A1***

$\mathrm{P}(T \mid G) \neq \mathrm{P}(T) \Rightarrow$ $G$ and $T$ are not independent ***R1***

| **Note**: Do not award ***A0R1.*** |
|---|

***[2 marks]***

***Total [8 marks]***

**8.** (a) use of inverse normal to find $z$-score ***(M1)***

$z = 2.0537...$

$2.0537... = \frac{82 - 75}{\sigma}$ ***(A1)***

$\sigma = 3.408401...$

$\sigma = 3.41$ ***A1***

***[3 marks]***

(b) evidence of identifying the correct area under the normal curve ***(M1)***

$\mathrm{P}(T > 80) = 0.071193...$

$\mathrm{P}(T > 80) = 0.0712$ ***A1***

***[2 marks]***

(c) recognition that $\mathrm{P}(80 < T < 82)$ is required ***(M1)***

$P(T < 82 | T > 80) = \frac{\mathrm{P}(80 < T < 82)}{\mathrm{P}(T > 80)} = \left(\frac{0.051193...}{0.071193...}\right)$ ***(M1)(A1)***

$= 0.719075...$

$= 0.719$ ***A1***

***[4 marks]***

*continued…*

*Question 8 continued*

(d) recognition of binomial probability ***(M1)***

$X \sim \mathrm{B}(64, 0.071193...)$ or $\mathrm{E}(X) = 64 \times 0.071193...$ ***(A1)***

$\mathrm{E}(X) = 4.556353...$

$\mathrm{E}(X) = 4.56$ (flights) ***A1***

***[3 marks]***

(e) $\mathrm{P}(X > 6) = P(X \geq 7) = 1 - \mathrm{P}(X \leq 6)$ ***(M1)***

$= 1 - 0.83088...$ ***(A1)***

$= 0.1691196...$

$= 0.169$ ***A1***

***[3 marks]***

***Total [15 marks]***

**9.** (a) considering that sum of probabilities is 1 ***(M1)***

$0.85+c+0.03+0.002+0.0001=1$

$0.1179$ ***A1***

***[2 marks]***

(b) valid attempt to find $\mathrm{E}(D)$ ***(M1)***

$\mathrm{E}(D)=(0\times0.85)+(2\times0.1179)+(10\times0.03)+(50\times0.002)+(1000\times0.0001)$

$\mathrm{E}(D)=0.7358$ ***A1***

No, not a fair game ***A1***

for a fair game, $\mathrm{E}(D)$ would be \$2 OR players expected winnings are 1.264 ***R1***

***[4 marks]***

(c) recognition of GP with $r=2$ ***(M1)***

$1000\times2^{n-1}$ OR $500\left(2^{n}\right)$ ***A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(d) recognizing $\mathrm{E}(D) > 2$ ***(M1)***

correct expression for $w^{\text{th}}$ week (or $n^{\text{th}}$ week) ***(A1)***

$(0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 2^{w-1} \times 0.0001)$

correct inequality (accept equation) ***(A1)***

$0.6358 + (1000 \times 2^{w-1} \times 0.0001) > 2$ OR $2^{n-1} > 13.642$

**EITHER**

$n - 1 > 3.76998$ OR $w = 4.76998...$ ***(A1)***

**OR**

$\mathrm{E}(D) = 1.4358$ in week 4 or $\mathrm{E}(D) = 2.2358$ in week 5 ***(A1)***

**THEN**

$w = 5$ ***A1***

expected profit per ticket $=$ their $\mathrm{E}(D) - 2$ ***(M1)***

$= 0.2358$ ***A1***

***[7 marks]***

***Total [15 marks]***

**4.** (a) valid approach to find $\mathrm{P}(R)$ ***(M1)***

tree diagram (must include probabilty of picking box) with correct required probabilities

OR $\mathrm{P}(R \cap B_1) + \mathrm{P}(R \cap B_2)$ OR $\mathrm{P}(R \mid B_1)\mathrm{P}(B_1) + \mathrm{P}(R \mid B_2)\mathrm{P}(B_2)$

$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2}$ ***(A1)***

$\mathrm{P}(R) = \frac{9}{14}$ ***A1***

***[3 marks]***

(b) events $A$ and $R$ are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box ***A2***

| **Note**: Both conclusion and reasoning are required. Do not split the ***A2***. |
|---|

***[2 marks]***

***Total [5 marks]***

**1.** (a) use of GDC to give ***(M1)***

$r = 0.883529...$

$r = 0.884$ ***A1***

**Note:** Award the ***(M1)*** for any correct value of $r, a, b$ or $r^2 = 0.780624...$ seen in part (a) or part (b).

***[2 marks]***

(b) $a = 1.36609...$ , $b = 64.5171...$

$a = 1.37$ , $b = 64.5$ ***A1***

***[1 mark]***

(c) attempt to find their difference ***(M1)***

$5 \times 1.36609...$ OR $1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$

$6.83045...$

$= 6.83$ $(6.85$ from $1.37)$

the student could have expected her score to increase by $7$ marks. ***A1***

**Note:** Accept an increase of $6$, $6.83$ or $6.85$.

***[2 marks]***

***Total [5 marks]***

**4.** (a) recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(30, 0.05)$

attempt to find $\mathrm{P}(X \geq 1)$ ***(M1)***

$1 - \mathrm{P}(X = 0)$ OR $1 - 0.95^{30}$ OR $1 - 0.214638...$ OR $0.785361...$

**Note:** The two ***M*** marks are independent of each other.

$\mathrm{P}(X \geq 1) = 0.785$ ***A1***

***[3 marks]***

(b) recognition of conditional probability ***(M1)***

$\mathrm{P}(X \leq 2 \mid X \geq 1)$ OR $\mathrm{P}(\text{at most 2 defective} \mid \text{at least 1 defective})$

**Note:** Recognition must be shown in context either in words or symbols but not just $\mathrm{P}(A \mid B)$.

$\frac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ OR $\frac{\mathrm{P}(X = 1) + \mathrm{P}(X = 2)}{\mathrm{P}(X \geq 1)}$ ***(A1)***

$\frac{0.597540...}{0.785361...}$ OR $\frac{0.812178... - 0.214638...}{0.785361...}$ OR $\frac{0.338903... + 0.258636...}{0.785361...}$ ***(A1)***

$= 0.760847...$

$\mathrm{P}(X \leq 2 \mid X \geq 1) = 0.761$ ***A1***

***[4 marks]***

***Total [7 marks]***

**9.** (a) $\mathrm{P}\left(\frac{\mu-1.5\sigma-\mu}{\sigma}<\frac{X-\mu}{\sigma}<\frac{\mu+1.5\sigma-\mu}{\sigma}\right)$ ***(M1)***

$P(-1.5<Z<1.5)$ OR $1-2\times\mathrm{P}(Z<-1.5)$ ***(A1)***

$\mathrm{P}(-1.5<Z<1.5)=0.866385...$

$\mathrm{P}(\mu-1.5\sigma<X<\mu+1.5\sigma)=0.866$ ***A1***

**Note:** Do not award any marks for use of their answers from part (b).

***[3 marks]***

(b) $z_1=-1.75068...$ and $z_2=1.30468...$ (seen anywhere) ***(A1)***

correct equations ***(A1)(A1)***

$\frac{106.2-\mu}{\sigma}=-1.75068...$, $\mu+1.30468...\sigma=182.6$

attempt to solve their equations involving z values ***(M1)***

$\mu=149.976...$, $\sigma=25.0051...$

$\mu=150$, $\sigma=25.0$ ***A1***

***[5 marks]***

(c) (i) new sample space is 96% (may be seen in (ii) or (iii)) ***(M1)***

$\mathrm{P}(\text{medium}|\text{not small})$ OR $\frac{0.576}{0.96}$

$\mathrm{P}(\text{Medium})=0.6$ ***A1***

(ii) $\mathrm{P}(\text{Large})=0.3$ ***A1***

(iii) $\mathrm{P}(\text{Premium})=0.1$ ***A1***

***[4 marks]***

*continued…*

*Question 9 continued.*

(d) attempt to express revenue from avocados ***(M1)***

$1.1\times0.6+1.29\times0.3+1.96\times0.1$ OR $1.243n$

correct inequality or equation for net profit in terms of $n$ ***(A1)***

$1.1\times0.6n+1.29\times0.3n+1.96\times0.1n-200\geq438$ OR $1.243n-200=438$

attempt to solve the inequality ***(M1)***

sketch OR $n=513.274...$

$n=514$ ***A1***

**Note:** Only award follow through in part (d) for 3 probabilities which add up to 1. FT of probabilities from c) that do not add up to 1 should only be awarded ***M*** marks, where appropriate, in d).

***[4 marks]***

***Total [16 marks]***

**2.** attempt to substitute into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

**Note:** Accept use of Venn diagram or other valid method.

$0.6 = 0.5 + 0.4 - \mathrm{P}(A \cap B)$ ***(A1)***

$\mathrm{P}(A \cap B) = 0.3$ (seen anywhere) ***A1***

attempt to substitute into $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$= \dfrac{0.3}{0.4}$

$\mathrm{P}(A \mid B) = 0.75 \left( = \dfrac{3}{4} \right)$ ***A1***

***Total [5 marks]***

**7.** (a) evidence of median position ***(M1)***
$80\text{th}$ employee
$40$ minutes ***A1***
***[2 marks]***

(b) valid attempt to find interval $(25-55)$ ***(M1)***
$18$ (employees), $142$ (employees) ***A1***
$124$ ***A1***
***[3 marks]***

(c) recognising that there are $16$ employees in the top $10\%$ ***(M1)***
$144$ employees travelled more than $k$ minutes ***(A1)***
$k = 56$ ***A1***
***[3 marks]***

(d) $b = 70$ ***A1***
***[1 mark]***

(e) (i) recognizing $a$ is first quartile value ***(M1)***
$40$ employees
$a = 33$ ***A1***

(ii) $47 - 33$ ***(M1)***
$\text{IQR} = 14$ ***A1***
***[4 marks]***

(f) attempt to find $1.5 \times$ **their** $\text{IQR}$ ***(M1)***
$33 - 21$
$12$ ***(A1)***
***[2 marks]***

***[Total 15 marks]***

**4.** (a) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(5,\ 0.7)$

attempt to find $\mathrm{P}(X \leq 3)$ ***M1***

$= 0.472 (= 0.47178)$ ***A1***

***[3 marks]***

(b) recognition of $2$ sixes in $4$ tosses ***(M1)***

$\mathrm{P}(\text{3rd six on the 5th toss}) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2\right] \times 0.7 (= 0.2646 \times 0.7)$ ***A1***

$= 0.185 (= 0.18522)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**5.** (a) $a = 1.29$ and $b = -10.4$ ***A1A1***

***[2 marks]***

(b) recognising both lines pass through the mean point ***(M1)***

$p = 28.7$, $q = 30.3$ ***A2***

***[3 marks]***

***Total [5 marks]***

**8.** (a) attempt to use the symmetry of the normal curve ***(M1)***

*eg* diagram, $0.5 - 0.1446$

$\mathrm{P}(24.15 < X < 25) = 0.3554$ ***A1***

***[2 marks]***

(b) (i) use of inverse normal to find z score ***(M1)***

$z = -1.0598$

correct substitution $\frac{24.15 - 25}{\sigma} = -1.0598$ ***(A1)***

$\sigma = 0.802$ ***A1***

(ii) $\mathrm{P}(X > 26) = 0.106$ ***(M1)A1***

***[5 marks]***

(c) recognizing binomial probability ***(M1)***

$\mathrm{E}(Y) = 10 \times 0.10621$ ***(A1)***

$= 1.06$ ***A1***

***[3 marks]***

(d) $\mathrm{P}(Y = 3)$ ***(M1)***

$= 0.0655$ ***A1***

***[2 marks]***

(e) recognizing conditional probability ***(M1)***

correct substitution ***A1***

$\frac{0.3554}{1 - 0.10621}$

$= 0.398$ ***A1***

***[3 marks]***

***Total [15 marks]***

**3.** (a) $\text{IQR} = 10 - 6 \, (= 4)$ ***(A1)***

attempt to find $Q_3 + 1.5 \times \text{IQR}$ ***(M1)***

$10 + 6$

16 ***A1***

***[3 marks]***

(b) (i) choosing $c = \frac{1}{2}a - 9$ ***(M1)***

$\frac{1}{2} \times 42 - 9$

= 12 (years old) ***A1***

(ii) attempt to solve system by substitution or elimination ***(M1)***

34 (years old) ***A1***

***[4 marks]***

***Total [7 marks]***

**9.** (a) uses $\sum \mathrm{P}(X = x) = 1$ to form a linear equation in $p$ and $q$ ***(M1)***

correct equation in terms of $p$ and $q$ from summing to $1$ ***A1***

$p + 0.3 + q + 0.1 = 1$ OR $p + q = 0.6$ (or equivalent)

uses $\mathrm{E}(X) = 2$ to form a linear equation in $p$ and $q$ ***(M1)***

correct equation in terms of $p$ and $q$ from $\mathrm{E}(X) = 2$ ***A1***

$p + 0.6 + 3q + 0.4 = 2$ OR $p + 3q = 1$ (or equivalent)

**Note:** The marks for using $\sum \mathrm{P}(X = x) = 1$ and the marks for using $\mathrm{E}(X) = 2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously ***A1***

for example, $2q = 0.4 \Rightarrow q = 0.2$ or $p + 3 \times (0.6 - p) = 1 \Rightarrow p = 0.4$

so $p = 0.4$ and $q = 0.2$ ***AG***

***[5 marks]***

(b) valid approach ***(M1)***

$\mathrm{P}(X > 2) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$ OR $\mathrm{P}(X > 2) = 1 - \mathrm{P}(X = 1) - \mathrm{P}(X = 2)$

$= 0.3$ ***A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(c) recognises at least one of the valid scores (6, 7, or 8) required to win the game ***(M1)***

**Note:** Award ***M0*** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let $T$ represent the score on the last two rolls

a score of $6$ is obtained by rolling $(2,4),(4,2)$ or $(3,3)$

$\mathrm{P}(T=6)=2(0.3)(0.1)+(0.2)^2 \ (=0.1)$ ***A1***

a score of $7$ is obtained by rolling $(3,4)$ or $(4,3)$

$\mathrm{P}(T=7)=2(0.2)(0.1) \ (=0.04)$ ***A1***

a score of $8$ is obtained by rolling $(4,4)$

$\mathrm{P}(T=8)=(0.1)^2 \ (=0.01)$ ***A1***

**Note:** The above 3 ***A1*** marks are independent of each other.

$\mathrm{P}(\text{Nicky wins})=0.1+0.04+0.01$

$=0.15$ ***A1***

***[5 marks]***

(d) $3+b=8$ ***(M1)***

$b=5$ ***A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(e) **METHOD 1**

**EITHER**

$\mathrm{P}(S=5)=\frac{4}{16}$

$\mathrm{P}(S=a+2)=\frac{4}{16}$ ***A1***

$\Rightarrow a+2=5$

**OR**

$\mathrm{P}(S=6)=\frac{3}{16}$

$\mathrm{P}(S=a+3)=\frac{2}{16}$ and $\mathrm{P}(S=5+1)=\frac{1}{16}$ ***A1***

$\Rightarrow a+3=6$

**OR**

$\mathrm{P}(S=4)=\frac{3}{16}$

$\mathrm{P}(S=a+1)=\frac{2}{16}$ and $\mathrm{P}(S=1+3)=\frac{1}{16}$ ***A1***

$\Rightarrow a+1=4$

**THEN**

$\Rightarrow a=3$ ***A1***

**Note:** Award ***A0A0*** for $a=3$ obtained without working/reasoning/justification.

***[2 marks]***

*continued…*

*Question 9 continued*

**METHOD 2**

**EITHER**

correctly lists a relevant part of the sample space ***A1***

for example, $\{S=4\}=\{(3,1),(1,a),(1,a)\}$ or $\{S=5\}=\{(2,a),(2,a),(2,a),(2,a)\}$

or $\{S=6\}=\{(3,a),(3,a),(1,5)\}$

$a+3=6$

**OR**

eliminates possibilities (exhaustion) for $a<5$

convincingly shows that $a\neq 2,4$ ***A1***

$a\neq 4$, for example, $\mathrm{P}(S=7)=\frac{2}{16}$ from $(2,5),(2,5)$ and so

$(3,a),(3,a)\Rightarrow a+3\neq 7$

**THEN**

$\Rightarrow a=3$ ***A1***

***[2 marks]***

***Total [16 marks]***

**2.** (a) **EITHER**

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) ***(A1)***

**OR**

$5+1+4=3+x$ ***(A1)***

**OR**

$\sum f = 20$ ***(A1)***

**THEN**

$x = 7$ ***A1***

***[2 marks]***

(b) **METHOD 1**

$1.58429\ldots$

$1.58$ ***A2***

**METHOD 2**

**EITHER**

$\sigma^2 = \frac{5\times(2-4.3)^2+1\times(3-4.3)^2+4\times(4-4.3)^2+3\times(5-4.3)^2+7\times(6-4.3)^2}{20}$ $(=2.51)$ ***(A1)***

**OR**

$\sigma^2 = \frac{5\times2^2+1\times3^2+4\times4^2+3\times5^2+7\times6^2}{20} - 4.3^2$ $(=2.51)$ ***(A1)***

**THEN**

$\sigma = \sqrt{2.51} = 1.58429...$

$= 1.58$ ***A1***

***[2 marks]***

***Total [4 marks]***

**4.** (a) $0.41+k-0.28+0.46+0.29-2k^2=1$ OR $k-2k^2+0.01=0.13$ (or equivalent) ***A1***

$2k^2-k+0.12=0$ ***AG***

***[1 mark]***

(b) one of $0.2$ OR $0.3$ ***(M1)***

$k=0.3$ ***A1***

reasoning to reject $k=0.2$ eg $\mathrm{P}(1)=k-0.28\geq 0$ therefore $k\neq 0.2$ ***R1***

***[3 marks]***

(c) attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X)=0\times 0.41+1\times(0.3-0.28)+2\times 0.46+3\times(0.29-2\times 0.3^2)$

$=1.27$ ***A1***

**Note:** Award ***M1A0*** if additional values are given.

***[2 marks]***

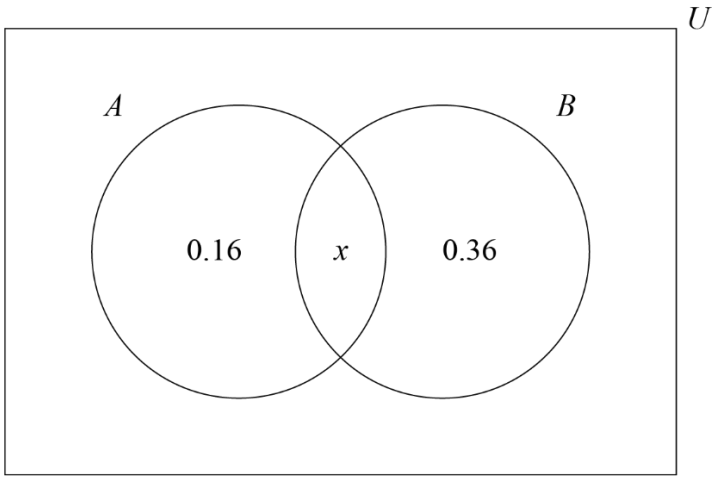
***Total [6 marks]***

**6.** (a) **METHOD 1**

**EITHER**

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ ***A1***

**OR**




***A1***

**THEN**

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ ***M1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ ***A1***

$x = 0.24$ ***A1***

**METHOD 2**

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence ***M1***

$\left(\mathrm{P}(A \cap B') = \mathrm{P}(A) \times \mathrm{P}(B') \Rightarrow\right)\ \mathrm{P}(A) \times \left(1 - \mathrm{P}(B)\right) = 0.16$ OR

$\left(\mathrm{P}(A' \cap B) = \mathrm{P}(A') \times \mathrm{P}(B) \Rightarrow\right)\ \left(1 - \mathrm{P}(A)\right) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ AND $\mathrm{P}(B) = 0.6$ ***A1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ ***(A1)***

$x = 0.24$ ***A1***

***[4 marks]***

*continued…*

*Question 6 continued*

(b) **METHOD 1**

recognising $\mathrm{P}(A' \mid B') = \mathrm{P}(A')$ ***(M1)***

$= 1 - 0.16 - 0.24$

$= 0.6$ ***A1***

**METHOD 2**

$\mathrm{P}(B) = 0.36 + 0.24 \left(= 0.6\right)$

$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')} \quad \left(= \dfrac{0.24}{0.4}\right)$ ***(A1)***

$= 0.6$ ***A1***

***[2 marks]***

***Total [6 marks]***

## Section B

**9.** (a) $\mathrm{P}(C < 61)$ ***(M1)***

$= 0.365112...$

$= 0.365$ ***A1***

***[2 marks]***

(b) recognition of binomial eg $X \sim \mathrm{B}(12, 0.365...)$ ***(M1)***

$\mathrm{P}(X = 5) = 0.213666...$

$= 0.214$ ***A1***

***[2 marks]***

(c) (i) Let $CM$ represent 'chocolate muffin' and $BM$ represent 'banana muffin'

$\mathrm{P}(B < 61) = 0.0197555...$ ***(A1)***

**EITHER**

$\mathrm{P}(CM) \times \mathrm{P}(C < 61 | CM) + \mathrm{P}(BM) \times \mathrm{P}(B < 61 | BM)$ (or equivalent in words) ***(M1)***

**OR**

tree diagram showing two ways to have a muffin weigh $< 61$ ***(M1)***

**THEN**

$(0.6 \times 0.365...) + (0.4 \times 0.0197...)$ ***(A1)***

$= 0.226969...$

$= 0.227$ ***A1***

(ii) recognizing conditional probability ***(M1)***

**Note:** Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A | B)$.

$\dfrac{0.6 \times 0.365112...}{0.226969...}$ ***(A1)***

$= 0.965183...$

$= 0.965$ ***A1***

***[7 marks]***

*continued…*

*Question 9 continued*

(d) **METHOD 1**

$\mathrm{P}(CM)\times\mathrm{P}(C<61|CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61|BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

attempt to solve for $\sigma$ using GDC ***(M1)***

**Note:** Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}(C<61)$ with a variable standard deviation.

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A2***

**METHOD 2**

$\mathrm{P}(CM)\times\mathrm{P}(C<61|CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61|BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

use of inverse normal to find $z$ score of their $\mathrm{P}(C<61)$ ***(M1)***

$z=-0.679229...$

correct substitution ***(A1)***

$\frac{61-62}{\sigma}=-0.679229...$

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A1***

***[5 marks]***

***Total [16 marks]***

**3.** (a) **METHOD 1**

using geometric sequence with $r = 1.02$ ***(M1)***

correct expression or listing terms correctly ***(A1)***

$45000 \times 1.02^{10}$ OR $45000 \times 1.02^{11-1}$ OR listing terms

Gemma's salary is \$54855 (must be to the nearest dollar) ***A1***

***[3 marks]***

**METHOD 2**

$N = 10$

$\text{PV} = \mp 45000$

$\text{I}\% = 2$

$\text{P/Y} = 1$

$\text{C/Y} = 1$

$FV = \pm 54854.7489...$ ***(M1)(A1)***

Gemma's salary is \$54855 (must be to the nearest dollar) ***A1***

***[3 marks]***

(b) finds $a = 1096.89...$ and $b = -2160753.8...$ (accept $b = -2.16 \times 10^6$) ***(A1)(A1)***

**Note:** Award ***(A1)(A1)*** for $S = 1096.89...x + 33028.49...$, or $S = 1096.89...x + 43997.4...$, or $S = 1096.89...x + 45094.3...$

Kaia's salary in 2021 is \$56063.21 (accept \$56817.09 from $b = -2.16 \times 10^6$) ***A1***

Kaia had a higher salary than Gemma in 2021 ***AG***

***[3 marks]***

***Total [6 marks]***

**4.** $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.68$

substitution of $\mathrm{P}(A) \cdot \mathrm{P}(B)$ for $\mathrm{P}(A \cap B)$ in $\mathrm{P}(A \cup B)$ ***(M1)***

$\mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ $(= 0.68)$

substitution of $3\mathrm{P}(B)$ for $\mathrm{P}(A)$ ***(M1)***

$3\mathrm{P}(B) + \mathrm{P}(B) - 3\mathrm{P}(B)\mathrm{P}(B) = 0.68$ (or equivalent) ***(A1)***

**Note**: The first two ***M*** marks are independent of each other.

attempts to solve their quadratic equation ***(M1)***

$\mathrm{P}(B) = 0.2, 1.133... \left(\frac{1}{5}, \frac{17}{15}\right)$

$\mathrm{P}(B) = 0.2 \left(= \frac{1}{5}\right)$ ***A2***

**Note**: Award ***A1*** if both answers are given as final answers for $\mathrm{P}(B)$.

***[6 marks]***

**5.** (a) $0.28$ (s) ***A1***

***[1 mark]***

(b) $\text{IQR} = 0.35 - 0.27 \ (= 0.08)$ (s) ***(A1)***

substituting **their** IQR into correct expression for upper fence ***(A1)***

$0.35 + 1.5 \times 0.08 \ (= 0.47)$ (s)

$0.46 < 0.47$ ***R1***

so $0.46$ (s) is not an outlier ***AG***

***[3 marks]***

(c) **EITHER**

the median is closer to the lower quartile (positively skewed) ***R1***

**OR**

The distribution is positively skewed ***R1***

**OR**

the range of reaction times below the median is smaller than the range of reaction times above the median ***R1***

**Note:** These are sample answers from a range of acceptable correct answers. Award ***R1*** for any correct statement that explains this.

Do not award ***R1*** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

***[1 mark]***

*continued…*

*Question 5 continued*

(d) **EITHER**

the distribution for ‘not sleeping well’ is centred at a higher reaction time ***R1***

**OR**

The median reaction time after not sleeping well is equal to the upper quartile reaction time after sleeping well ***R1***

**OR**

75% of reaction times are <0.35 seconds after sleeping well, compared with 50% after not sleeping well ***R1***

**OR**

the sample size of 9 is too small to draw any conclusions ***R1***

**Note:** These are sample answers from a range of acceptable correct answers. Accept any relevant correct statement **that relates to the median and/or quartiles shown in the box plots**. **Do not accept** a comparison of means. Do not award ***R1*** if there is also an incorrect statement, even if another statement in the answer is correct.
Award ***R0*** to “correlation does not imply causation”.

***[1 mark]***

***Total [6 marks]***

**9.** (a) **METHOD 1**

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

attempt to solve for $\sigma$ graphically or numerically using the GDC ***(M1)***

graph of normal curve $T \sim \mathrm{N}(35, \sigma^2)$ for $\mathrm{P}(T > 40)$ and $y = 0.25$ OR $\mathrm{P}(T < 40)$ and $y = 0.75$ OR table of values for $\mathrm{P}(T < 40)$ or $\mathrm{P}(T > 40)$

$\sigma = 7.413011...$

$\sigma = 7.41$ (min) ***A2***

**METHOD 2**

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

$z = 0.674489...$ ***(A1)***

valid equation using their $z$-score (clearly identified as z-score and not a probability) ***(M1)***

$\frac{40 - 35}{\sigma} = 0.674489...$ OR $5 = 0.674489...\sigma$

$7.413011\ldots$

$\sigma = 7.41$ (min) ***A1***

***[4 marks]***

(b) $\mathrm{P}(T > 45)$ ***(M1)***

$= 0.0886718...$

$= 0.0887$ ***A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(c) recognizing binomial probability ***(M1)***

$L \sim \mathrm{B}(5,\ 0.0886718...)$

$\mathrm{P}(L \geq 1) = 1 - \mathrm{P}(L = 0)$ OR

$\mathrm{P}(L \geq 1) = \mathrm{P}(L = 1) + \mathrm{P}(L = 2) + \mathrm{P}(L = 3) + \mathrm{P}(L = 4) + \mathrm{P}(L = 5)$ ***(M1)***

$0.371400\ldots$

$\mathrm{P}(L \geq 1) = 0.371$ ***A1***

***[3 marks]***

(d) recognizing conditional probability in context ***(M1)***

finding $\{L < 3\} \cap \{L \geq 1\} = \{L = 1,\ L = 2\}$ (may be seen in conditional probability) ***(A1)***

$\mathrm{P}(L = 1) + \mathrm{P}(L = 2) = 0.36532...$ (may be seen in conditional probability) ***(A1)***

$\mathrm{P}(L < 3 \mid L \geq 1) = \dfrac{0.36532...}{0.37140...}$ ***(A1)***

$0.983636\ldots$

$0.984$ ***A1***

***[5 marks]***

*continued…*

*Question 9 continued*

(e) **METHOD 1**

recognizing that Suzi can be late no more than once (in the remaining six days) ***(M1)***

$X \sim B(6, 0.0886718...)$, where $X$ is the number of days late ***(A1)***

$P(X \leq 1) = P(X = 0) + P(X = 1)$ ***(M1)***

$= 0.907294...$

$P(\text{Suzi gets a bonus}) = 0.907$ ***A1***

**Note:** The first two marks may be awarded independently.

**METHOD 2**

recognizing that Suzi must be on time at least five times (of the remaining six days) ***(M1)***

$X \sim B(6, 0.911328...)$, where $X$ is the number of days on time ***(A1)***

$P(X \geq 5) = 1 - P(X \leq 4)$ OR $1 - 0.0927052...$ OR $P(X = 5) + (X = 6)$ OR $0.334434... + 0.572860...$ ***(M1)***

$= 0.907294...$

$P(\text{Suzi gets a bonus}) = 0.907$ ***A1***

**Note:** The first two marks may be awarded independently.

***[4 marks]***

***Total [18 marks]***

**6.** (a) $\mathrm{P}(A \cap B) = 0.24$ ***A1***

***[1 mark]***

(b) $\mathrm{P}(A \cup B) = 1.1 - \mathrm{P}(A \cap B)$ ***(A1)***

$(0 \leq)\mathrm{P}(A \cup B) \leq 1$ ***(M1)***

**Note:** This may be conveyed in a clearly labelled diagram or written explanation where $\mathrm{P}(A \cup B) = 1$

the minimum value of $\mathrm{P}(A \cap B)$ is $0.1$ ***A1***

***[3 marks]***

(c) A is a subset of B (so $\mathrm{P}(A \cap B) = \mathrm{P}(A)$). ***R1***

**Note:** This may be conveyed in a clearly labelled diagram where $A$ is completely inside $B$, or in a written explanation indicating that $\mathrm{P}(A \cap B) = \mathrm{P}(A)$

so the maximum value of $\mathrm{P}(A \cap B)$ is $0.3$ ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***

***Total [6 marks]***

## Section B

**1.** (a) $1.01206\ldots, 2.45230\ldots$

$a = 1.01$ , $b = 2.45$ $(1.01x + 2.45)$ ***A1A1***

***[2 marks]***

(b) $0.981464\ldots$

$r = 0.981$ ***A1***

**Note:** A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700...$, $b = 2.09814...$ and $r = 0.980888...$ Some candidates may write the 3 sf answers, ie. $a = 1.02$, $b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0$, $b = 2.1$ and $r = 0.98$. In these cases award ***A0A0*** for part (a) and ***A0*** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

***[1 mark]***

(c) correct substitution of 78 into **their** regression equation ***(M1)***

$81.3930\ldots$ $81.23$ from 3 sf answer

$81$ ***A1***

***[2 marks]***

***Total [5 marks]***

**9.**

**Note**: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for $\mathrm{P}(T < 55 \mid T > 40)$ accept $\mathrm{P}(T \leq 55 \mid T > 40)$, $\mathrm{P}(T \leq 55 \mid T \geq 40)$, etc.

(a) recognising to find $\mathrm{P}(T > 40)$ ***(M1)***

$\mathrm{P}(T > 40) = 0.574136...$

$\mathrm{P}(T > 40) = 0.574$ ***A1***

***[2 marks]***

(b) attempt to multiply four independent probabilities using their $\mathrm{P}(T > 40)$ and $\mathrm{P}(T < 40)$ ***(M1)***

$(1-p)^3 \cdot p$ OR $(1-0.574136...)^3 \cdot 0.574136...$ OR $(0.425863...)^3 \cdot 0.574136...$ ***(A1)***

$0.0443430...$

$0.0443$ , $0.0444$ from 3 sf values ***A1***

***[3 marks]***

*continued…*

*Question 9 continued*

(c) (i) recognizing conditional probability ***(M1)***

$\mathrm{P}(T < 55 \mid T > 40)$

**Note:** Award ***(M1)*** for an expression or description in context. Accept $\mathrm{P}(T > 40 \mid T < 55)$ but do not accept just $\mathrm{P}(A \mid B)$.

$\frac{\mathrm{P}(40 < T < 55)}{\mathrm{P}(T > 40)}$ ***(A1)***

$\frac{0.461944...}{0.574136...}$ ***(A1)***

$\mathrm{P}(T < 55 \mid T > 40) = 0.804590...$

$= 0.805$ ***A1***

(ii) recognizing binomial probability ***(M1)***

$X \sim \mathrm{B}(n,\ p)$

$n = 10$ and $p = 0.804589...$ ***(A1)***

$0.0242111..., 0.0240188...$ using $p = 0.805$

$\mathrm{P}(X = 5) = 0.0242$ ***A1***

***[7 marks]***

*continued…*

*Question 9 continued*

(d) Let $\mathrm{P}(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere) ***(M1)***

**EITHER**

expressing the three regions in one variable ***(M1)***

$x + 0.904 + 2x$ OR $\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a)$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b)$

OR $x$ and $2x$ correctly indicated on labelled bell diagram

$\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a) = 1$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b) = 1$ (or equivalent) ***(A1)***

**OR**

expressing either $\mathrm{P}(T < a)$ or $\mathrm{P}(T > b)$ only in terms of $\mathrm{P}(a \le T \le b)$ ***(M1)***

$(\mathrm{P}(T < a) =)\frac{1}{3}(1 - \mathrm{P}(a \le T \le b))$ OR $(\mathrm{P}(T > b) =)\frac{2}{3} \cdot (1 - \mathrm{P}(a \le T \le b))$

$x = \frac{1}{3}(1 - 0.904)(= 0.032)$ OR $\mathrm{P}(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$ ***(A1)***

**THEN**

$\mathrm{P}(T < a) = 0.032$

$a = 22.18167...$

$a = 22.2$ accept 22.1 ***A1***

***[4 marks]***

***Total [16 marks]***

**3.** (a) (i) summing frequencies of riders or finding complement ***(M1)***

probability $= \frac{34}{40}$ ***A1***

(ii) attempt to find expected value ***(M1)***

$\frac{16}{40}+\left(2\times\frac{13}{40}\right)+\left(3\times\frac{2}{40}\right)+\left(4\times\frac{3}{40}\right)$

$\frac{60}{40}\left(=1.5\right)$ ***A1***

***[4 marks]***

(b) evidence of **their** rides/visitor $\times 1000$ or $\div 10$ ***(M1)***

$1500$ OR $0.15$

$150$ (times) ***A1***

***[2 marks]***

***Total [6 marks]***

**2.** (a) $u_1 + 3d = u_4$ ***(M1)***

$0.6 + 3d = 0.15$

$d = -0.15$ ***A1***

***[2 marks]***

(b) **METHOD 1**

$u_2 = 0.45$ or $u_3 = 0.3$ (may be seen in their equation) ***(A1)***

summing their probabilities to 1 (seen anywhere) ***(M1)***

$\frac{0.6}{k} + \frac{u_2}{k} + \frac{u_3}{k} + \frac{0.15}{k} = 1$

$\frac{0.6}{k} + \frac{0.45}{k} + \frac{0.3}{k} + \frac{0.15}{k} = 1$ (or equivalent) ***(A1)***

$\frac{1.5}{k} = 1$

$k = 1.5$ ***A1***

***[4 marks]***

**METHOD 2** (using $S_n$ formula)

$S_4 = \frac{4}{2}\left(2(0.6) + (4-1)(-0.15)\right)$ OR $S_4 = \frac{4}{2}\left(2\left(\frac{0.6}{k}\right) + (4-1)\left(\frac{-0.15}{k}\right)\right)$

OR $S_4 = \frac{4}{2}(0.6 + 0.15)$ OR $S_4 = \frac{4}{2}\left(\frac{0.6}{k} + \frac{0.15}{k}\right)$ (or equivalent) ***(A1)***

summing their probabilities to 1 (seen anywhere) ***(M1)***

$\frac{u_1}{k} + \frac{u_2}{k} + \frac{u_3}{k} + \frac{u_4}{k} = 1$ OR $u_1 + u_2 + u_3 + u_4 = k$ OR $S_4 = 1$ OR $S_4 = k$

$\frac{4}{2}\left(2(0.6) + (4-1)(-0.15)\right) = k$ (or equivalent) ***(A1)***

$k = 1.5$ ***A1***

***[4 marks]***

***Total [6 marks]***

**4.** substitutes into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ to form

$0.55 = 0.4 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (or equivalent) ***(A1)***

substitutes into $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ to form $0.25 = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ (or equivalent) ***(A1)***

attempts to combine their two probability equations to form an equation in $\mathrm{P}(B)$ ***(M1)***

**Note:** The above two ***A*** marks are awarded independently.

correct equation in $\mathrm{P}(B)$ ***A1***

$0.55 = 0.4 + \mathrm{P}(B) - 0.25\mathrm{P}(B)$ OR $\dfrac{\mathrm{P}(B) - 0.15}{\mathrm{P}(B)} = 0.25$ OR $\mathrm{P}(B) - 0.15 = 0.25\mathrm{P}(B)$

(or equivalent)

$\mathrm{P}(B) = \dfrac{15}{75} \left( = \dfrac{1}{5} = 0.2 \right)$ ***A1***

***Total [5 marks]***

**4.** (a) recognising to find $y(25)$ ***(M1)***

$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$

$= 310$ (children) ***A1***

***[2 marks]***

(b) recognizing $x$ on $y$ is required ***(M1)***

$0.0935114\ldots$ and $7.43053\ldots$ ***(A1)***

$x = 0.0935y + 7.43$ ***A1***

***[3 marks]***

*continued…*

*Question 4 continued*

(c) attempt to substitute their answer to part (a) into their regression equation for either $x$ or $y$ ***(M1)***

$x = 0.0935114... \times 310 + 7.43053... (= 36.4190...)$

36 (accept 37 or 36.4) ***A1***

**Note:** Award ***(M1)A1FT*** for $x = 37$ found from using $y = 9.39x - 41.5$.

Award ***(M1)A0FT*** for a correct ***FT*** answer that lies outside $[15, 46]$.

***[2 marks]***

***Total [7 marks]***

**5. METHOD 1**

$Q_1 = 31.86$ OR $Q_3 = 32.14$ ***(A1)***

recognition that the area under the normal curve below $Q_1$ or above $Q_3$ is $0.25$ OR the area between $Q_1$ and $Q_3$ is $0.5$ (seen anywhere including on a diagram) ***(M1)***

**EITHER**

equating an appropriate correct normal CDF function to its correct probability ($0.25$ or $0.5$ or $0.75$) ***(A2)***

**OR**

$z = -0.674489...$ OR $z = 0.674489...$ (seen anywhere) ***(A1)***

$-0.674489... = \frac{31.86 - 32}{\sigma}$ OR $0.674489... = \frac{32.14 - 32}{\sigma}$ ***(A1)***

**THEN**

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

**METHOD 2**

recognition that the area under the normal curve below $Q_1$ or above $Q_3$ is $0.25$ OR the area between $Q_1$ and $Q_3$ is $0.5$ (seen anywhere including on a diagram) ***(M1)***

$z = -0.674489...$ OR $z = 0.674489...$ ***(A1)***

$(Q_1 =) 32 - 0.674489...\sigma$ OR $(Q_3 =) 32 + 0.674489...\sigma$ ***(A1)***

$(Q_3 - Q_1 =) 2 \times 0.674489...\sigma$

$2 \times 0.674489...\sigma = 0.28$ ***(A1)***

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

***Total [5 marks]***

**8.** (a) (i) $p = 12$ ***A1***

(ii) $q = 100$ ***A1***

***[2 marks]***

(b) $\mathrm{P}(\text{Adult}) = \frac{100}{160}$ $\left(= 0.625\right)$ (seen anywhere) ***(A1)***

**Note:** Award ***A1*** for $\left(X \sim\right) \mathrm{B}(3, 0.625)$ or $\left(\frac{100}{160}\right)^3$ but no further marks.

recognition that choice of adults is without replacement (may be seen in tree diagram) ***(M1)***

$\frac{100}{160} \times \frac{99}{159} \times \frac{98}{158}$ ***(A1)***

$0.241372\ldots$

$0.241$ ***A1***

***[4 marks]***

(c) (i) $\frac{x}{48+x}\left(= \frac{1}{3}\right)$ OR $\frac{\frac{x}{160}}{\frac{48+x}{160}}$ ***(A1)(A1)***

**Note:** Award ***A1*** for correct numerator and ***A1*** for correct denominator.

$x = 24$ ***A1***

(ii) $\mathrm{P}\left(A \cap M\right) = \frac{24}{160}\left(= \frac{3}{20}\right)$ ***A1***

***[4 marks]***

*continued…*

*Question 8 continued*

(d) **METHOD 1 (using $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ )**

recognition that A and M are independent if $\mathrm{P}(A \mid M) = \mathrm{P}(A)$ ***(M1)***

$\frac{1}{3} \neq \frac{100}{160}$ ***R1***

so they are not independent ***A1***

**METHOD 2 (using $\mathrm{P}(A) \cdot \mathrm{P}(B) = \mathrm{P}(A \cap B)$ )**

attempt to find the product $\mathrm{P}(A) \times \mathrm{P}(M)$ OR $\mathrm{P}(A) \times \mathrm{P}(D)$ ***(M1)***

$\frac{100}{160} \times \frac{72}{160} \neq \frac{24}{160}$ $\left(\frac{9}{32} \neq \frac{3}{20}\right)$ OR $\frac{100}{160} \times \frac{88}{160} \neq \frac{76}{160}$ $\left(\frac{11}{32} \neq \frac{19}{40}\right)$ ***R1***

so they are not independent ***A1***

**Note:** Do not award ***R0A1***.

***[3 marks]***

(e) P(dark chocolate) $= \frac{88}{160}$ $(= 0.55)$ (maybe seen in part (d)) ***(A1)***

recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.55)$

recognition that $\mathrm{P}(X \geq 5)$ or $1 - \mathrm{P}(X \leq 4)$ is required ***(M1)***

**Note:** These two M marks are independent of each other.

0.738437…

0.738 ***A1***

***[4 marks]***

***Total [17 marks]***

**1.** (a) $a = 1.93258...,\ b = 7.21662...$

$a = 1.93,\ b = 7.22$ ***A1A1***

***[2 marks]***

(b) $r = 0.991087...$

$r = 0.991$ ***A1***

***[1 mark]***

(c) attempt to substitute $d = 20$ into their equation ***(M1)***

$\text{height} = 45.8683...$

$\text{height} = 45.9$ (cm) ***A1***

***[2 marks]***

***Total [5 marks]***

**4.** (a) evidence of attempting to find correct area under normal curve ***(M1)***

$\mathrm{P}(W > 210)$ OR sketch

$\mathrm{P}(W > 210) = 0.115069...$

$\mathrm{P}(W > 210) = 0.115$ ***A1***

***[2 marks]***

(b) recognizing $\mathrm{P}(W < w) = 1 - \mathrm{P}(w < W < 210) - \mathrm{P}(W > 210)$ ***(M1)***

$\mathrm{P}(W < w) = 1 - 0.8 - 0.115069...$

$\mathrm{P}(W < w) = 0.084930...$

$\mathrm{P}(W < w) = 0.0849$ ***A1***

***[2 marks]***

(c) evidence of attempting to use inverse normal function ***(M1)***

$w = 197.136...$

$w = 197$ (grams) ***A1***

***[2 marks]***

(d) recognition of binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.0849302...)$

$\mathrm{P}(X = 1) = 0.382076...$

$\mathrm{P}(X = 1) = 0.382$ ***A1***

***[2 marks]***

***Total [8 marks]***

**9.** (a) (i) $\frac{10}{n}$ ***A1***

(ii) multiplying probabilities for *GG* ***(M1)***

$$\mathrm{P}(GG) = \frac{10}{n} \times \frac{9}{n-1}$$

$$\mathrm{P}(GG) = \frac{90}{n^2 - n} \left( \text{accept} \frac{90}{n(n-1)} \right)$$ ***A1***

***[3 marks]***

(b) $\mathrm{P}(\text{First red}) = \frac{15}{25}$ and $\mathrm{P}(\text{Second red}) = \frac{14}{24}$ (seen anywhere) ***(A1)***

$\mathrm{P}(RR) = \frac{15}{25} \times \frac{14}{24}$ (or equivalent) ***A1***

$= 0.35$ ***AG***

***[2 marks]***

(c) $\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23}$ OR $0.35 \times \frac{13}{23}$ ***(A1)***

$0.197826...$

$\mathrm{P}(\text{three red}) = 0.198$ (exact answer is $\frac{91}{460}$) ***A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(d) $\text{P(at least one green)} = 1 - \text{P(three red)}$ OR

P(at least one G)=P(one G)+P(two G)+P(three G) ***(M1)***

$1-\left(\frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\right)$ OR $3\left(\frac{10}{25}\times\frac{15}{24}\times\frac{14}{23}\right)+3\left(\frac{10}{25}\times\frac{9}{24}\times\frac{15}{23}\right)+\left(\frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}\right)$

$0.802173...$

$\text{P(at least one green)} = 0.802$ (exact answer is $\frac{369}{460}$) ***A1***

***[2 marks]***

(e) $\text{P(first green on third draw)} = \frac{15}{25}\times\frac{14}{24}\times\frac{10}{23}\times\frac{22}{22}\left(=\frac{7}{46}=0.152173...\right)$ ***(A1)***

$\text{P}\left(\text{first green on fourth draw}\right) = \frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\times\frac{10}{22}\left(=\frac{91}{1012}=0.0899209...\right)$ ***(A1)***

**Note**: The first two ***(A1)*** are independent.

attempt to substitute their probabilities into expected value formula ***(M1)***

expected points per game $=10\times\frac{7}{46}+50\times\frac{91}{1012}\left(=\frac{3045}{506}=6.01778...\right)$ ***(A1)***

setting up inequality or equation in $k$ ***(M1)***

$\frac{3045}{506}k > 100$

$k > 16.6174...\left(=\frac{10120}{609}\right)$

Millie must play at least 17 times. ***A1***

***[6 marks]***

***Total [15 marks]***

**6.** $\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.6$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

**Note:** The first two ***A*** marks are independent of each other.

**EITHER** (finding intersections of functions)

attempt to make $a$ the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.6 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

**OR** (solving algebraically)

attempt to solve their equations algebraically to find a cubic in $k$ ***(M1)***

$k^3 - k^2 - 2k + 0.4 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.6 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in $k$ ***(M1)***

**THEN**

$a = 0.773073...$ OR $k = 0.185928...$

(Other solutions to the cubic are: $k = 1.92921...$, $k = -1.11514...$)

$a = 0.773$ ***A1***

***Total [5 marks]***

## Section B

**9.** (a) recognizing probabilities sum to $1$ ***(M1)***

$0.213 + \mathrm{P}(82.4 < X < 87.3) + 0.409 = 1$

$\mathrm{P}(82.4 < X < 87.3) = 0.378$ ***A1***

***[2 marks]***

(b) **METHOD 1**

recognizing the need to use inverse normal with $0.213,\ (1 - 0.409)$ or $0.409$ ***(M1)***

$m + \text{invNorm}(0.213)s = 82.4,\ m + \text{invNorm}(1 - 0.409)s = 87.3$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either $z$-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 86.2011...,\ \sigma = 4.77502...$

$\mu = 86.2,\ \sigma = 4.78$ ***A1***

**Note:** Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

**METHOD 2**

use of inverse normal to find at least one $z$-score for $P(Z < z) = 0.213$, or $\mathrm{P}(Z < z) = 1 - 0.409$ ***(M1)***

$z_1 = -0.796055...$ OR $z_2 = 0.230118...$

$\frac{82.4 - \mu}{\sigma} = -0.796055...,\ \frac{87.3 - \mu}{\sigma} = 0.230118...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve $z$-values rather than probabilities) ***(M1)***

$m = 86.2011...,\ s = 4.77502...$

$m = 86.2,\ s = 4.78$ ***A1***

***[5 marks]***

*continued…*

*Question 9 continued*

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \text{B}(100, 0.409)$

$\text{P}(X = 32) = 0.0157931...$

$= 0.0158$ ***A1***

(ii) $\text{P}(X < 44) = 0.702975...$(seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

**Note:** recognition must be shown in context, either in symbols eg $\text{P}(X = 32 \mid X < 44)$, or in words eg $\text{P}(32 \text{ plants} \mid \text{less than } 44 \text{ plants})$, not only as $\text{P}(A \mid B)$.

$\left(\text{P}(X = 32 \mid X < 44) = \right) \dfrac{\text{P}(X = 32)}{\text{P}(X < 44)}$ OR $\dfrac{\text{P}(X = 32)}{\text{P}(X \leq 43)} \left( = \dfrac{0.0157931...}{0.702975...} \right)$ ***(A1)***

$= 0.0224661...$

$\text{P}(X = 32 \mid X < 44) = 0.0225$ ***A1***

***[6 marks]***

*continued…*

*Question 9 continued*

(d) $Q_1 = 90.54$ OR $Q_3 = 95.06$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(90.54 < F < 95.06) = 0.5$ OR $P(F < 90.54) = 0.25$ OR $P(F < 95.06) = 0.75$ (or equivalent)

**EITHER**

attempt to find $d$ using graph or table ***(M1)***

**OR**

$1 - 2P\left(Z < -\frac{2.26}{d}\right) = 0.5$ OR $P(Z < -\frac{2.26}{d}) = 0.25$ OR $P(Z < \frac{2.26}{d}) = 0.75$

OR $P(-\frac{2.26}{d} < Z < \frac{2.26}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.26}{d} = -0.674489...$ OR $\frac{2.26}{d} = 0.674489...$

**THEN**

$3.35068...$

$d = 3.35$ ***A1***

***[3 marks]***

***Total [16 marks]***

**6.** $\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.3$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

**Note:** The first two ***A*** marks are independent of each other.

**EITHER** (finding intersections of functions)

attempt to make $a$ the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.3 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

**OR** (solving algebraically)

attempt to solve their equations algebraically to find a cubic in $k$ ***(M1)***

$k^3 - k^2 - 2k + 0.7 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.3 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in $k$ ***(M1)***

**THEN**

$a = 0.552839...$ OR $k = 0.315870...$ (other solutions to cubic are $k = -1.18538..., 1.86951...$ )

$a = 0.553$ ***A1***

***Total [5 marks]***

## Section B

**9.** (a) recognizing probabilities sum to $1$ ***(M1)***

$0.288 + \mathrm{P}(94.6 < X < 98.1) + 0.434 = 1$

$\mathrm{P}(94.6 < X < 98.1) = 0.278$ ***A1***

***[2 marks]***

(b) **METHOD 1**

recognizing the need to use inverse normal with $0.288$, $(1-0.434)$ or $0.434$ ***(M1)***

$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1-0.434)\sigma = 98.1$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either $z$-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 97.2981..., \sigma = 4.82468...$

$\mu = 97.3, \sigma = 4.82$ ***A1***

**Note:** Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

**METHOD 2**

use of inverse normal to find at least one $z$-score for $\mathrm{P}(Z < z) = 0.288$ or $\mathrm{P}(Z < z) = 1 - 0.434$ ***(M1)***

$z_1 = -0.559236...$ OR $z_2 = 0.166199...$

$\frac{94.6-\mu}{\sigma} = -0.559236...$, $\frac{98.1-\mu}{\sigma} = 0.166199...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve $z$-values rather than probabilities) ***(M1)***

$\mu = 97.2981..., \sigma = 4.82468...$

$\mu = 97.3, \sigma = 4.82$ ***A1***

***[5 marks]***

*Question 9 continued*

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.434)$

$\mathrm{P}(X = 34) = 0.0133198...$

$= 0.0133$ ***A1***

(ii) $\mathrm{P}(X < 49) = 0.848218...$ (seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

**Note:** recognition must be shown in context, either in symbols eg $\mathrm{P}(X = 34 \mid X < 49)$, or in words eg $\mathrm{P}(34 \text{ plants} \mid \text{less than } 49 \text{ plants})$, not only as $\mathrm{P}(A \mid B)$.

$\left(\mathrm{P}(X = 34 \mid X < 49) =\right) \dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X < 49)}$ OR $\dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X \leq 48)} \left(= \dfrac{0.0133198...}{0.848218...}\right)$ ***(A1)***

$= 0.0157033...$

$\mathrm{P}(X = 34 \mid X < 49) = 0.0157$ ***A1***

***[6 marks]***

*continued…*

*Question 9 continued*

(d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$ (or equivalent)

**EITHER**

attempt to find $d$ using graph or table ***(M1)***

**OR**

$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5$ OR $P(Z < -\frac{2.41}{d}) = 0.25$ OR $P(Z < \frac{2.41}{d}) = 0.75$

OR $P(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.41}{d} = -0.674489...$ OR $\frac{2.41}{d} = 0.674489...$

**THEN**

$3.57307...$

$d = 3.57$ ***A1***

***[3 marks]***

***Total [16 marks]***

**2.** (a) attempt to form equation for the sum of frequencies$=16$ or mean$=3$ ***(M1)***

$p+q+4+2+3=16\left(\Rightarrow p+q=7\right)$ ***A1***

$\frac{p+2q+12+8+18}{16}=3\left(\Rightarrow p+2q=10\right)$ OR $\frac{p+2q+12+8+18}{9+p+q}=3\left(\Rightarrow 2p+q=11\right)$ ***A1***

attempt to eliminate one variable from their equations ***(M1)***

$p+2\left(7-p\right)+38=48$ OR $2\left(7-q\right)+q=11$

$p=4$ and $q=3$ ***A1***

**Note:** Award ***M1A0A0M0A1*** for $p=4, q=3$ with no working.

***[5 marks]***

(b) mean final score $=30$ ***A1***

***[1 mark]***

***Total [6 marks]***

**9.** (a) evidence of understanding that there are now $3$R and $2$B ***(M1)***

$p = \frac{3}{5},\ q = \frac{2}{5}$ ***A1***

***[2 marks]***

(b) attempt to add two products ***(M1)***

P(same) = P(RR or BB) = $\frac{3}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{2}{5}$ ***A1***

$= \frac{14}{20}$

$= \frac{7}{10}$ ***AG***

***[2 marks]***

(c) attempt to use conditional probability formula in context ***(M1)***

$\mathrm{P(RR|same)} = \frac{\mathrm{P(RR)}}{\mathrm{P(same)}}$

**Note:** Award ***M0*** if candidate only writes $\mathrm{P}(A|B)$ formula and nothing else.

$= \frac{\left(\frac{12}{20}\right)}{\left(\frac{14}{20}\right)}$ ***(A1)***

$= \frac{12}{14}\left(= \frac{6}{7}\right)$ ***A1***

***[3 marks]***

*continued…*

*Question 9 continued*

(d) $a = \frac{6}{20}\left(= \frac{3}{10}\right),\ b = \frac{12}{20}\left(= \frac{6}{10}\right)$ ***A1A1***

***[2 marks]***

(e) attempt to use the formula for $\mathrm{E}(X)$ ***(M1)***

$\mathrm{E}(X) = 0 \times \frac{1}{10} + 1 \times \frac{6}{20} + 2 \times \frac{12}{20}$

$= \frac{30}{20}\left(= \frac{3}{2}\right)$ ***A1***

***[2 marks]***

(f) $\frac{1}{6}$ ***A1***

***[1 mark]***

(g) **METHOD 1**

$\mathrm{P}(n-1 \text{ reds }) = \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{n+1}{n+2}\ \left(= \frac{3}{n+2}\right)$ (***A1)***

$\mathrm{P}(\text{next one blue}) = \frac{1}{n+3}$ (***A1)***

$\mathrm{P}(n-1 \text{ reds then 1 blue}) = \mathrm{P}(n-1 \text{ reds}) \times \mathrm{P}(\text{next one blue})$ ***(M1)***

$\frac{3}{n+2} \times \frac{1}{n+3} = \frac{3}{56}$ (***A1)***

$(n+2)(n+3) = 56$

$n = 5$ ***A1***

**Note:** If no working shown, award ***M1A0A0A0A1*** for $n = 5$.

*continued…*

*Question 9 continued*

**METHOD 2**

Let $X$ be the number of selections in total made when first blue picked

attempt to establish pattern for $X = 1, 2, 3, \ldots$ with at least $3$ cases ***(M1)***

$\mathrm{P}(X = 1) = \frac{1}{4}$ and $\mathrm{P}(\text{second pick}) = \frac{3}{4} \times \frac{1}{5}$ ***(A1)***

$\mathrm{P}(X = 3)\left(= \frac{3}{4} \times \frac{4}{5} \times \frac{1}{6}\right) = \frac{3}{5} \times \frac{1}{6}$ ***(A1)***

$\mathrm{P}(X = 5) = \frac{3}{7} \times \frac{1}{8}\left(= \frac{3}{56}\right)$ ***(A1)***

so $n = 5$ ***A1***

**METHOD 3**

$\mathrm{P}(\text{next one blue}) = \frac{1}{n+3}$ ***(A1)***

recognising $\mathrm{P}(n-1 \text{ R then } 1\text{B}) = \mathrm{P}(n-1 \text{ R}) \times \mathrm{P}(\text{next one B})$ OR $\frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{1}{n+3}$ ***(M1)***

$\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{1}{8}\left(= \frac{3}{56}\right)$ ***(A1)(A1)***

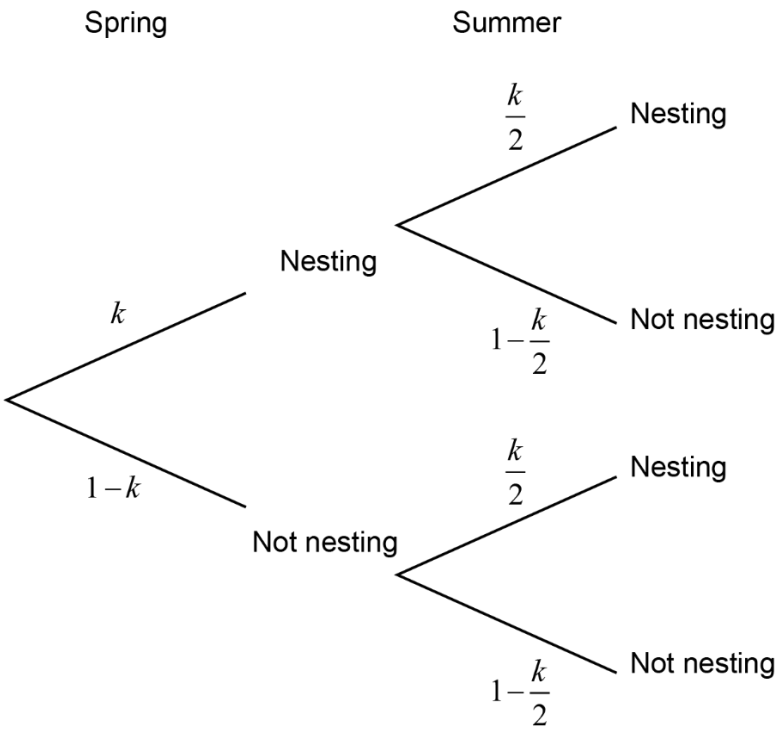
**Note:** Award ***A1*** for $\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$ (seen anywhere) and ***A1*** for $\times \frac{1}{8}$.

so $n = 5$ ***A1***

***[5 marks]***

***Total [17 marks]***

**5.** (a)




$1-k$ for Spring ***A1***

$1-\frac{k}{2}$ for both Summers ***A1***

***[2 marks]***

*continued…*

*Question 5 continued.*

(b) (i) multiplying the two correct branches ***(A1)***

$(1-k)\left(1-\frac{k}{2}\right)$

attempt to expand and equate to $\frac{5}{9}$ ***(M1)***

$1-k-\frac{k}{2}+\frac{k^2}{2}=\frac{5}{9}$

$18-18k-9k+9k^2=10$ OR $\frac{k^2}{2}-\frac{3k}{2}+\frac{4}{9}=0$ OR $\frac{9k^2}{2}-\frac{27k}{2}+4=0$ ***A1***

$9k^2-27k+8=0$ ***AG***

(ii) ($k=\frac{1}{3}$ is the only valid solution as) $\frac{8}{3}>1$ ***R1***

**Note:** Accept any valid reasoning indicating that any probability cannot be greater than 1 and/or probability cannot be less than 0.

***[4 marks]***

***Total [6 marks]***

**1.** (a) median = $75$ ***A1***

(upper quartile =) $84$ OR (lower quartile =) $68$ ***(A1)***

Interquartile range = $16$ ***A1***

***[3 marks]***

(b)

**Note**: In this part, their reasoning and answer must be consistent with their values in part (a).

In both part (i) and part (ii), award ***R0A1*** for a correct answer with no reasoning.

(i) $80 > 75$ OR Australia has a higher median OR Spain's median is lower ***R1***

**Note**: Award ***R1*** for correct reasoning based on a comparison of their medians.

in general (on average), rabbits in Australia have longer ears (than rabbits in Spain)

OR

in the top $50\%$ of each distribution, some rabbits in Spain have smaller ears than those in Australia

OR

in the bottom $50\%$ of each distribution, some rabbits in Australia have longer ears than those in Spain ***A1***

(ii) $16 > 11$ OR higher IQR in Spain OR lower IQR in Australia ***R1***

**Note**: Award ***R1*** for correct reasoning based on a comparison of their IQR's.

(the IQR's suggest that) there is more variation/spread in (the middle 50% of) rabbit ears in Spain (than those in Australia) (or equivalent) ***A1***

**Note**: Award ***A1*** for any correct answer which uses the IQR's to compare each distribution.

***[4 marks]***

***Total [7 marks]***



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**4.** (a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) ***(M1)***

$\left(\mathrm{P}\left(X > 13\right) =\right)\ 0.0668072...$

$= 0.0668$ ***A1***

***[2 marks]***

(b) **EITHER**

equating an appropriate correct normal CDF function to $0.1$ or $0.9$ ***(M1)***

$\mathrm{P}\left(X > 10 + 2k\right) = 0.1$ OR $\mathrm{P}\left(Z < k\right) = 0.9$ OR $\mathrm{P}\left(X < 10 - 2k\right) = 0.1$ OR $\mathrm{P}\left(Z < -k\right) = 0.1$

**OR**

recognising need to use inverse normal with $0.1$ or $0.9$ ***(M1)***

**THEN**

$1.28155...$

$k = 1.28$ ***A1***

***Total [4 marks]***

**6.** (a) $r = 0.901017\ldots$

$r = 0.901$ ***A2***

***[2 marks]***

(b) Student $11$ Test B: should not extrapolate ***R1***

***[1 mark]***

(c) (i) Student $12$ Test A: should not use line of $y$ on $x$ to predict $x$ from $y$ (or equivalent) ***R1***

(ii) attempt to find the equation of the regression line of $x$ on $y$ ***(M1)***

$(x =)\ 0.987124\ldots y - 3.21970\ldots\ \ ((x =)\ 0.987y - 3.22)$ ***A1***

$(x =)\ 0.987124\ldots(90) - 3.21970\ldots\ \ (= 85.6214...)$ ***A1***

$= 86$ to nearest integer. ***AG***

**Note**: Condone notation for $x$ and $y$ switched if values are correct.

***[4 marks]***

***Total [7 marks]***

## Section B

**1.** (a) (i) $4.45$ (hours) ***A1***

(ii) one correct quartile either $Q_1 = 1.9$ or $Q_3 = 5.7$ ***(A1)***

$\text{IQR} = 5.7 - 1.9$

$= 3.8$ (hours) ***A1***

***[3 marks]***

(b) attempts to find the upper fence value ***(M1)***

upper fence $= 11.4$ ***(A1)***

$11.7 > 11.4$ ***R1***

$\Rightarrow 11.7$ is an outlier ***AG***

***[3 marks]***

***Total [6 marks]***



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**3.** (a) **EITHER**

$\overline{y} = 2.1875 \times 7 + 0.6875$ ***A1***

**OR**

$\overline{y} = 15.3125 + 0.6875$ ***A1***

**THEN**

$\overline{y} = 16$ ***AG***

***[1 mark]***

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in $p$ and $q$ ***(M1)***

$16 = \frac{9 + 13 + p + q + 21}{5}$ $\left(80 = p + q + 43 \Rightarrow p + q = 37\right)$ ***(A1)***

attempts to solve two linear equations simultaneously for $p$ and $q$ (one of which is $q = p + 3$) ***(M1)***

$16 = \frac{9 + 13 + p + p + 3 + 21}{5}$ $\left(80 = 2p + 46\right)$

$p = 17$ and $q = 20$ ***A1***

***[4 marks]***

***Total [5 marks]***

**6.**

**METHOD 1**

correct inequality or equation involving $\mathrm{P}(X=0)$ ***(A1)***

$1-\mathrm{P}(X=0)>0.99$ OR $\mathrm{P}(X=0)<0.01$ OR $1-\mathrm{P}(X=0)=0.99$ OR $\mathrm{P}(X=0)=0.01$

attempts to solve their inequality (equality) involving $0.75^n$ for $n$ ***(M1)***

$1-0.75^n>0.99$ OR $0.75^n<0.01$ OR $0.75^n=0.01$ OR $1-0.75^n=0.99$

**Note:** Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

**EITHER**

$n>16.0078...$ OR $n=16.0078...$ ***(A2)***

the least value of $n$ is $17$ ***A1***

**OR**

$\mathrm{P}(X=0)=0.010022...\ (>0.01)$ (corresponding to $n=16$) ***(A1)***

$\mathrm{P}(X=0)=0.0075169...\ (<0.01)$ ***(A1)***

corresponding to $n=17$ (which is the least value of $n$) ***A1***

*continued…*

*Question 6 continued.*

**METHOD 2 (TABLE ONLY APPROACH)**

attempts to use binomial cdf to calculate a correct value of $\mathrm{P}(X \geq 1)$ for one value of $n$ ***(M1)***

calculates correct values of $\mathrm{P}(X \geq 1)$ for at least one value of $n$ ***(A1)***

$\mathrm{P}(X \geq 1) = 0.989977... \ (< 0.99)$ (corresponding to $n = 16$) ***(A1)***

$\mathrm{P}(X \geq 1) = 0.992483... \ (> 0.99)$ ***(A1)***

corresponding to $n = 17$ (which is the least value of $n$) ***A1***

***[5 marks]***

**SECTION B**

**7.** (a) recognition to add $\mu$ and $\sigma$ ***(M1)***

$49.8$ (cm) ***A1***

***[2 marks]***

(b) $\mathrm{P}(L > 48)$ ***(M1)***

**Note:** Award ***(M1)*** for a clearly labelled diagram.

$= 0.283854...$

$= 0.284$ ***A1***

***[2 marks]***

(c) $\mathrm{P}(L > x) = 0.99$ OR $\mathrm{P}(L < x) = 0.01$ ***(M1)***

**Note:** Award ***(M1)*** for a clearly labelled diagram or the use of inverse normal.

$x = 35.8293...$

$x = 35.8$ (cm) ***A1***

***[2 marks]***

*continued…*

*Question 7 continued.*

(d) (i) $\mathrm{P}(40 < L < 56) = 0.902149...$ (may be seen in part (ii)) ***(A1)***

attempts to find $100 \times \mathrm{P}(40 < L < 56)$ with their probability ***(M1)***

$= 90.2149...$

$= 90.2$ ***A1***

**Note:** Accept $90$ or $91$.

(ii) recognizes binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.902149...)$

$\mathrm{P}(X = 95) = 0.038105...$

$= 0.0381$ ***A1***

***[5 marks]***

*continued…*

*Question 7 continued.*

(e) $\mathrm{P}(45.55 \leq L < 45.65)$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for any reasonable interval centred on $45.6$, no wider than $\mathrm{P}(45.5 \leq L < 45.7)$.

Accept either of $\mathrm{P}(45.55 \leq L \leq 45.65)$ or $\mathrm{P}(45.55 < L < 45.65)$.

$= 0.009498...$

$= 0.00950$ ***A1***

***[3 marks]***

***Total [14 marks]***

**1.** (a) (i) $27$ ***A1***

(ii) $30$ ***A1***

(iii) $35$ ***A1***

***[3 marks]***

(b) $Q_1 = 31$ and $Q_3 = 46$ ***(A1)***

attempt to subtract their upper and lower quartiles ***(M1)***

**Note:** Award ***M1*** only for correct values, or for values clearly indicated as candidate's $Q_1$ and $Q_3$.

$46 - 31$

$\text{IQR} = 15$ ***A1***

***[3 marks]***

***[Total: 6 marks]***



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**3.** (a) use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

$\mathrm{P}(A \cap B) = 0.45 + 0.65 - 0.8$ (or equivalent) ***(A1)***

$= 0.3$ ***A1***

***[3 marks]***

(b) $\mathrm{P}(A' \cap B') = 0.2$ (may be seen in Venn diagram) ***(A1)***

attempt to substitute their values into $\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ ***(M1)***

$\mathrm{P}(A' \mid B') = \dfrac{0.2}{0.35}$

$= \dfrac{20}{35}\left(= \dfrac{4}{7}\right)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**1.** (a) (i) $22$ ***A1***

(ii) $20$ ***A1***

(iii) $30$ ***A1***

***[3 marks]***

(b) $Q_1 = 26$ and $Q_3 = 38$ ***(A1)***

attempt to subtract their upper and lower quartiles ***(M1)***

**Note:** Award ***M1*** only for correct values, or for values clearly indicated as candidate's Q1 and Q3.

$38 - 26$

$\text{IQR} = 12$ ***A1***

***[3 marks]***

***[Total: 6 marks]***

**3.** (a) use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

$\mathrm{P}(A \cap B) = 0.65 + 0.45 - 0.85$ (or equivalent) ***(A1)***

$= 0.25$ ***A1***

***[3 marks]***

(b) $\mathrm{P}(A' \cap B') = 0.15$ (may be seen in Venn diagram) ***(A1)***

attempt to substitute their values into $\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ ***(M1)***

$= \dfrac{15}{55}\left(= \dfrac{3}{11}\right)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**4.** (a) recognition of sum of probabilities equals $1$ ***(M1)***

$\frac{2k}{15}+\frac{4k}{15}+\frac{7k}{15}+\frac{10k}{15}=1$

$k=0.652173...$

$k=0.652\left(=\frac{15}{23}\right)$ ***A1***

***[2 marks]***

(b) correct probabilities: $\frac{2}{23},\frac{4}{23},\frac{7}{23},\frac{10}{23}$ OR $0.0870,\ 0.174,\ 0.304,\ 0.435$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$2\times\frac{2}{23}+4\times\frac{4}{23}+7\times\frac{7}{23}+10\times\frac{10}{23}$ OR $\frac{169k}{15}$

$=7.34782...$

$\mathrm{E}(X)=7.35\left(=\frac{169}{23}\right)$ (same 3sf from previous 3sf answer) ***A1***

***[3 marks]***

***Total [5 marks]***

**7.** (a) $16.7037\ldots$

$16.7(\%)$ ***A1***

***[1 mark]***

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.167...)$

attempt to find $\mathrm{P}(X < 150)$ OR $\mathrm{P}(X \leq 149)$ ***(M1)***

$0.0669378...$

$\mathrm{P}(X < 150) = 0.0669$ (accept $0.0673$ from previous 3sf) ***A1***

***[3 marks]***

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \geq 75 \mid \text{age} \geq 55)$ OR $\dfrac{\mathrm{P}(\text{age} \geq 55 \cap \text{age} \geq 75)}{\mathrm{P}(\text{age} \geq 55)}$

**Note**: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \geq 75)}{\mathrm{P}(\text{age} \geq 55)}$ ***(A1)***

$\dfrac{2.1}{(2.1+12.9)}$ OR $\dfrac{0.0233853...}{0.167}$ ***(A1)***

$0.14$ (exact) $\left(= \dfrac{7}{50}\right)$ ***A1***

**Note**: Condone use of "$a$" or "$X$" or any letter for age.

***[4 marks]***

(d) $15, 26.5, 45, 65, 85$ ***A1***

***[1 mark]***

*continued…*

*Question 7 continued*

(e) (i) $38.0740\ldots$

mean = $38.1$ ***A1***

(ii) variance is square of standard deviation ***(M1)***

$16.3639...^2$

$267.778...$

variance = $268$ ***A1***

***[3 marks]***

(f)

(i) Graph D ***A1***

**Note**: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) SmallTalk ***A1***

**Note**: Do not award ***A0A1***.

***[2 marks]***

***Total [14 marks]***

**4.** (a) recognition of sum of probabilities equals $1$ ***(M1)***

$\frac{3k}{20}+\frac{5k}{20}+\frac{8k}{20}+\frac{11k}{20}=1$

$k=0.740740$

$k=0.741\left(=\frac{20}{27}\right)$ ***A1***

***[2 marks]***

(b) correct probabilities: $\frac{3}{27},\frac{5}{27},\frac{8}{27},\frac{11}{27}$ OR $0.111,\ 0.185,\ 0.296,\ 0.407$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$3\times\frac{3}{27}+5\times\frac{5}{27}+8\times\frac{8}{27}+11\times\frac{11}{27}$ OR $\frac{219k}{20}$

$=8.11111...$

$\mathrm{E}(X)=8.11\left(=\frac{219}{27}=\frac{73}{9}\right)$ (same 3sf from previous 3sf answer) ***A1***

***[3 marks]***

***Total [5 marks]***

**7.** (a) $22.4358\ldots$

$22.4(\%)$ ***A1***

***[1 mark]***

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.224...)$

attempt to find $\mathrm{P}(X < 200)$ OR $\mathrm{P}(X \leq 199)$ ***(M1)***

$0.0284945...$

$\mathrm{P}(X < 200) = 0.0285$ (accept $0.0303$ from previous 3sf) ***A1***

***[3 marks]***

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \geq 65 \mid \text{age} \geq 45)$ OR $\dfrac{\mathrm{P}(\text{age} \geq 45 \cap \text{age} \geq 65)}{\mathrm{P}(\text{age} \geq 45)}$

**Note**: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \geq 65)}{\mathrm{P}(\text{age} \geq 45)}$ ***(A1)***

$\dfrac{3.7}{(3.7+17.3)}$ OR $\dfrac{0.0395}{0.224}$ ***(A1)***

$0.176190...$

$0.176 \left(= \dfrac{37}{210}\right)$ ***A1***

**Note**: Condone use of "$a$" or "$X$" or any letter for age.

***[4 marks]***

*continued…*

*Question 7 continued.*

(d) $15.5, 21.5, 35, 55, 75$ ***A1***

***[1 mark]***

(e) (i) $35.7280\ldots$

mean = $35.7$ ***A1***

(ii) variance is square of standard deviation ***(M1)***

$14.3159...^2$

$204.945...$

variance = $205$ ***A1***

***[3 marks]***

(f)

(i) Graph C ***A1***

**Note**: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) MyLife ***A1***

**Note**: Do not award ***A0A1***.

***[2 marks]***

***Total [14 marks]***

**5.** (a) $\mathrm{P}(A) = \frac{1}{4}$ ***(A1)***

attempt to use $\mathrm{P}(B|A) = \frac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)}$ ***(M1)***

$\frac{2}{3} = \frac{\mathrm{P}(B \cap A)}{\left(\frac{1}{4}\right)}$

$\mathrm{P}(A \cap B) = \frac{2}{3}\left(\frac{1}{4}\right)$

$= \frac{2}{12}\left(= \frac{1}{6}\right)$ ***A1***

***[3 marks]***

(b) attempt to use $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ OR a Venn diagram, with their values of $\mathrm{P}(A)$ and $\mathrm{P}(B \cap A)$ ***M1***

$\frac{3}{4} = \frac{1}{4} + \mathrm{P}(B) - \frac{1}{6}$

$\mathrm{P}(B) = \frac{1}{2} + \frac{1}{6}$

$= \frac{4}{6}\left(= \frac{2}{3}\right)$ ***A1***

$\mathrm{P}(B|A) = \mathrm{P}(B)$ OR $\mathrm{P}(A)\mathrm{P}(B) = \frac{1}{6}$ so $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ (hence $A$ and $B$ are independent) ***R1***

**Note:** The ***R1*** is dependent on all previous marks

***[3 marks]***

***Total [6 marks]***

**7.** (a) recognise sum of probabilities is 1 (seen anywhere) ***(M1)***

$k+3k^2+2k^2+k^2=1$ OR $\sum \mathrm{P}(X=x)=1$

$6k^2+k-1=0$ OR $6k^2+k=1$ ***A1***

valid attempt to solve their quadratic $=0$ ***(M1)***

$(3k-1)(2k+1)$ OR $k=\dfrac{-1\pm\sqrt{1-4(6)(-1)}}{2(6)}$

$k=\dfrac{1}{3}$ and $k=-\dfrac{1}{2}$ ***A1***

$0<k<1$ (so, disregard $k=-\dfrac{1}{2}$) ***R1***

**Note:** accept $k>0$ or $k\neq 0$ or $k$ is a probability.

Do not award ***R1*** unless the previous mark has been awarded.

$k=\dfrac{1}{3}$ ***AG***

**Note:** If a candidate uses $k=\dfrac{1}{3}$ to verify that the probabilities sum to 1, award ***M1A0M0A0R0***.

***[5 marks]***

(b) $1-\mathrm{P}(X=3a)$ OR $\mathrm{P}(X=0)+\mathrm{P}(X=a)+\mathrm{P}(X=2a)$ ***(M1)***

$1-k^2\ \left(=1-\dfrac{1}{9}\right)$ OR $5k^2+k\ \left(=\dfrac{5}{9}+\dfrac{1}{3}\right)$ OR $\dfrac{1}{3}+\dfrac{3}{9}+\dfrac{2}{9}$

$\mathrm{P}(X<3a)=\dfrac{8}{9}$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued.*

(c) valid attempt to identify the correct required outcomes ***(M1)***

$\frac{\mathrm{P}(X \geq a \cap X < 3a)}{\mathrm{P}(X < 3a)}$ OR $\mathrm{P}(a \leq X < 3a)$ OR $\mathrm{P}(X = a) + \mathrm{P}(X = 2a)$ OR

$X = a$ and $2a$ OR $\mathrm{P}(X < 3a) - \mathrm{P}(X < a)$

correct numerator $3k^2 + 2k^2 \left(= 5k^2\right)$ OR $\frac{3}{9} + \frac{2}{9}$ OR $\frac{8}{9} - \frac{1}{3} \left(= \frac{5}{9}\right)$ ***(A1)***

$\frac{\frac{5}{9}}{\frac{8}{9}}$

$\mathrm{P}(X \geq a \,|\, X < 3a) = \frac{5}{8}$ ***A1***

***[3 marks]***

(d) attempt to use the expected value formula ***(M1)***

$0 \times \frac{1}{3} + a \times \frac{1}{3} + 2a \times \frac{2}{9} + 3a \times \frac{1}{9} \ (= 20)$

$\frac{10a}{9} = 20$ OR $3a + 4a + 3a = 180$ (or equivalent) ***A1***

$a = 18$ ***A1***

***[3 marks]***

***Total [13 marks]***

**1.** (a) $68$ **OR** $74$ ***(A1)***

$74-68$

$=6$ ***A1***

***[2 marks]***

(b) recognizing $25\%$ of golfers scored between $70$ and $74$ ***(M1)***

$25\%\times 80$ **OR** $60-40$

$=20$ ***A1***

***[2 marks]***

***Total [4 marks]***

**6.** (a) recognizing constant change from $Y$ to $X$ and/or comparing the two distributions

$b-7=22-19$ **OR** $b=7+(22-19)$ **OR** $\frac{b-7}{a}=\frac{22-19}{a}$ ***(M1)***

$b=10$ ***A1***

***[2 marks]***

(b) $\left(\mathrm{P}(7-a<X<7+a)=\right)\ 0.68$ ***A1***

***[1 mark]***

(c) **EITHER**

recognizing that $22$ is one standard deviation above the mean ***(M1)***

**OR**

recognizing symmetry of the normal curve and total area $=1$ ***(M1)***

**THEN**

$0.16$ or $0.34$ (or equivalent) seen in correct sketch or probability statement ***(A1)***

$\left(\mathrm{P}(Y<22)=\right)1-\frac{0.32}{2}$ **OR** $0.5+0.34$ **OR** $0.68+0.16$

$0.84$ ***A1***

***[3 marks]***

***Total [6 marks]***

## Section B

**5.** (a) attempt to substitute $F = 19.8$ into the regression line for $A$ on $F$ ***(M1)***

$A = 2.89(19.8) + 99.3$

$= 156.522 \text{ (cm)}$

arm span = $157 \text{ (cm)}$ ***A1***

| ***Note:*** Award ***M0A0*** for choosing the wrong regression line to get $A = 156.417...$ so $A = 156$. |
|---|

***[2 marks]***

(b) recognition that the lines intersect at the mean point (may be seen on a sketch) ***(M1)***

$2.89F + 99.3 = \frac{F + 32.6}{0.335}$ OR $0.335A - 32.6 = \frac{A - 99.3}{2.89}$

$159.686...$ or $20.8948...$

the mean arm span $= 160$ (cm) , the mean foot length $= 20.9$ (cm) ***A1A1***

***[3 marks]***

(c) **METHOD 1**

recognition of symmetry of interval around mean (may be seen on a sketch) ***(M1)***

$P(H < 153) = 0.06$ OR $P(H < 173) = 0.94$ OR equivalent

$\frac{153 - 163}{\sigma} = -1.55477...$ OR $\frac{173 - 163}{\sigma} = 1.55477...$ (***A1)***

$\sigma = 6.43181...$

$\sigma = 6.43 \text{ (cm)}$ ***A1***

**METHOD 2**

attempt to find $\sigma$ by equating an appropriate correct normal CDF function to $0.88$ (or e.g. $0.06$ or $0.94$) ***(M1)***

$\sigma = 6.43181...$

$\sigma = 6.43 \text{ (cm)}$ ***A2***

| **Note:** Accept use of calculator notation e.g. $normcdf(153, 173, 163, \sigma) = 0.88$ |
|---|

***[3 marks]***

***Total [8 marks]***

**7.** (a) $\mathrm{P}(\text{scores } 5 \text{ points}) = \frac{10}{36}$ ***A1A1***

**Note:** Award ***A1*** for numerator of 10 and ***A1*** for denominator of $36$.

$= \frac{5}{18}$ ***AG***

***[2 marks]***

(b) **METHOD 1**

attempts to list or count all outcomes with a difference of more than one ***(M1)***

$4+3+3+3+3+4\left(=20\right)$

$\mathrm{P}(\text{scores } 0 \text{ points}) = \frac{20}{36}\left(=\frac{5}{9}=0.555555...=0.556\right)$ ***A1***

**METHOD 2**

attempts to subtract P(scores $5$ points) and P(scores $10$ points) from $1$ ***(M1)***

$\mathrm{P}(\text{scores } 0 \text{ points}) = 1-\frac{10}{36}-\frac{6}{36}$

$= \frac{20}{36}\left(=\frac{5}{9}=0.555555...=0.556\right)$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued.*

(c) $\mathrm{P}(\text{scores } 10 \text{ points}) = \frac{6}{36}\left(=\frac{1}{6}\right)$ (may be seen in a table) ***(A1)***

| $x$ | 0 | 5 | 10 |
|---|---|---|---|
| $\mathrm{P}(X = x)$ | $\frac{20}{36}$ | $\frac{10}{36}$ | $\frac{6}{36}$ |

attempt to use formula for expected value ***(M1)***

$E(X) = \left(0 \times \frac{20}{36} + \right)5 \times \frac{10}{36} + 10 \times \frac{6}{36}$ ***(A1)***

$= \frac{110}{36}\left(= \frac{55}{18} = 3.05555... = 3.06\right)$ ***A1***

***[4 marks]***

(d) recognition that their part c) must be multiplied by $90$ ***(M1)***

$\frac{110}{36} \times 90$

$= 275$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued.*

(e) recognise that round scores must be five 10s or four 10s and a 5 ***(M1)***

$\mathrm{P(five\ 10s)} = \left(\frac{1}{6}\right)^5$, $\mathrm{P(four\ 10s\ and\ a\ 5)} = 5\left(\frac{1}{6}\right)^4\left(\frac{5}{18}\right)$ ***(A1) (A1)***

$\mathrm{P(Lynn\ wins\ a\ prize)} = 0.000128601... + 0.00107167...$

$= 0.00120027...\left(= 1.20027... \times 10^{-3}\right)$

$= 0.00120\left(= 1.20 \times 10^{-3} = \frac{7}{5832}\right)$ (accept $0.0012$) ***A1***

***[4 marks]***

***Total [14 marks]***

**1.**

(a) $1.97651..., 40.2286...$

$a = 1.98, b = 40.2 \quad ((y =)1.98x + 40.2)$ ***A1A1***

***[2 marks]***

(b) attempt to substitute $x = 20$ into their regression equation ***(M1)***

$y = 1.97651... \times 20 + 40.2286...$

$79.7589...$

$79.8$ (hours) ***A1***

***[2 marks]***

***Total [4 marks]***



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**8.**

(a) recognising to find $\mathrm{P}(W < 170)$ ***(M1)***

$\mathrm{P}(W < 170) = 0.265985\ldots$

$\mathrm{P}(W < 170) = 0.266$ (accept 26.6%) ***A1***

***[2 marks]***

(b) recognising $\mathrm{P}(W > w) = 0.2$ OR labelled sketch (or equivalent) ***(M1)***

$w = 181.732\ldots$

$w = 181.7$ (grams) (must be 4 sf) ***A1***

***[2 marks]***

(c) recognizing the need to find $\mathrm{P}(170 < W < 185)$ ***(M1)***

diagram, $\mathrm{P}(170 < W < 185) = 0.628(364\ldots)$

62.8% ***A1***

***[2 marks]***

(d) recognising binomial probability ***(M1)***

$X \sim \mathrm{B}(40, 0.628364\ldots)$ OR $n = 40$ and $p = 0.628364\ldots$ ***(A1)***

$\mathrm{P}(X \geq 30) = 0.073861\ldots$

$\mathrm{P}(X \geq 30) = 0.0739$ (accept 7.39%) ***A1***

***[3 marks]***

*continued…*

*Question 8 continued.*

(e) $Y \sim \mathrm{B}(10, 0.073861\ldots)$ ***(A1)***

$\mathrm{P}(Y = 4) = 0.003944\ldots$

$\mathrm{P}(Y = 4) = 0.00394$ (accept $0.394\%$) ***A1***

***[2 marks]***

(f) $\mathrm{P}(W < 170) = 0.12$ OR $\mathrm{P}(W > 185) = 0.06$ ***(A1)***

use of inverse normal to find $z$ score for their probability ***(M1)***

$z = -1.17(4986\ldots)$, $z = 1.55(4773\ldots)$ ***(A1)(A1)***

attempt to solve their two equations ***(M1)***

$$-1.174986\ldots = \frac{170 - \mu}{\sigma}$$

$$1.554773\ldots = \frac{185 - \mu}{\sigma}$$

$\mu = 176.456\ldots$

$\mu = 176$ (grams) ***A1***

***[6 marks]***

***Total [17 marks]***

**4.** (a) (i) summing probabilities and equating to $1$ ***(M1)***

$1.5a + 2a + 0.281 + a + 0.026 = 1$

$(a =)\ 0.154$ ***A1***

(ii) $2$ (days) ***A1***

***[3 marks]***

(b) using expected value formula ***(M1)***

(mean =) $2.44$ ($2.436$ (exact)) ***A1***

***[2 marks]***

(c) convenience (sampling) ***A1***

***[1 mark]***

***Total [6 marks]***

**8.** (a) fixed number of trials

each trial has two possible outcomes ( $p$, $q$ with $\mathrm{P}(p)+\mathrm{P}(q)=1$)

outcome of each trial is independent of all the others OR probability of success constant. ***A1A1***

**Note:** Award ***A1*** for each correct condition.

***[2 marks]***

(b) $1900\times 0.37$ ***(A1)***

$703$ (people) ***A1***

***[2 marks]***

(c) (i) Let $D$ be the number of people who will ride *Daifong*

recognition of binomial distribution ***(M1)***

$\mathrm{P}(D=712)=0.0172556...$

$=0.0173$ ***A1***

(ii) recognizing the need to find $\mathrm{P}(684\le D\le 712)$ ***(M1)***

$P(D\le 712)=0.674739...$

$P(D\le 683)=0.177146...$

$\mathrm{P}(684 £ D £ 712) = 0.497593...$

$=0.498$ ***A1***

**Note:** If the normal approximation to the binomial distribution is used, award marks as appropriate.

***[4 marks]***

*continued…*

*Question 8 continued*

(d) recognizing conditional probability ***(M1)***

**Note:** Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$.

$\mathrm{P}(D \leq 692 \mid 684 \leq D \leq 712)\left(= \dfrac{\mathrm{P}(684 \leq D \leq 692)}{\mathrm{P}(684 \leq D \leq 712)}\right)$ or equivalent in words

$\mathrm{P}(684 \leq D \leq 692) = 0.132318...$ (seen anywhere) ***(A1)***

$\left(\dfrac{\mathrm{P}(684 \leq D \leq 692)}{\mathrm{P}(684 \leq D \leq 712)} =\right) \dfrac{0.132318...}{0.497593...}$ ***(A1)***

$= 0.265917...$

$= 0.266$ ***A1***

***[4 marks]***

(e) substituting into formula for independent events ***(M1)***

$= 0.37 \times 0.61$

$= 0.226$ $(= 0.2257$ (exact)) ***A1***

***[2 marks]***

(f) (let $T$ be the number of people who went on *Torbellino*)

use of tables, guess-and-check or inverse binomial ***(M1)***

**Note:** Award ***(M1)*** for at least one correct value of $\mathrm{P}(T \leq 500)$ for a value of $n$.

Award ***(M1)*** for an attempt to set up the normal distribution with $\mathrm{z} = 0.504$.

$n = 809$ (accept $n = 808$) ***A2***

***[3 marks]***

***Total [17 marks]***

**3.** (a) $0.5+0.6-0.4$ ***(A1)***

$\mathrm{P}(A\cup B)=0.7$ ***A1***

***[2 marks]***

(b) $\left(\mathrm{P}(A)\times\mathrm{P}(B)=\right)0.3\neq 0.4\left(=\mathrm{P}(A\cap B)\right)$ ***A1***

hence $A$ and $B$ are not independent ***AG***

**Note:** Accept $0.5\times 0.6\neq 0.4$ or $0.3\neq \mathrm{P}(A\cap B)$.

***[1 mark]***

(c) $\mathrm{P}(A\cap B')=0.1$ (seen anywhere, including Venn diagram) ***(A1)***

attempt to use conditional probability with $A$ and $B'$ (seen anywhere) ***(M1)***

$\frac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$, $\frac{0.1}{0.4}$

**Note:** Accept using their $\mathrm{P}(A\cap B')$ and $\mathrm{P}(B')$ values, which may be seen in their Venn diagram.

$P(A\mid B')=\frac{1}{4}$ ***A1***

***[3 marks]***

***Total [6 marks]***

**1.** (a) recognition that sum of probabilities $=1$ ***(M1)***

$0.2+0.3+k+0.1=1$

$k=0.4$ ***A1***

***[2 marks]***

(b) recognition that $\mathrm{P}(X>1.5)=\mathrm{P}(X=2)+\mathrm{P}(X=3)+\mathrm{P}(X=4)$ OR $1-\mathrm{P}(X=1)$ ***(M1)***

$0.3+0.4+0.1$ OR $1-0.2$

$=0.8$ ***A1***

**Note:** Award ***(M1)A1FT*** only if $0<k<1$ and their '$0.8$' is $<1$.

***[2 marks]***

(c) attempt to substitute into the formula for $\mathrm{E}(X)$ ***(M1)***

$=(1\times 0.2)+(2\times 0.3)+3k+(4\times 0.1)$

$=2.4$ ***A1***

**Note:** Award ***(M1)A1FT*** only if $0<k<1$.

***[2 marks]***

***Total [6 marks]***



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**4.** (a) attempt to use $\mathrm{P}(A \cap B) = \mathrm{P}(A \mid B)\mathrm{P}(B)\left(= \frac{1}{4} \times \frac{1}{3}\right)$ ***(M1)***

$= \frac{1}{12}$ ***A1***

***[2 marks]***

(b) $\mathrm{P}(B') = \frac{2}{3}$ or $\mathrm{P}(A \cap B') = \frac{5}{12}$ ***(A1)***

attempt to use $\mathrm{P}(A \mid B') = \frac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')}$ ***(M1)***

$= \frac{\left(\frac{5}{12}\right)}{\left(\frac{2}{3}\right)}$ ***(A1)***

$= \frac{5}{8}\left(= \frac{15}{24}\right)$ ***A1***

***[4 marks]***

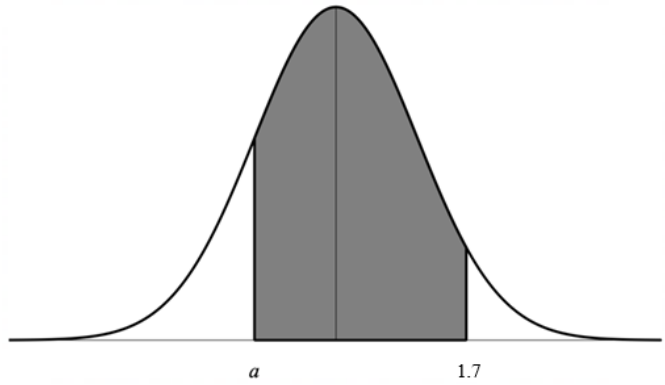
***Total [6 marks]***

**2.** (a) $0.955434\ldots$

$0.955$ ***A1***

***[1 mark]***

(b)




***A1***

**Note:** Award ***A1*** only if $a$ is indicated to the left of the mean AND the correct area is indicated either by shading, or by labelling with $0.7865$.

***[1 mark]***

(c) attempt to solve $\mathrm{P}(a < Z < 1.7) = 0.7865$ for $\mathrm{P}(Z < a)$ or $a$ ***(M1)***

their $\mathrm{P}(Z < 1.7) - \mathrm{P}(Z < a) = 0.7865 \left(\Rightarrow \mathrm{P}(Z < a) = 0.955434\ldots - 0.7865 = 0.168934...\right)$

**Note:** Accept calculator notation such as $\mathrm{normcdf}(a, 1.7, 0, 1) = 0.7865$.

$a = -0.958384...$

$a = -0.958$ (accept $-0.960$ or $-0.96$ from previous 3 sf value) ***A1***

***[2 marks]***

***Total [4 marks]***

**3.** (a) attempt to find mean of $x$-values ***(M1)***

$\overline{x} = 6.45$ ***A1***

***[2 marks]***

(b) (i) attempt to substitute their $\overline{x}$ into regression equation ***(M1)***

$\overline{y} = 0.014375 + 2.1625 \times 6.45$

$\overline{y} = 14.0 \ (13.9625 \text{ exact})$ ***A1***

(ii) attempt to use formula for the mean ***(M1)***

$\sum_{i=1}^{8} y_i = 8 \times 13.9625... \left(\Rightarrow p + 93.2 = 111.7\right)$

$p = 18.5$ ***A1***

**Note:** Accept $p = 18.8$ from the use of 3 sf in part (b)(i).
Some candidates are attempting part (b)(ii) before (b)(i), and substitute $8.9$ into the given regression equation to find $p$. They then use their value of $p$ to find $\overline{y}$. In this case, do not award any marks in this part. Allow ***FT*** from their incorrect $p$ in part (c).

***[4 marks]***

(c) $0.953958...$

$r = 0.954$ ***A1***

***[1 mark]***

***Total [7 marks]***

**6.** (a) recognition of binomial distribution ***(M1)***

$\binom{10}{2}(p)^2(1-p)^8,\ \mathrm{B}(10, p)$

**Note:** The ***(M1)*** may be awarded for just stating 'binomial', parameters are not required, accept incorrect parameters. Do not award the ***(M1)*** if candidates only state $np$. Accept the use of calculator notation, such as binompdf.

attempt to consider both machines using $p = 0.1$ AND $p = 0.05$ ***(M1)***

$0.5\times(0.193710...)+0.5\times(0.0746347...)$ ***(A1)***

$0.134172...$

$0.134$ ***A1***

***[4 marks]***

*continued…*

*Question 6 continued.*

(b) recognition of conditional probability ***(M1)***

$\mathrm{P}(A \mid 2 \text{ defective})$ OR $\dfrac{\mathrm{P}(\mathrm{A} \cap 2 \text{ defective})}{\mathrm{P}(2 \text{ defective})}$

**Note:** recognition must be shown in context, either in words or symbols, not just $\mathrm{P}(A \mid B)$.

$\dfrac{0.0968...}{0.134...}$ ***(A1)***

$0.72187...$

$0.722$ ***A1***

**Note:** Accept $0.723$ or $0.724$ from using 3 sf values from part (a).

***[3 marks]***

***Total [7 marks]***

## Section B

**1**. (a) (i) $\overline{x} = 235.714...$ and $\overline{y} = 33.4285...$

$\overline{x} = 236$ and $\overline{y} = 33.4$ ***A1***

(ii) recognition that the gradient is $\frac{\overline{y}}{\overline{x}}$ ***(M1)***

$m = 0.141818...\left(\frac{33.4285...}{235.714...}\right)$

$m = 0.142$

$y = 0.142x$ ***A1***

***[3 marks]***

(b) valid attempt to solve ***(M1)***

$a = 0.125515....$ OR $b = 3.84278...$

$a = 0.126$, $b = 3.84$ ***A1***

***[2 marks]***

(c) attempt to substitute $350$ into the regression equation from (b) ***(M1)***

$y = 0.125515... \times 350 + 3.84278...$

$y = 47.7731...$

$y = 47.8$. ***A1***

***[2 marks]***

***Total [7 marks]***



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**4.**

(a) the median is the middle value and as Aiden's score is below $10.5$ and Brett's score is above $10.5$, hence the middle value stays in the same place ***R1***

**Note:** Award the ***R1*** for any valid reasoning referring to the scores being on either side of the median.

***[1 mark]***

(b) attempt to use that the sum of scores is $n \times \text{mean}$ ***(M1)***

$10.5 \times 28$ OR $10.6 \times 30$

sum of Aiden and Brett's scores = $24$ ($24 = B + A$) (seen anywhere) ***(A1)***

difference between Aiden and Brett's scores = $14$ ($14 = B - A$) (seen anywhere) ***(A1)***

attempt to solve their two simultaneous equations or by trial and improvement ***(M1)***

(Aiden's score =) $5$, (Brett's score =) $19$ ***A1***

**Note:** The first two ***A*** marks are independent of each other.

***[5 marks]***

***Total [6 marks]***

**6.** (a) recognition that $\mathrm{P}(\text{height of large sunflowers} > 180) = 0.25$. ***(M1)***

$\frac{180-\mu}{\sigma} = 0.674489...$ OR $\mu + 0.674\sigma = 180$ ***(A1)***

$A = 0.674489\ldots$

$A = 0.674$ ***A1***

***[3 marks]***

(b) correct parameters for the height of giant sunflowers ***(A1)***

$\mu + 35$ and $2\sigma$ OR $\mu + 35$ and $(2\sigma)^2$

$\frac{180-\mu-35}{2\sigma} = -2.05374...$ ***(A1)***

attempt to solve their two simultaneous equations in two unknowns, $\mu$ and $\sigma$. ***(M1)***

$\mu = 175.063..., \sigma = 7.31913....$

$\mu = 175,\ \sigma = 7.32$ ***A1***

***[4 marks]***

***Total [7 marks]***

## Section B

APOLLO

# Calculus

83 questions

**5.** (a) $f'(x) = -2(x-h)$ ***A1***

***[1 mark]***

(b) $g'(x) = \mathrm{e}^{x-2}$ OR $g'(3) = \mathrm{e}^{3-2}$ (may be seen anywhere) ***A1***

**Note:** The derivative of $g$ must be explicitly seen, either in terms of $x$ or 3.

recognizing $f'(3) = g'(3)$ ***(M1)***

$-2(3-h) = \mathrm{e}^{3-2} \ (= e)$

$-6+2h = \mathrm{e}$ OR $3-h = -\frac{\mathrm{e}}{2}$ ***A1***

**Note:** The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \frac{\mathrm{e}+6}{2}$ ***AG***

***[3 marks]***

(c) $f(3) = g(3)$ ***(M1)***

$-(3-h)^2 + 2k = \mathrm{e}^{3-2} + k$

correct equation in $k$

**EITHER**

$-\left(3-\frac{\mathrm{e}+6}{2}\right)^2 + 2k = \mathrm{e}^{3-2} + k$ ***A1***

$k = \mathrm{e} + \left(\frac{6-\mathrm{e}-6}{2}\right)^2 \left(= \mathrm{e} + \left(\frac{-\mathrm{e}}{2}\right)^2\right)$ ***A1***

**OR**

$k = \mathrm{e} + \left(3-\frac{\mathrm{e}+6}{2}\right)^2$ ***A1***

$k = \mathrm{e} + 9 - 3\mathrm{e} - 18 + \frac{\mathrm{e}^2 + 12\mathrm{e} + 36}{4}$ ***A1***

**THEN**

$k = \mathrm{e} + \frac{\mathrm{e}^2}{4}$ ***AG***

***[3 marks]***
***Total [7 marks]***

**8.** (a) attempt to use quotient or product rule ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^4\left(\frac{1}{x}\right) - (\ln x)\left(4x^3\right)}{\left(x^4\right)^2}$ OR $(\ln x)\left(-4x^{-5}\right) + \left(x^{-4}\right)\left(\frac{1}{x}\right)$ ***A1***

correct working ***A1***

$= \frac{x^3(1 - 4\ln x)}{x^8}$ OR cancelling $x^3$ OR $\frac{-4\ln x}{x^5} + \frac{1}{x^5}$

$= \frac{1 - 4\ln x}{x^5}$ ***AG***

***[3 marks]***

(b) $f'(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***(M1)***

$\frac{1 - 4\ln x}{x^5} = 0$

$\ln x = \frac{1}{4}$ ***(A1)***

$x = \mathrm{e}^{\frac{1}{4}}$ ***A1***

substitution of their $x$ to find $y$ ***(M1)***

$y = \frac{\ln \mathrm{e}^{\frac{1}{4}}}{\left(\mathrm{e}^{\frac{1}{4}}\right)^4}$

$= \frac{1}{4\mathrm{e}}\left(= \frac{1}{4}\mathrm{e}^{-1}\right)$ ***A1***

$\mathrm{P}\left(\mathrm{e}^{\frac{1}{4}}, \frac{1}{4\mathrm{e}}\right)$

***[5 marks]***

*continued…*

*Question 8 continued*

(c) $f''\left(e^{\frac{1}{4}}\right) = \dfrac{20\ln e^{\frac{1}{4}} - 9}{\left(e^{\frac{1}{4}}\right)^6}$ ***(M1)***

$= \dfrac{5-9}{e^{1.5}} \quad \left(= -\dfrac{4}{e^{1.5}}\right)$ ***A1***

which is negative ***R1***

hence P is a local maximum ***AG***

**Note:** The ***R1*** is dependent on the previous ***A1*** being awarded.

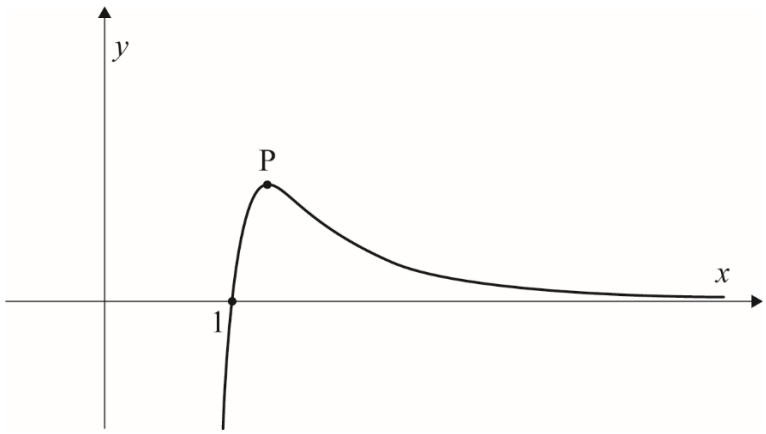
***[3 marks]***

(d) $\ln x > 0$ ***(A1)***

$x > 1$ ***A1***

***[2 marks]***

(e)




***A1A1A1***

**Note**: Award ***A1*** for one $x$-intercept only, located at 1
***A1*** for local maximum, P, in approximately correct position
***A1*** for curve approaching $x$-axis as $x \to \infty$ (including change in concavity).

***[3 marks]***
***Total [16 marks]***

**5.** (a) $\ln\left(x^2-16\right)=0$ ***(M1)***

$e^0=x^2-16\left(=1\right)$

$x^2=17$ OR $x=\pm\sqrt{17}$ ***(A1)***

$a=\sqrt{17}$ ***A1***

***[3 marks]***

(b) attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2-16}$) ***(M1)***

$f'(x)=\frac{2x}{x^2-16}$ ***A1***

setting their derivative $=\frac{1}{3}$ ***M1***

$\frac{2x}{x^2-16}=\frac{1}{3}$

$x^2-16=6x$ OR $x^2-6x-16=0$ (or equivalent) ***A1***

valid attempt to solve their quadratic ***(M1)***

$x=8$ ***A1***

**Note:** Award ***A0*** if the candidate's final answer includes additional solutions (such as $x=-2, 8$).

***[6 marks]***

***Total [9 marks]***

**8.** (a) $6+6\cos x=0$ (or setting their $f'(x)=0$) ***(M1)***

$\cos x=-1$ (or $\sin x=0$)

$x=\pi,\ x=3\pi$ ***A1A1***

***[3 marks]***

(b) attempt to integrate $\int_{\pi}^{3\pi}(6+6\cos x)\mathrm{d}x$ ***(M1)***

$=\left[6x+6\sin x\right]_{\pi}^{3\pi}$ ***A1A1***

substitute their limits into their integrated expression and subtract ***(M1)***

$=(18\pi+6\sin 3\pi)-(6\pi+6\sin\pi)$

$=(6(3\pi)+0)-(6\pi+0)\ \ (=18\pi-6\pi)$ ***A1***

area $=12\pi$ ***AG***

***[5 marks]***

*continued…*

*Question 8 continued*

(c) attempt to substitute into formula for surface area (including base) ***(M1)***

$\pi\left(2^2\right)+\pi(2)(l)=12\pi$ ***(A1)***

$4\pi+2\pi l=12\pi$

$2\pi l=8\pi$

$l=4$ ***A1***

***[3 marks]***

(d) valid attempt to find the height of the cone ***(M1)***

e.g. $2^2+h^2=(\text{their } l)^2$

$h=\sqrt{12}\ \left(=2\sqrt{3}\right)$ ***(A1)***

attempt to use $V=\frac{1}{3}\pi r^2 h$ with their values substituted ***M1***

$\left(\frac{1}{3}\pi\left(2^2\right)\left(\sqrt{12}\right)\right)$

$\text{volume}=\frac{4\pi\sqrt{12}}{3}\left(=\frac{8\pi\sqrt{3}}{3}=\frac{8\pi}{\sqrt{3}}\right)$ ***A1***

***[4 marks]***

***Total [15 marks]***

**9.** (a) setting $s(t) = 0$ ***(M1)***

$8t - t^2 = 0$

$t(8 - t) = 0$

$p = 8$ (accept $t = 8$, $(8, 0)$) ***A1***

**Note:** Award ***A0*** if the candidate's final answer includes additional solutions (such as $p = 0, 8$).

***[2 marks]***

(b) (i) recognition that when particle changes direction $v = 0$ OR local maximum on graph of $s$ OR vertex of parabola ***(M1)***

$q = 4$ (accept $t = 4$) ***A1***

(ii) substituting their value of $q$ into $s(t)$ OR integrating $v(t)$ from $t = 0$ to $t = 4$ ***(M1)***

displacement $= 16$ (m) ***A1***

***[4 marks]***

(c) $s(10) = -20$ OR distance $= |s(t)|$ OR integrating $v(t)$ from $t = 0$ to $t = 10$ ***(M1)***

distance $= 20$ (m) ***A1***

***[2 marks]***

(d) 16 forward + 36 backward OR $16 + 16 + 20$ OR $\int_0^{10} |v(t)| dt$ ***(M1)***

$d = 52$ (m) ***A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(e) **METHOD 1**

graphical method with triangles on $v(t)$ graph ***M1***

$49+\left(\frac{x(2x)}{2}\right)$ ***(A1)***

$49+x^2=52,\ x=\sqrt{3}$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

***[4 marks]***

**METHOD 2**

recognition that $\text{distance}=\int|v(t)|\,\mathrm{d}t$ ***M1***

$\int_0^7(14-2t)\,\mathrm{d}t+\int_7^k(2t-14)\,\mathrm{d}t$

$\left[14t-t^2\right]_0^7+\left[t^2-14t\right]_7^k$ ***(A1)***

$14(7)-7^2+\left(\left(k^2-14k\right)-\left(7^2-14(7)\right)\right)=52$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

***[4 marks]***

***Total [14 marks]***

**1.** (a) correct integration $3x^2 + 7x + c$ ***A1A1A1***

| **Note:** Award ***A1*** for $3x^2$, ***A1*** for $7x$ and ***A1*** for $+c$ |
|---|

***[3 marks]***

(b) recognition that $f(x) = \int f'(x)\mathrm{d}x$ ***(M1)***

$3(1.2)^2 + 7(1.2) + c = 7.32$ ***(A1)***

$c = -5.4$

$f(x) = 3x^2 + 7x - 5.4$ ***A1***

***[3 marks]***

***Total [6 marks]***

**5.** (a) recognising $v = 0$ ***(M1)***

$t = 6.74416...$

$= 6.74$ (sec) ***A1***

**Note:** Do not award ***A1*** if additional values are given.

***[2 marks]***

(b) $\int_0^{10} |v(t)| \mathrm{d}t$ OR $-\int_0^{6.74416...} v(t)\,\mathrm{d}t + \int_{6.74416...}^{9.08837...} v(t)\,\mathrm{d}t - \int_{9.08837...}^{10} v(t)\,\mathrm{d}t$ ***(A1)***

$= 37.0968...$

$= 37.1$ (m) ***A1***

***[2 marks]***

(c) recognizing acceleration at $t = 7$ is given by $v'(7)$ ***(M1)***

acceleration $= 5.93430...$

$= 5.93$ ($ms^{-2}$) ***A1***

***[2 marks]***

***Total [6 marks]***

**9.** (a) Attempt to find the point of intersection of the graph of $f$ and the line $y = x$ ***(M1)***

$x = 5.56619...$

$= 5.57$ ***A1***

***[2 marks]***

(b) $f'(x) = -45e^{-0.5x}$ ***A1***

attempt to set the gradient of $f$ equal to $-1$ ***(M1)***

$-45e^{-0.5x} = -1$

Q has coordinates $(2\ln 45, 2)$ (accept $(-2\ln\frac{1}{45}, 2)$ ***A1A1***

**Note**: Award ***A1*** for each value, even if the answer is not given as a coordinate pair.

Do not accept $\frac{\ln\frac{1}{45}}{-0.5}$ or $\frac{\ln 45}{0.5}$ as a final value for $x$. Do not accept 2.0 or 2.00 as a final value for $y$.

***[4 marks]***

(c) attempt to substitute coordinates of Q (in any order) into an appropriate equation ***(M1)***

$y - 2 = -(x - 2\ln 45)$ OR $2 = -2\ln 45 + c$ ***A1***

equation of $L$ is $y = -x + 2\ln 45 + 2$ ***AG***

***[2 marks]***

(d) (i) $x = \ln 45 + 1 (= 4.81)$ ***A1***

(ii) appropriate method to find the sum of two areas using integrals of the difference of two functions ***(M1)***

**Note**: Allow absence of incorrect limits.

$\int_{4.806...}^{5.566...} (x - (-x + 2\ln 45 + 2))\,dx + \int_{5.566...}^{7.613...} (90e^{-0.5x} - (-x + 2\ln 45 + 2))\,dx$ ***(A1)(A1)***

**Note**: Award ***A1*** for one correct integral expression including correct limits and integrand.
Award ***A1*** for a second correct integral expression including correct limits and integrand.

$= 1.52196...$

$= 1.52$ ***A1***

***[5 marks]***

(e) by symmetry $2 \times 1.52...$ ***(M1)***

$= 3.04$ ***A1***

**Note**: Accept any answer that rounds to 3.0 (but do not accept 3).

***[2 marks]***
***Total [15 marks]***

**2.** recognition that $y = \int \cos\left(x - \frac{\pi}{4}\right) \mathrm{d}x$ ***(M1)***

$y = \sin\left(x - \frac{\pi}{4}\right) \ (+c)$ ***(A1)***

substitute both $x$ and $y$ values into their integrated expression including $c$ (***M1)***

$2 = \sin\frac{\pi}{2} + c$

$c = 1$

$y = \sin\left(x - \frac{\pi}{4}\right) + 1$ ***A1***

***[4 marks]***

**5**. (a) $f'(4) = 6$ ***A1***

***[1 mark]***

(b) $f(4) = 6 \times 4 - 1 = 23$ ***A1***

***[1 mark]***

(c) $h(4) = f\left(g(4)\right)$ ***(M1)***

$h(4) = f(4^2 - 3 \times 4) = f(4)$

$h(4) = 23$ ***A1***

***[2 marks]***

(d) attempt to use chain rule to find $h'$ ***(M1)***

$f'\left(g\left(x\right)\right) \times g'\left(x\right)$ OR $\left(x^2 - 3x\right)' \times f'\left(x^2 - 3x\right)$

$h'(4) = \left(2 \times 4 - 3\right) f'\left(4^2 - 3 \times 4\right)$ ***A1***

$= 30$

$y - 23 = 30\left(x - 4\right)$ OR $y = 30x - 97$ ***A1***

***[3 marks]***

***Total [7 marks]***

**7.** (a) (i) valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4-6t=0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t=\frac{2}{3}$ (s) ***A1***

(ii) attempt to integrate $v$ ***(M1)***

$\int v\ \mathrm{d}t = \int\left(4+4t-3t^2\right)\ \mathrm{d}t = 4t+2t^2-t^3\left(+c\right)$ ***A1A1***

**Note:** Award ***A1*** for $4t+2t^2$, ***A1*** for $-t^3$.

attempt to substitute their $t$ into their solution for the integral ***(M1)***

distance $=4\left(\frac{2}{3}\right)+2\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^3$

$=\frac{8}{3}+\frac{8}{9}-\frac{8}{27}$ (or equivalent) ***A1***

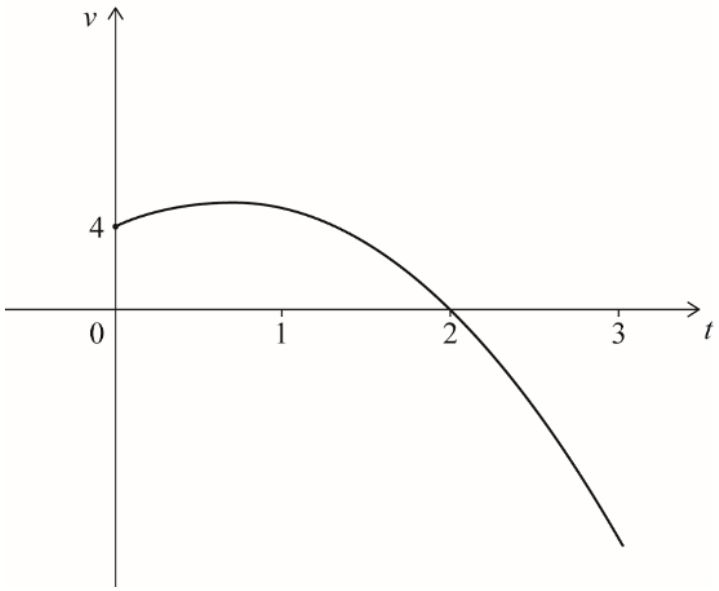
$=\frac{88}{27}$ (m) ***AG***

***[7 marks]***

*continued…*

*Question 7 continued.*

(b)




valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\dfrac{-4\pm\sqrt{16+48}}{-6}$

correct $x$- intercept on the graph at $t=2$ ***A1***

**Note**: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

**Note:** The 3 must be clearly indicated.

vertex in approximately correct place for $t=\dfrac{2}{3}$ and $v>4$ ***A1***

***[4 marks]***

*continued…*

*Question 7 continued.*

(c) recognising to integrate between $0$ and $2$, or $2$ and $3$ OR $\int_0^3 \left|4+4t-3t^2\right| \, \mathrm{d}t$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \, \mathrm{d}t$

$=8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \, \mathrm{d}t$

$=-5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, \mathrm{d}t - \int_2^3 v \, \mathrm{d}t$ OR $\int_0^2 v \, \mathrm{d}t + \left|\int_2^3 v \, \mathrm{d}t\right|$

total distance travelled $=13$ (m) ***A1***

***[5 marks]***

***Total [16 marks]***

(a)

**9.** (a) $f$ increases when $p < x < 0$ ***A1***

$f$ increases when $f'(x) > 0$ OR $f'$ is above the $x$-axis ***R1***

**Note:** Do not award ***A0R1.***

***[2 marks]***

(b) $x = 0$ ***A1***

***[1 mark]***

(c) (i) $f$ is minimum when $x = p$ ***A1***

because $f'(p) = 0$, $f'(x) < 0$ when $x < p$ and $f'(x) > 0$ when $x > p$

(may be seen in a sign diagram clearly labelled as $f'$)

OR because $f'$ changes from negative to positive at $x = p$

OR $f'(p) = 0$ and slope of $f'$ is positive at $x = p$ ***R1***

**Note:** Do not award ***A0 R1***

(ii) $f$ has points of inflexion when $x = q$, $x = r$ and $x = t$ ***A2***

$f'$ has turning points at $x = q$, $x = r$ and $x = t$

OR

$f''(q) = 0$, $f''(r) = 0$ and $f''(t) = 0$ and $f'$ changes from increasing to decreasing or vice versa at each of these $x$-values (may be seen in a sign diagram clearly labelled as $f''$ and $f'$) ***R1***

**Note:** Award ***A0*** if any incorrect answers are given. Do not award ***A0R1.***

***[5 marks]***

*continued…*

*Question 9 continued.*

(d) recognizing area from $p$ to $t$ (seen anywhere) ***M1***

$\int_p^t |f'(x)|\,dx$

recognizing to negate integral for area below $x$-axis ***(M1)***

$\int_p^0 f'(x)\,dx - \int_0^t f'(x)\,dx$ OR $\int_p^0 f'(x)\,dx + \int_t^0 f'(x)\,dx$

$\int_m^n f'(x)\,dx = f(n) - f(m)$ (for any integral) ***(M1)***

$f(0) - f(p) - [f(t) - f(0)]$ OR $f(0) - f(p) + f(0) - f(t)$ ***(A1)***

$2f(0) - [f(t) + f(p)] = 20$, $2f(0) - 4 = 20$ ***(A1)***

$f(0) = 12$ ***A1***

***[6 marks]***
***Total [14 marks]***

**7.** (a) (i) $\frac{\text{AP}}{42}$ OR $\frac{215}{84}$ OR $\frac{65}{42}+\frac{215}{84}$ ***(M1)***

time = $4.10714\ldots$ (hours)

time = $4.11$ (hours) ***A1***

(ii) $\text{AB}=\sqrt{215^2+65^2}\ (=224.610...)$ ***(A1)***

time = $5.34787\ldots$ (hours)

time = $5.35$ (hours) ***A1***

***[4 marks]***

(b) (i) $\text{AD}=\sqrt{(215-x)^2+65^2}$ ***(A1)***

$t=\dfrac{\sqrt{(215-x)^2+65^2}}{42}$ ***(A1)***

$T=\dfrac{\sqrt{(215-x)^2+65^2}}{42}+\dfrac{x}{84}\left(=\dfrac{\sqrt{x^2-430x+50450}}{42}+\dfrac{x}{84}\right)$ ***A1***

(ii) valid approach to find the minimum for $T$ (may be seen in (iii)) ***(M1)***

graph of $T$ OR $T'=0$ OR graph of $T'$

$x=177.472...$ km

$x=177$ km ***A1***

(iii) $T=3.89980...$

$T=3.90$ (hours) ***A1***

**Note:** Only allow ***FT*** in (b)(ii) and (iii) for $0<x<215$ and a function $T$ that has a minimum in that interval.

***[6 marks]***

*continue…*

*Question 7 continued.*

(c) (i) $C = 200 \cdot \frac{\sqrt{(215-x)^2 + 65^2}}{42} + 150 \cdot \frac{x}{84}$ ***(A1)***

valid approach to find the minimum for $C(x)$ (may be seen in (ii)) ***(M1)***

graph of $C$ OR $C' = 0$ OR graph of $C'$

$x = 188.706... \text{ km}$

$x = 189 \text{ km}$ ***A1***

**Note:** Only allow ***FT*** from (b) if the function $T$ has a minimum in $0 < x < 215$.

(ii) $C = 670.864$

$C = \$671$ ***A1***

**Note:** Only allow ***FT*** from (c)(i) if the function $C$ has a minimum in $0 < x < 215$.

***[4 marks]***

***Total [14 marks]***

**4.** attempt to integrate ***(M1)***

$u = 2x^2 + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4x$

$\int \frac{8x}{\sqrt{2x^2+1}}\mathrm{d}x = \int \frac{2}{\sqrt{u}}\mathrm{d}u$ ***(A1)***

***EITHER***

$= 4\sqrt{u}\,(+C)$ ***A1***

***OR***

$= 4\sqrt{2x^2+1}\,(+C)$ ***A1***

***THEN***

correct substitution into **their** integrated function (must have $C$) ***(M1)***

$5 = 4 + C \Rightarrow C = 1$

$f(x) = 4\sqrt{2x^2+1} + 1$ ***A1***

***Total [5 marks]***

**8.** (a) $f'(x)=x^2+2x-15$ ***(M1)A1***

***[2 marks]***

(b) correct reasoning that $f'(x) = 0$ (seen anywhere) ***(M1)***

$x^2 + 2x - 15 = 0$

valid approach to solve quadratic ***M1***

$(x-3)(x+5)$, quadratic formula

correct values for $x$

$3, -5$

correct values for $a$ and $b$

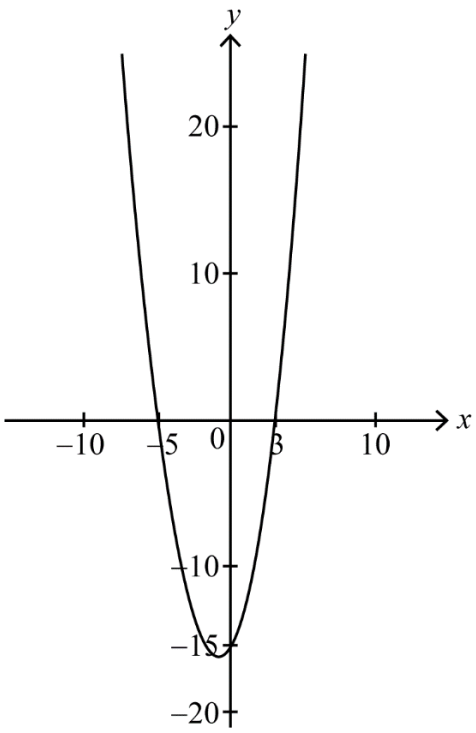
$a = -5$ and $b = 3$ ***A1***

***[3 marks]***

*continued…*

*Question 8 continued*

(c) (i)




***A1***

(ii) first derivative changes from positive to negative at $x=a$ ***A1***

so local maximum at $x=a$ ***AG***

***[2 marks]***

(d) (i) $f''(x) = 2x + 2$ ***A1***

substituting **their** $b$ into **their** second derivative ***(M1)***

$f''(3) = 2 \times 3 + 2$

$f''(b) = 8$ ***(A1)***

(ii) $f''(b)$ is positive so graph is concave up ***R1***

so local minimum at $x = b$ ***AG***

***[4 marks]***

(e) normal to $f$ at $x=a$ is $x = -5$ (seen anywhere) ***(A1)***

attempt to find $y$-coordinate at their value of $b$ ***(M1)***

$f(3) = -10$ ***(A1)***

tangent at $x = b$ has equation $y = -10$ (seen anywhere) ***A1***

intersection at $(-5, -10)$

$p = -5$ and $q = -10$ ***A1***

***[5 marks]***

***[Total 16 marks]***

**9.** (a) attempt to use quotient rule ***(M1)***

correct substitution into quotient rule

$f'(x) = \dfrac{5kx\left(\dfrac{1}{5x}\right) - k\ln 5x}{(kx)^2}$ (or equivalent) ***A1***

$= \dfrac{k - k\ln 5x}{k^2x^2}, (k \in \mathbb{R}^+)$ ***A1***

$= \dfrac{1 - \ln 5x}{kx^2}$ ***AG***

***[3 marks]***

(b) $f'(x) = 0$ ***M1***

$\dfrac{1 - \ln 5x}{kx^2} = 0$

$\ln 5x = 1$ ***(A1)***

$x = \dfrac{e}{5}$ ***A1***

***[3 marks]***

(c) $f''(x) = 0$ ***M1***

$\dfrac{2\ln 5x - 3}{kx^3} = 0$

$\ln 5x = \dfrac{3}{2}$ ***A1***

$5x = e^{\frac{3}{2}}$ ***A1***

so the point of inflexion occurs at $x = \dfrac{1}{5}e^{\frac{3}{2}}$ ***AG***

***[3 marks]***

*continued…*

*Question 9 continued*

(d) attempt to integrate ***(M1)***

$u = \ln 5x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\int \frac{\ln 5x}{kx}\mathrm{d}x = \frac{1}{k}\int u \ \mathrm{d}u$ ***(A1)***

***EITHER***

$= \frac{u^2}{2k}$ ***A1***

so $\frac{1}{k}\int_1^{\frac{3}{2}} u \ \mathrm{d}u = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1***

***OR***

$= \frac{(\ln 5x)^2}{2k}$ ***A1***

so $\int_{\frac{\mathrm{e}}{5}}^{\frac{1}{5}\mathrm{e}^{\frac{3}{2}}} \frac{\ln 5x}{kx}\mathrm{d}x = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{\mathrm{e}}{5}}^{\frac{1}{5}\mathrm{e}^{\frac{3}{2}}}$ ***A1***

***THEN***

$= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$

$= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to $3$ ***M1***

$\frac{5}{8k} = 3$

$k = \frac{5}{24}$ ***A1***

***[7 marks]***

***Total [16 marks]***

**6.** (a) use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is $16.5\,\text{cm}$ (to the left of O) ***A1***

***[3 marks]***

(b) attempt to find time when particle changes direction *eg* considering the first maximum on the graph of $s$ or the first $t$ – intercept on the graph of $s'$. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of $s'$ for **their** value of *t*, $s''(1.51986...)$ ***(M1)***

$= -8.92\ (\text{cm/s}^2)$ ***A1***

***[4 marks]***

***Total [7 marks]***



## Section B

**2.** (a) $\frac{3\sqrt{x}-5}{\sqrt{x}} = 3-5x^{-\frac{1}{2}}$ ***A1***

$p = -\frac{1}{2}$

***[1 mark]***

(b) $\int \frac{3\sqrt{x}-5}{\sqrt{x}}\mathrm{d}x = 3x-10x^{\frac{1}{2}}\ (+c)$ ***A1A1***

substituting limits into their integrated function and subtracting ***(M1)***

$3(9)-10(9)^{\frac{1}{2}}-\left(3(1)-10(1)^{\frac{1}{2}}\right)$ OR $27-10\times 3-(3-10)$

$= 4$ ***A1***

***[4 marks]***

***Total [5 marks]***

**5.** evidence of using product rule ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)\times\left(k\mathrm{e}^{kx}\right)+2\times\mathrm{e}^{kx} \quad \left(=\mathrm{e}^{kx}(2kx-k+2)\right)$ ***A1***

correct working for one of (seen anywhere) ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=1 \Rightarrow k\mathrm{e}^{k}+2\mathrm{e}^{k}$

**OR**

slope of tangent is $5\mathrm{e}^{k}$

their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=1$ equals the *slope* of $y=5\mathrm{e}^{k}x$ (= $5\mathrm{e}^{k}$) (seen anywhere) ***(M1)***

$k\mathrm{e}^{k}+2\mathrm{e}^{k}=5\mathrm{e}^{k}$

$k=3$ ***A1***

***[5 marks]***

**7.** (a) **EITHER**

attempt to use $x = -\frac{b}{2a}$ ***(M1)***

$q = -\frac{-12}{2 \times 3}$

**OR**

attempt to complete the square ***(M1)***

$3(x-2)^2 - 12 + p$

**OR**

attempt to differentiate and equate to 0 ***(M1)***

$f''(x) = 6x - 12 = 0$

**THEN**

$q = 2$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued*

(b) (i) discriminant $= 0$ ***A1***

(ii) **EITHER**

attempt to substitute into $b^2 - 4ac$ ***(M1)***

$(-12)^2 - 4 \times 3 \times p = 0$ ***A1***

**OR**

$f'(2) = 0$ ***(M1)***

$-12 + p = 0$ ***A1***

**THEN**

$p = 12$ ***A1***

***[4 marks]***

(c) $f''(x) = 6x - 12$ ***A1***

attempt to find $f''(0)$ ***(M1)***

$= 6 \times 0 - 12$

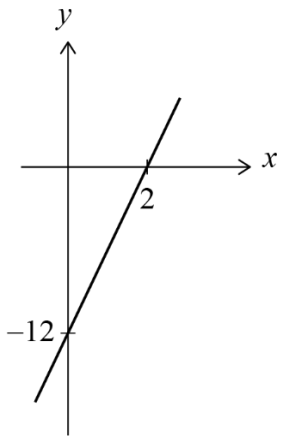
gradient = $-12$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued*

(d)




***A1A1***

| **Note:** Award ***A1*** for line with positive gradient, ***A1*** for correct intercepts. |
|---|

***[2 marks]***

(e) (i) $a = 2$ ***A1***

(ii) $x < 2$ ***A1***

$f''(x) < 0$ (for $x < 2$) OR the $f''$ is below the *x*-axis (for $x < 2$)

OR $\overset{-}{\longleftarrow}\overset{+}{\underset{2}{\longrightarrow}}\ f''$ (sign diagram must be labelled $f''$) ***R1***

***[3 marks]***

***Total [14 marks]***

**9.** (a) (i) **EITHER**

attempt to use binomial expansion ***(M1)***

$1+{}^3C_1\times 1\times(-a)+{}^3C_2\times 1\times(-a)^2+1\times(-a)^3$

**OR**

$(1-a)(1-a)(1-a)$

$=(1-a)(1-2a+a^2)$ ***(M1)***

**THEN**

$=1-3a+3a^2-a^3$ ***A1***

(ii) $a=\cos 2x$ ***(A1)***

So, $1-3\cos 2x+3\cos^2 2x-\cos^3 2x=$

$(1-\cos 2x)^3$ ***A1***

attempt to substitute any double angle rule for $\cos 2x$ into $(1-\cos 2x)^3$ ***(M1)***

$=(2\sin^2 x)^3$ ***A1***

$=8\sin^6 x$ ***AG***

**Note:** Allow working RHS to LHS.

***[6 marks]***

*continued…*

*Question 9 continued*

(b) (i) recognizing to integrate $\int\left(4\cos x\times 8\sin^6 x\right)\mathrm{d}x$ ***(M1)***

**EITHER**

applies integration by inspection ***(M1)***

$32\int\left(\cos x\times(\sin x)^6\right)\mathrm{d}x$

$=\frac{32}{7}\sin^7 x\,(+c)$ ***A1***

$\left[\frac{32}{7}\sin^7 x\right]_0^m \quad \left(=\frac{32}{7}\sin^7 m-\frac{32}{7}\sin^7 0\right)$ ***A1***

**OR**

$u=\sin x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\cos x$ ***(M1)***

$\int 32\cos x\,(\sin^6 x)\,\mathrm{d}x=\int 32u^6\mathrm{d}u$

$=\frac{32}{7}u^7\,(+c)$ ***A1***

$\left[\frac{32}{7}\sin^7 x\right]_0^m$ OR $\left[\frac{32}{7}u^7\right]_0^{\sin m} \quad \left(=\frac{32}{7}\sin^7 m-\frac{32}{7}\sin^7 0\right)$ ***A1***

**THEN**

$=\frac{32}{7}\sin^7 m$ ***AG***

*continued…*

*Question 9 continued*

(ii) **EITHER**

$\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x \left( = \left[ \frac{32}{7}\sin^7 x \right]_m^{\frac{\pi}{2}} \right) = \frac{32}{7}\sin^7 \frac{\pi}{2} - \frac{32}{7}\sin^7 m$ ***M1***

$\frac{32}{7}\sin^7 \frac{\pi}{2} - \frac{32}{7}\sin^7 m = \frac{127}{28}$ OR $\frac{32}{7}\left(1 - \sin^7 m\right) = \frac{127}{28}$ ***(M1)***

**OR**

$\int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x = \int_0^m f(x)\,\mathrm{d}x + \int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x$ ***M1***

$\frac{32}{7} = \frac{32}{7}\sin^7 m + \frac{127}{28}$ ***(M1)***

**THEN**

$\sin^7 m = \frac{1}{128} \left( = \frac{1}{2^7} \right)$ ***(A1)***

$\sin m = \frac{1}{2}$ ***(A1)***

$m = \frac{\pi}{6}$ ***A1***

***[9 marks]***

***Total [15 marks]***

**7.** (a) $x = 3$ ***A1***

**Note:** Must be an equation in the form "$x =$ ". Do not accept 3 or $\frac{-b}{2a} = 3$.

***[1 mark]***

(b) (i) $h = 3,\ k = 4$ (accept $a(x-3)^2 + 4$) ***A1A1***

(ii) attempt to substitute coordinates of Q ***(M1)***

$12 = a(5-3)^2 + 4,\ 4a + 4 = 12$

$a = 2$ ***A1***

***[4 marks]***

(c) recognize need to find derivative of $f$ ***(M1)***

$f'(x) = 4(x-3)$ or $f'(x) = 4x - 12$ ***A1***

$f'(5) = 8$ (may be seen as gradient in their equation) ***(A1)***

$y - 12 = 8(x-5)$ or $y = 8x - 28$ ***A1***

**Note:** Award ***A0*** for $L = 8x - 28$.

***[4 marks]***

*continued…*

*Question 7 continued*

(d) **METHOD 1**

Recognizing that for $g$ to be increasing, $f(x)-d>0$, or $g'>0$ ***(M1)***

The vertex must be above the $x$-axis, $4-d>0$, $d-4<0$ ***(R1)***

$d<4$ ***A1***

***[3 marks]***

**METHOD 2**

attempting to find discriminant of $g'$ ***(M1)***

$(-12)^2-4(2)(22-d)$

recognizing discriminant must be negative ***(R1)***

$-32+8d<0$ OR $\Delta<0$

$d<4$ ***A1***

***[3 marks]***

(e) recognizing that for g to be concave up, $g''>0$ ***(M1)***

$g''>0$ when $f'>0$, $4x-12>0$, $x-3>0$ ***(R1)***

$x>3$ ***A1***

***[3 marks]***

***Total [15 marks]***

**8.** (a) $\frac{1}{x-4}+1=x-3$ ***(M1)***

$x^2-8x+15=0$ OR $(x-4)^2=1$ ***(A1)***

valid attempt to solve **their** quadratic ***(M1)***

$(x-3)(x-5)=0$ OR $x=\frac{8\pm\sqrt{8^2-4(1)(15)}}{2(1)}$ OR $(x-4)=\pm1$

$x=5$ $(x=3,\ x=5)$ (may be seen in answer) ***A1***

$\mathrm{B}(5,\ 2)$ (accept $x=5,\ y=2$) ***A1***

***[5 marks]***

*continued…*

*Question 8 continued*

(b) recognizing two correct regions from $x=3$ to $x=5$ and from $x=5$ to $x=k$ ***(R1)***

$\text{triangle}+\int_5^k f(x)\mathrm{d}x$ OR $\int_3^5 g(x)\mathrm{d}x+\int_5^k f(x)\mathrm{d}x$ OR $\int_3^5 (x-3)\mathrm{d}x+\int_5^k \left(\frac{1}{x-4}+1\right)\mathrm{d}x$

area of triangle is $2$ OR $\frac{2\cdot 2}{2}$ OR $\left(\frac{5^2}{2}-3(5)\right)-\left(\frac{3^2}{2}-3(3)\right)$ ***(A1)***

correct integration ***(A1)(A1)***

$\int\left(\frac{1}{x-4}+1\right)\mathrm{d}x=\ln(x-4)+x\ (+C)$

**Note:** Award ***A1*** for $\ln(x-4)$ and ***A1*** for $x$.

**Note:** The first three ***A*** marks may be awarded independently of the ***R*** mark.

substitution of **their** limits (for $x$) into **their** integrated function (in terms of $x$) ***(M1)***

$\ln(k-4)+k-(\ln 1+5)$

$\left[\ln(x-4)+x\right]_5^k=\ln(k-4)+k-5$ ***A1***

adding **their** two areas (in terms of $k$) and equating to $\ln p+8$ ***(M1)***

$2+\ln(k-4)+k-5=\ln p+8$

equating **their** non-log terms to 8 (equation must be in terms of $k$) ***(M1)***

$k-3=8$

$k=11$ ***A1***

$11-4=p$

$p=7$ ***A1***

***[10 marks]***

***Total [15 marks]***

**5.** (a) recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692...$

$t = 3.35$ (seconds) ***A1***

**Note:** Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

***[2 marks]***

(b) recognizing particle changes direction when $v = 0$ OR when $t = 3.34692...$ ***(M1)***

$a = -4.71439...$

$a = -4.71\,(\text{ms}^{-2})$ ***A2***

***[3 marks]***

(c) distance travelled = $\int_0^6 |v|\,\mathrm{d}t$ OR

$$\int_0^{3.34...} \left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t - \int_{3.34...}^6 \left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t \;\; (= 14.3104... + 6.44300...) \quad \textit{(A1)}$$

$= 20.7534...$

= 20.8 (metres) ***A1***

***[2 marks]***

***Total [7 marks]***

**2. METHOD 1**

recognises that $g(x) = \int \left(3x^2 + 5\mathrm{e}^x\right) \mathrm{d}x$ ***(M1)***

$g(x) = x^3 + 5\mathrm{e}^x (+C)$ ***(A1)(A1)***

**Note:** Award ***A1*** for each integrated term.

substitutes $x = 0$ and $y = 4$ into their integrated function (must involve $+C$) ***(M1)***

$4 = 0 + 5 + C \Rightarrow C = -1$

$g(x) = x^3 + 5\mathrm{e}^x - 1$ ***A1***

**METHOD 2**

attempts to write both sides in the form of a definite integral ***(M1)***

$\int_0^x \mathrm{g}'(t)\, \mathrm{d}t = \int_0^x \left(3t^2 + 5\mathrm{e}^t\right) \mathrm{d}t$ ***(A1)***

$g(x) - 4 = x^3 + 5\mathrm{e}^x - 5\mathrm{e}^0$ ***(A1)(A1)***

**Note:** Award ***A1*** for $g(x) - 4$ and ***A1*** for $x^3 + 5\mathrm{e}^x - 5\mathrm{e}^0$.

$g(x) = x^3 + 5\mathrm{e}^x - 1$ ***A1***

***[5 marks]***

**6.** (a) recognises the need to find the value of $t$ when $v = 0$ ***(M1)***

$t = 1.57079... \left( = \frac{\pi}{2} \right)$

$t = 1.57 \left( = \frac{\pi}{2} \right)$ (s) ***A1***

***[2 marks]***

(b) recognises that $a(t) = v'(t)$ ***(M1)***

$t_1 = 2.26277...$, $t_2 = 2.95736...$

$t_1 = 2.26$, $t_2 = 2.96$ (s) ***A1A1***

**Note:** Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \le t \le 3$.

***[3 marks]***

(c) speed is greatest at $t = 3$ ***(A1)***

$a = -1.83778...$

$a = -1.84$ (m s$^{-2}$) ***A1***

***[2 marks]***

***Total [7 marks]***

## Section B

**8.** (a) (i) $32$ (cm) ***A1***

(ii) $h_A(0) = \sin(6) + 27$ ***(M1)***

$= 26.7205...$

$= 26.7$ (cm) ***A1***

***[3 marks]***

(b) attempts to solve $h_A(t) = h_B(t)$ for $t$ ***(M1)***

$t = 4.00746..., 4.70343..., 5.88332...$

$t = 4.01, 4.70, 5.88$ (weeks) ***A2***

***[3 marks]***

*continued…*

*Question 8 continued*

(c) recognises that $h_A'(t)$ and $h_B'(t)$ are required ***(M1)***

attempts to solve $h_A'(t) = h_B'(t)$ for $t$ ***(M1)***

$t = 1.18879...$ and $2.23598...$ OR $4.33038...$ and $5.37758...$ OR $7.47197...$ and $8.51917...$ ***(A1)***

**Note:** Award full marks for $t = \frac{4\pi}{3} - 3, \frac{5\pi}{3} - 3, \left(\frac{7\pi}{3} - 3, \frac{8\pi}{3} - 3, \frac{10\pi}{3} - 3, \frac{11\pi}{3} - 3\right)$.
*Award* subsequent marks for correct use of these exact values.

$1.18879... < t < 2.23598...$ OR $4.33038... < t < 5.37758...$ OR
$7.47197... < t < 8.51917...$ ***(A1)***

attempts to calculate the total amount of time ***(M1)***

$3(2.2359... - 1.1887...)$ $\left(= 3\left(\left(\frac{5\pi}{3} - 3\right) - \left(\frac{4\pi}{3} - 3\right)\right)\right)$

$= 3.14$ $(= \pi)$ (weeks) ***A1***

***[6 marks]***

***Total [12 marks]***

**2.** $g'(x) = 2x\mathrm{e}^{x^2+1}$ ***(A2)***

substitute $x = -1$ into **their** derivative ***(M1)***

$g'(-1) = -2\mathrm{e}^2$ ***A1***

**Note:** Award ***A0M0A0*** in cases where candidate's incorrect derivative is $g'(x) = \mathrm{e}^{x^2+1}$.

***[4 marks]***

**5.** recognizing need to integrate ***(M1)***

$\int \frac{6x}{x^2+1}\mathrm{d}x$ OR $u = x^2+1$ OR $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

$\int \frac{3}{u}\mathrm{d}u$ OR $3\int \frac{2x}{x^2+1}\mathrm{d}x$ ***(A1)***

$= 3\ln\left(x^2+1\right)(+c)$ or $3\ln u\,(+c)$ ***A1***

correct substitution of $x = 1$ and $f(x) = 5$ or $x = 1$ and $u = 2$ into equation

using **their** integrated expression (must involve *c*) ***(M1)***

$5 = 3\ln 2 + c$

$f(x) = 3\ln\left(x^2+1\right) + 5 - 3\ln 2 \left(= 3\ln\left(x^2+1\right) + 5 - \ln 8 = 3\ln\left(\frac{x^2+1}{2}\right) + 5\right)$

(or equivalent) ***A1***

**Note:** Accept the use of the modulus sign in working and the final answer.

***[5 marks]***

**3.** (a) $(0.708519..., 0.639580...)$

$(0.709, 0.640)$ $(x = 0.709,\ y = 0.640)$ ***A1A1***

***[2 marks]***

(b) $1.09885...$

$x = 1.10$ accept $(1.10, 0)$ ***A1***

***[1 mark]***

(c) **METHOD 1**

$\int_0^2 |f(x)|\,dx$ ***(A1)***

$4.61117...$

area = $4.61$ ***A2***

**METHOD 2**

$-\int_{1.09885...}^{2} f(x)\,dx$ OR $\int_{1.09885...}^{2} |f(x)|\,dx$ OR $4.17527\ldots$ ***(A1)***

$\int_0^{1.09885...} f(x)\,dx - \int_{1.09885...}^{2} f(x)\,dx$ OR $0.435901\ldots + 4.17527\ldots$ ***(A1)***

$4.61117...$

area = $4.61$ ***A1***

***[3 marks]***

***Total [6 marks]***

**7.** (a) initial displacement is $s(0)$ ***(M1)***

$6\ (\text{m})$ ***A1***

***[2 marks]***

(b) velocity is $s'$ ***(M1)***

$-2.29920$

$-2.30\ (\text{m/s})$ ***A1***

***[2 marks]***

(c) attempting to find $t$ when the particle changes direction ***(M1)***

$t = 0.433007...\text{AND}\ 3.25575...\text{AND}\ 6.33965...$ (may be seen on a graph) ***(A1)***

particle travels away from P when $v > 0$ OR when $s' > 0$ ***(M1)***

$0 \le t < 0.433007...,\ 3.25575... < t < 6.33965...$

$0 \le t < 0.433,\ 3.26 < t < 6.34$ ***A1A1***

***[5 marks]***

*continued…*

*Question 7 continued*

(d) recognizing that acceleration is $a(t) = v'(t)$ OR $a(t) = s''(t)$ ***(M1)***

attempting to find max/min on graph of velocity OR finding zeros on graph of acceleration ***(M1)***

$b = 1.23140...$, $c = 5.68959...$

$b = 1.23$, $c = 5.69$ ***A1A1***

***[4 marks]***

(e) **METHOD 1** (using integral of velocity)

correct integral (accept absence of $\mathrm{d}t$) ***(A1)***

$\int_{1.23140...}^{5.68959...} |v(t)| \, \mathrm{d}t$ OR $\int_{b}^{c} |s'(t)| \, \mathrm{d}t$ OR $-\int_{1.23140...}^{3.25575...} v(t) \, \mathrm{d}t + \int_{3.25575...}^{5.68959...} v(t) \, \mathrm{d}t$ OR

$3.8560 + 15.696$

$19.5525\ldots$

total distance $= 19.6$ (m) ***A2***

*continued…*

*Question 7 continued*

**METHOD 2** (using differences in displacement)

finding displacement at $b, c$ **and** local min on displacement graph ***(A1)***

$(b,\ 4.43306),\ (c,\ 16.2734), (3.25575,\ 0.577001)$ OR $4.43306, 0.577001,\ 16.2734$

correct approach ***(A1)***

$(4.43306 – 0.577001) + (16.2734 – 0.577001)$ OR towards $\mathrm{P}\ 3.85606$ + away from $\mathrm{P}\ 15.696$

$19.5525\ldots$

total distance $= 19.6\ (\mathrm{m})$ ***A1***

***[3 marks]***

***Total [16 marks]***

**7.** (a) substitution of $x = 0$ ***(M1)***

$(y =)\ 3$ (accept $(0, 3)$) ***A1***

***[2 marks]***

(b) evidence of using the product rule ***(M1)***

$h'(x) = 2e^x + 2xe^x$ ***A1***

***[2 marks]***

(c) setting their derivative equal to zero ***(M1)***

correct working ***(A1)***

$2e^x(1+x)\ (=0)$ OR $-2x = 2$

$x = -1$ (seen anywhere, and must follow on from their derivative) ***A1***

substituting their value of $x$ into $h(x)$ ***(M1)***

$y = -\frac{2}{e} + 3\ \left(= -2e^{-1} + 3\right)$ ***A1***

$A\left(-1, -\frac{2}{e} + 3\right)$

***[5 marks]***

*continued…*

*Question 7 continued*

(d) (i) $h''(x) = 2e^x + 2e^x + 2xe^x$ OR $2e^x + 2e^x(1+x)$ ***A1A1***

**Note:** Award ***A1*** for $\left(2e^x\right)' = 2e^x$ , ***A1*** for $2e^x + 2xe^x$ or $\left(2x+2\right)e^x$

$h''(x) = \left(2x+4\right)e^x$ ***AG***

(ii) recognition that $h'' > 0$ OR attempt to find point of inflexion ***(M1)***

since $e^x > 0$, $2x+4>0$ OR $2x+4=0$ $\left(\Rightarrow x=-2\right)$

$x > -2$ ***A1***

***[4 marks]***

***Total [13 marks]***

**9.** (a) attempt to integrate $v$ (integration of at least one term) ***(M1)***

$\left(s(t)=\right)-\frac{1}{4}t^4+\frac{7}{6}t^3-t^2+6t\ \ (+C)$ ***A2***

| **Note**: Award ***A1*** for at least two correct terms. |
|---|

substitution of $t=1$ into their integrated expression ***(M1)***

displacement = $5\frac{11}{12}\left(=\frac{71}{12}\right)$ (m) ***A1***

***[5 marks]***

(b) attempt to differentiate $v$ (differentiation of at least one term) ***(M1)***

$a(t)=-3t^2+7t-2$ ***A1***

***[2 marks]***

(c) setting their $v'(t)=0$ ***(M1)***

$-3t^2+7t-2=0$

valid attempt to solve quadratic ***(M1)***

$(3t-1)(t-2)=0$ OR $\frac{-7\pm\sqrt{49-4(-3)(-2)}}{-6}$

$t=\frac{1}{3},2$ ($t=\frac{1}{3}$ may be omitted) ***(A1)***

substitute their largest positive $t$-value into $v(t)$ ***(M1)***

greatest speed is $8$ ($ms^{-1}$) ***A1***

***[5 marks]***

*continued…*

*Question 9 continued*

(d) attempt to check other boundary value at $t = 4$ ***(M1)***

$v(4) = -64 + 56 - 8 + 6 \ \ (= -10)$

greatest speed is $10$ ms$^{-1}$ ***A1***

***[2 marks]***

(e) identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)***

$t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$

$\int_{\frac{1}{3}}^{2} v(t)\,\mathrm{d}t + \int_{k}^{4} |v(t)|\,\mathrm{d}t$ OR $\int_{\frac{1}{3}}^{2} v\,\mathrm{d}t + \left|\int_{k}^{4} v\,\mathrm{d}t\right|$ OR $\int_{\frac{1}{3}}^{2} v(t)\,\mathrm{d}t - \int_{k}^{4} v(t)\,\mathrm{d}t$ ***A1***

**Note**: Condone missing $\mathrm{d}t$ .

***[3 marks]***

***Total [17 marks]***

**5.** $A = \int_0^c \frac{x}{x^2+2}\,\mathrm{d}x$

**EITHER**

attempts to integrate by inspection or substitution using $u = x^2 + 2$ or $u = x^2$ ***(M1)***

**Note:** If candidate simply states $u = x^2 + 2$ or $u = x^2$, but does not attempt to integrate, do not award the ***(M1)***.

**Note:** If candidate does not explicitly state the $u$-substitution, award the ***(M1)*** only for expressions of the form $k \ln u$ or $k \ln(u+2)$.

$\left[\frac{1}{2}\ln u\right]_2^{c^2+2}$ OR $\left[\frac{1}{2}\ln(u+2)\right]_0^{c^2}$ OR $\left[\frac{1}{2}\ln\left(x^2+2\right)\right]_0^{c}$ ***A1***

**Note:** Limits may be seen in the substitution step.

**OR**

attempts to integrate by inspection ***(M1)***

**Note:** Award the ***(M1)*** only for expressions of the form $k \ln\left(x^2+2\right)$.

$\left[\frac{1}{2}\ln\left(x^2+2\right)\right]_0^{c}$ ***A1***

**Note:** Limits may be seen in the substitution step.

**THEN**

correctly substitutes their limits into their integrated expression ***(M1)***

$\frac{1}{2}\left(\ln\left(c^2+2\right) - \ln 2\right)\left(= \ln 3\right)$ OR $\frac{1}{2}\ln\left(c^2+2\right) - \frac{1}{2}\ln 2\left(= \ln 3\right)$

*continued…*

*Question 5 continued*

correctly applies at least one log law to their expression ***(M1)***

$\frac{1}{2}\ln\left(\frac{c^2+2}{2}\right)(=\ln 3)$ OR $\ln\sqrt{c^2+2}-\ln\sqrt{2}(=\ln 3)$ OR $\ln\left(\frac{c^2+2}{2}\right)=\ln 9$

**OR** $\ln(c^2+2)-\ln 2=\ln 9$ OR $\ln\sqrt{\frac{c^2+2}{2}}(=\ln 3)$ OR $\ln\frac{\sqrt{c^2+2}}{\sqrt{2}}(=\ln 3)$

**Note:** Condone the absence of $\ln 3$ up to this stage.

$\frac{c^2+2}{2}=9$ **OR** $\sqrt{\frac{c^2+2}{2}}=3$ ***A1***

$c^2=16$

$c=4$ ***A1***

**Note:** Award ***A0*** for $c=\pm 4$ as a final answer.

***Total [6 marks]***

**9.** (a) $y^2 = 9 - x^2$ OR $y = \pm\sqrt{9-x^2}$ ***A1***

(since $y > 0$) $\Rightarrow y = \sqrt{9-x^2}$ ***AG***

***[1 mark]***

(b) $b = 2y \left(= 2\sqrt{9-x^2}\right)$ or $h = x + 3$ ***(A1)***

attempts to substitute their base expression and height expression into $A = \frac{1}{2}bh$ ***(M1)***

$A = \sqrt{9-x^2}(x+3)$ (or equivalent) $\left(= \frac{2(x+3)\sqrt{9-x^2}}{2} = x\sqrt{9-x^2} + 3\sqrt{9-x^2}\right)$ ***A1***

***[3 marks]***

(c) attempts to use the product rule to find $\frac{\mathrm{d}A}{\mathrm{d}x}$ ***(M1)***

attempts to use the chain rule to find $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{9-x^2}$ ***(M1)***

$\left(\frac{\mathrm{d}A}{\mathrm{d}x} = \right) \sqrt{9-x^2} + (3+x)\left(\frac{1}{2}\right)\left(9-x^2\right)^{-\frac{1}{2}}(-2x) \left(= \sqrt{9-x^2} - \frac{x^2+3x}{\sqrt{9-x^2}}\right)$ ***A1***

$\left(\frac{\mathrm{d}A}{\mathrm{d}x} = \right) \frac{9-x^2}{\sqrt{9-x^2}} - \frac{x^2+3x}{\sqrt{9-x^2}} \left(= \frac{9-x^2-\left(x^2+3x\right)}{\sqrt{9-x^2}}\right)$ ***A1***

$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{9-3x-2x^2}{\sqrt{9-x^2}}$ ***AG***

***[4 marks]***

*continued…*

*Question 9 continued*

(d) $\frac{\mathrm{d}A}{\mathrm{d}x} = 0 \left( \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}} = 0 \right)$ ***(M1)***

attempts to solve $9 - 3x - 2x^2 = 0$ (or equivalent) ***(M1)***

$-(2x - 3)(x + 3)(= 0)$ OR $x = \frac{3 \pm \sqrt{(-3)^2 - 4(-2)(9)}}{2(-2)}$ (or equivalent) ***(A1)***

$x = \frac{3}{2}$ ***A1***

**Note:** Award the above ***A1*** if $x = -3$ is also given.

substitutes their value of $x$ into either $y = \sqrt{9 - x^2}$ or $y = -\sqrt{9 - x^2}$ ***(M1)***

**Note:** Do not award the above ***(M1)*** if $x \leq 0$.

$y = -\sqrt{9 - \left(\frac{3}{2}\right)^2}$

$= -\frac{\sqrt{27}}{2} \left( = -\frac{3\sqrt{3}}{2}, = -\sqrt{\frac{27}{4}}, = -\sqrt{6.75} \right)$ ***A1***

***[6 marks]***

***Total [14 marks]***

**7.** (a) (i) $96\,(°)$ (exact) ***A1***

(ii) $79.9970...$

$80.0\,(°)$ (accept $80$) ***A1***

***[2 marks]***

(b) $-4.71976...$

$-4.72$ (°C $min^{-1}$) ***A2***

***[2 mark]***

(c) 3 valid descriptors, in any order: ***A2***

- at 3 minutes (or when $t = 3$)
- cooling/decreasing (do not accept "changing")
- 4.72 °C $min^{-1}$ (must include units) (accept approximately 5 deg/min)

***[2 marks]***

*continued…*

*Question 7 continued*

(d) **METHOD 1**

valid attempt to solve $H(t) = 67$ (accept an inequality) ***(M1)***

eg intersection of graphs, use of logarithms.

6.11058… ***(A1)***

7 (min) ***A1***

**METHOD 2**

valid attempt to find crossover values ***(M1)***

(6, 67.4087…) and (7, 63.8406…) ***(A1)***

7 (min) ***A1***

***[3 marks]***

(e) recognition that $t \to \infty$ ***(M1)***

$21(°C)$ ***A1***

***[2 marks]***

(f) **METHOD 1 (working with slopes of *H*)**

valid attempt to analyse progression of slopes of *H* ***(M1)***

$\lim_{t \to \infty} H'(t) = 0$ ***A1***

**METHOD 2 (working with $H'$)**

valid attempt to use $H'$ and large values of $t$. ***(M1)***

$\lim_{t \to \infty} H'(t) = 0$ ***A1***

***[2 marks]***

***Total [13 marks]***

**6.** (a) recognize that acceleration is zero when $v'(t) = 0$ OR at a local maximum on the graph of $v$ ***(M1)***

$t_1 = 0.394791...$

$t_1 = 0.395 \left( = \arctan\left(\frac{5}{12}\right)\right)$ (seconds) ***A1***

***[2 marks]***

(b) recognition that $v = 0$ ***(M1)***

sketch OR $t = 4.71238...$ OR $t = 10.9955...$

$t_2 = 10.9955...$

$t_2 = 11.0 \left( = \frac{7\pi}{2}\right)$ ***A1***

***[2 marks]***

(c) $\int_{t_1}^{t_2} |v| \, dt$ OR $\int_{0.394791...}^{10.9955...} |v| \, dt$ OR $\int_{0.394791...}^{4.71238...} v \, dt + \int_{4.71238...}^{10.9955...} |v| \, dt \left(= 6.53806... + 1.29313...\right)$

OR $\int_{0.394791...}^{4.71238...} v \, dt - \int_{4.71238...}^{10.9955...} v \, dt \left(= 6.53806... - \left(-1.29313...\right)\right)$ ***(A1)***

distance = $7.83118\ldots$

= $7.83$ (m) ***A1***

***[2 marks]***

***Total [6 marks]***

## Section B

**1.** (a) (i) recognizing that $f'$ is needed ***(M1)***

$(f'(4) =) -2$ ***A1***

(ii) $\frac{1}{2}$ ***A1***

***[3 marks]***

(b) $y + 24 = \frac{1}{2}(x - 4)$ $(y = 0.5x - 26)$ ***A1***

***[1 mark]***

(c) attempt to find intersection of curve and their normal either graphically or analytically ***(M1)***

sketch showing intersection OR $x^2 - 10x = 0.5x - 26$

$(6.5, -22.8)$ (exact answer is $(6.5, -22.75)$) ***A1A1***

***[3 marks]***

***Total [7 marks]***



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**5.** (a) recognizing at rest when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ OR $s$ is a minimum ***(M1)***

$q = 4.05165...$

$= 4.05$ ***A1***

***[2 marks]***

(b) **METHOD 1**

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{4.05...} |v(t)| \mathrm{d}t$ OR $\int_0^{4.05...} \left|\frac{\mathrm{d}}{\mathrm{d}t} s(t)\right| \mathrm{d}t$ OR $\left|\int_0^{4.05...} v(t) \mathrm{d}t\right|$ OR $-\int_0^{4.05...} v(t)\,\mathrm{d}t$ ***(A1)***

**Note:** Only accept $\left|\int_0^q v(t)\,\mathrm{d}t\right|$ if their value of $q$ does not result in the particle changing direction in the first $q$ seconds.

$= 8.51841...$

$= 8.52 \text{ (m)}$ ***A1***

**METHOD 2**

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.31841...$ AND at minimum is $-5.2$ ***(A1)***

total distance travelled = $3.31841... - (-5.2)$

$= 8.51841...$

$= 8.52 \text{ (m)}$ ***A1***

***[3 marks]***

***Total [5 marks]***

**7.** (a) $\pi x^2 h = 41$ ***(A1)***

attempt to rearrange AND substitute their $h$ into the expression for the total surface area ***(M1)***

$S = 2\pi x\left(\frac{41}{\pi x^2}\right) + 4\pi x^2$ ***A1***

$S = \frac{82}{x} + 4\pi x^2$ ***AG***

***[3 marks]***

(b) (i) $\frac{\mathrm{d}S}{\mathrm{d}x} = -\frac{82}{x^2} + 8\pi x$ (or equivalent) ***A1A1***

**Note:** Award ***A1*** for each correct term.
Award ***A1A0*** if additional terms are given.

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ ***(M1)***

$-\frac{82}{a^2} + 8\pi a = 0$ ***(A1)***

$(a =)\left(\frac{82}{8\pi}\right)^{\frac{1}{3}}$ ***A1***

***[5 marks]***

*continued…*

*Question 7 continued*

(c) (i) $\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 164x^{-3} + 8\pi$ (or equivalent) ***A1A1***

**Note:** Award ***A1*** for each correct term.
Award ***A1A0*** if additional terms are given.

(ii) **EITHER**

substituting their value of $x$ into their $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$ ***(M1)***

$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 75.3982...$

$= 75.4\ (=24\pi) > 0$ (75.7 from a=1.48) ***A1***

**OR**

sketch of the graph of $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$ with their value of $x$ clearly indicated ***(M1)***

$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0$ at $x = a$ ***A1***

**THEN**

therefore $S$ is a minimum ***AG***

(iii) attempt to substitute their value of $a$ into $S$ OR use of graph of $S$ ***(M1)***

$82.9304...$

minimum surface area = $82.9$ (cm$^2$) ***A1***

***[6 marks]***

***Total [14 marks]***

**1.** (a) (i) recognizing that $f'$ is needed ***(M1)***

$\left(f'(3) =\right) -2$ ***A1***

(ii) $\frac{1}{2}$ ***A1***

***[3 marks]***

(b) $y+15=\frac{1}{2}(x-3)$ $(y=0.5x-16.5)$ ***A1***

***[1 mark]***

(c) attempt to find intersection of curve and their normal either graphically or analytically ***(M1)***

sketch showing intersection OR $x^2-8x=0.5x-16.5$

$(5.5,-13.8)$ (exact answer is $(5.5,-13.75)$) ***A1A1***

| |
|---|
| **Note:** Award ***A1*** for $5.5$ and ***A1*** for $-13.8$. |

***[3 marks]***

***Total [7 marks]***



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**5.** (a) recognizing at rest when $\frac{\mathrm{d}s}{\mathrm{d}t}=0$ OR $s$ is a minimum ***(M1)***

$q=5.73553...$

$=5.74$ ***A1***

***[2 marks]***

(b) **METHOD 1**

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{5.73...}|v(t)|\,\mathrm{d}t$ OR $\int_0^{5.73...}\left|\frac{\mathrm{d}}{\mathrm{d}t}s(t)\right|\mathrm{d}t$ OR $\left|\int_0^{5.73...}v(t)\,\mathrm{d}t\right|$ OR $-\int_0^{5.73...}v(t)\,\mathrm{d}t$ ***(A1)***

**Note:** Only accept $\left|\int_0^{q}v(t)\,\mathrm{d}t\right|$ if their value of $q$ does not result in the particle changing direction in the first $q$ seconds.

$=7.68302...$

$=7.68\ (\mathrm{m})$ ***A1***

**METHOD 2**

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.38302...$ AND at minimum is $-4.3$ ***(A1)***

total distance travelled = $3.38302...-(-4.3)$

$=7.68302...$

$=7.68\ (\mathrm{m})$ ***A1***

***[3 marks]***

***Total [5 marks]***

**7.** (a) $\pi x^2 h = 45$ ***(A1)***

attempt to rearrange AND substitute their $h$ into the expression for the total surface area ***(M1)***

$S = 2\pi x\left(\frac{45}{\pi x^2}\right) + 4\pi x^2$ ***A1***

$S = \frac{90}{x} + 4\pi x^2$ ***AG***

***[3 marks]***

(b) (i) $\frac{\mathrm{d}S}{\mathrm{d}x} = -\frac{90}{x^2} + 8\pi x$ (or equivalent) ***A1A1***

| |
|---|
| **Note:** Award ***A1*** for each correct term.<br>Award ***A1A0*** if additional terms are given. |

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ ***(M1)***

$-\frac{90}{a^2} + 8\pi a = 0$ ***A1***

$(a =)\left(\frac{90}{8\pi}\right)^{\frac{1}{3}}$ ***A1***

***[5 marks]***

*continued…*

*Question 7 continued*

(c) (i) $\frac{d^2S}{dx^2} = 180x^{-3} + 8\pi$ (or equivalent) ***A1A1***

| |
|---|
| **Note:** Award ***A1*** for each correct term.<br>Award ***A1A0*** if additional terms are given. |

(ii) **EITHER**

substituting their value of $x$ into their $\frac{d^2S}{dx^2}$ ***(M1)***

$\frac{d^2S}{dx^2} = 75.39822...$

$= 75.4 \ (=24\pi) > 0$ ***A1***

**OR**

sketch of the graph of $\frac{d^2S}{dx^2}$ with their value of $x$ clearly indicated ***(M1)***

$\frac{d^2S}{dx^2} > 0$ at $x = a$ ***A1***

**THEN**

therefore $S$ is a minimum ***AG***

(iii) attempt to substitute their value of $a$ into $S$ OR use of graph of $S$ ***(M1)***

$88.2401...$

minimum surface area = $88.2$ (cm$^2$) ***A1***

***[6 marks]***

***Total [14 marks]***

**5.** (a) recognising $\cos x = 2\sin x \cos x$ ***(M1)***

$(\cos x \neq 0)$ so $\sin x = \frac{1}{2}$ OR one correct value (accept degrees) ***(A1)***

$x$ - coordinates $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ ***A1***

**Note**: Award ***(M1)(A1)A0*** for solutions of $30^\circ$ and $150^\circ$.

***[3 marks]***

(b) **METHOD 1**

attempt to integrate $\pm(\cos x - \sin 2x)$ ***(M1)***

$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (\cos x - \sin 2x)\,dx$ OR $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (\cos x - 2\sin x \cos x)\,dx$

$= \left[\sin x + \frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ OR $= \left[\sin x - \sin^2 x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ ***A1***

**Note**: Award ***A1*** for $\pm$ correct integration. Condone incorrect or absent limits up to this point.

attempt to substitute their limits into their integral and subtract ***M1***

$= \left(\sin\left(\frac{5\pi}{6}\right) + \frac{1}{2}\cos\left(\frac{5\pi}{3}\right)\right) - \left(\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\cos(\pi)\right)$ OR $\left(\sin\left(\frac{5\pi}{6}\right) - \sin^2\left(\frac{5\pi}{6}\right)\right) - \left(\sin\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{2}\right)\right)$

$= \left(\frac{1}{2} + \frac{1}{4}\right) - \left(1 - \frac{1}{2}\right)$ OR $= \left(\frac{1}{2} - \frac{1}{4}\right) - (1 - 1)$

area $= \frac{1}{4}$ ***A1***

**Note**: Award all corresponding marks as appropriate for finding the area between A and B.
Accept work done in degrees.

*continued…*

*Question 5 continued*

**METHOD 2**

$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \cos x \mathrm{d}x = \left[\sin x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ and $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sin 2x \mathrm{d}x = \left[-\frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ ***A1***

**Note**: Award ***A1*** for correct integration. Condone incorrect or absent limits up to this point.

attempt to substitute their limits into their integral and subtract (for both integrals) ***M1***

$\sin\left(\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{2}\right)$ and $-\frac{1}{2}\cos\left(\frac{5\pi}{3}\right) + \frac{1}{2}\cos(\pi)$

attempt to subtract the two integrals in either order (seen anywhere) ***(M1)***

$\left(\sin\left(\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{2}\right)\right) - \left(-\frac{1}{2}\cos\left(\frac{5\pi}{3}\right) + \frac{1}{2}\cos(\pi)\right)$ OR $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \cos x \mathrm{d}x - \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sin 2x \mathrm{d}x$

$= \left(\frac{1}{2} - 1\right) - \left(\frac{1}{4} - \frac{1}{2}\right) \quad \left(= -\frac{1}{4}\right)$

area $= \frac{1}{4}$ ***A1***

**Note**: Award all corresponding marks as appropriate for finding the area between A and B.
Accept work done in degrees.

***[4 marks]***

***Total [7 marks]***

**7.** (a) (i) $\frac{dy}{dx} = 3x^2 - 2x - 1$ ***A1A1***

**Note:** Award ***A1*** for $3x^2 - 2x$ and ***A1*** for $-1$.

(ii) $\frac{d^2y}{dx^2} = 6x - 2$ ***A1***

***[3 marks]***

(b) setting their (quadratic) $\frac{dy}{dx}$ to 0 and solve using valid method ***(M1)***

$(3x+1)(x-1) = 0$ OR $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} \left( = \frac{2 \pm \sqrt{4+12}}{6} \right)$

$x = -\frac{1}{3}$ ( OR $x = 1$ ) ***A1***

substituting one of their $x$ values into $\frac{d^2y}{dx^2}$ ***M1***

**EITHER**

at $x = -\frac{1}{3}$, $\frac{d^2y}{dx^2} = 6\left(-\frac{1}{3}\right) - 2 \left(= -4\right) < 0$ so local max ***R1***

**OR**

at $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 2(=4) > 0$ so local min hence local max at $x = -\frac{1}{3}$ ***R1***

**Note:** Award ***R1*** only if the previous ***M1*** has been awarded and there is reference to $< 0$ or $> 0$, as appropriate.

*continued…*

*Question 7 continued*

**THEN**

substituting their $x$-coordinate of A into $y$ ***(M1)***

$$y=\left(-\frac{1}{3}\right)^3-\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)+1$$

$$y=\frac{32}{27}$$ ***A1***

so coordinates of A are $\left(-\frac{1}{3},\frac{32}{27}\right)$

**Note:** This ***(M1)A1*** may be awarded independently of the previous ***M1R1***.

***[6 marks]***

(c) setting their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to $0$ ***(M1)***

$$6x-2=0$$

$$x=\frac{1}{3}$$ ***A1***

***[2 marks]***

(d) gradient of tangent = $-1$ ***(A1)***

negative reciprocal of their gradient ***(M1)***

gradient of normal = $1$

equation is $y=x+1$ (accept point/slope form $y-1=(x-0)$) ***A1***

**Note:** Do not accept $L=x+1$.

***[3 marks]***

***Total [14 marks]***

**8.** (a) (i) **METHOD 1**

$x^2 + 2x + 2 = (x+1)^2 + 1$ ***A1***

$(x+1)^2 \geq 0$ ***R1***

$(x+1)^2 + 1 > 0$ ***AG***

**METHOD 2**

discriminant $\Delta = 4 - 8(= -4)$ ***A1***

$\Delta < 0$ and concave up OR $\Delta < 0$ and coefficient of $x^2 > 0$ (may be seen in diagram) ***R1***

hence $x^2 + 2x + 2 > 0$ ***AG***

(ii) $x > -1$ (accept $x \geq -1$) ***A1***

***[3 marks]***

*continued…*

*Question 8 continued.*

(b) (i) $x=-1$ ***A1***

(ii) attempt to use the quotient rule ***(M1)***

$$f''(x)=\frac{2\left(x^2+2x+2\right)-(2x+2)(2x+2)}{(x^2+2x+2)^2}$$ ***A1A1***

| |
|---|
| ***Note:*** Award ***A1*** for first term in numerator, ***A1*** for second term in numerator (including the minus sign). If the numerator is correct, but the denominator is incorrect or missing, award ***M1A1A0***. |

$$f''(x)=\frac{2x^2+4x+4-4x^2-8x-4}{(x^2+2x+2)^2} \text{ OR } f''(x)=\frac{2x^2+4x+4-\left(4x^2+8x+4\right)}{(x^2+2x+2)^2}$$ ***A1***

$$f''(x)=\frac{-2x^2-4x}{(x^2+2x+2)^2}$$ ***AG***

*continued…*

*Question 8 continued.*

(iii) substituting $x = -1$ into $f''(x)$, clearly leading to positive numerator ***A1***

$$f''(-1) = \left(\frac{-2+4}{1}\right) \ (= 2)$$

$f''(-1) > 0$ ***R1***

therefore this is a local minimum point ***AG***

| **Note**: Do not award ***A0R1***. |
|---|

***[7 marks]***

*continued…*

*Question 8 continued.*

(c) recognition to integrate $f'(x)$ ***(M1)***

$f(x)=\int\frac{2x+2}{x^2+2x+2}\mathrm{d}x$ OR $\int\frac{1}{u}\mathrm{d}u$ with $u=x^2+2x+2$

$f(x)=\ln\left(x^2+2x+2\right)(+c)$ ***A1***

using initial conditions $x=2, y=3+\ln 10$ to find $c$ ***(M1)***

$f(2)=\ln(10)+c=3+\ln(10)\Rightarrow c=3$

$f(x)=\ln\left(x^2+2x+2\right)+3$ ***A1***

***[4 marks]***

(d) $f'(2)=\frac{6}{10}$ ***A1***

attempt to take the negative reciprocal of their $f'(2)$ ***M1***

gradient of normal $=-\frac{10}{6}$

$y-(\ln 10+3)=-\frac{5}{3}(x-2)$ $\left(y=-\frac{5}{3}x+\frac{19}{3}+\ln 10\right)$ $\left(\frac{y-\ln 10-3}{x-2}=-\frac{5}{3}\right)$ ***A1***

***[3 marks]***

***Total [17 marks]***

**5.** (a) recognition that velocity is zero ***(M1)***

$v = 2\sin(0.5t) + 0.3t - 2 = 0$

$t = 1.68694\ldots$

$t = 1.69$ ***A1***

***[2 marks]***

(b) recognition that $v > 0$ ***(M1)***

$1.68694... < t < 6.11857...$

$1.69 < t < 6.12$ ***A1***

***[2 marks]***

(c) attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of $\mathrm{d}t$) ***(M1)***

$\int_0^{10}(2\sin(0.5t) + 0.3t - 2)\,\mathrm{d}t$ OR $\int_0^{10} v(t)\,\mathrm{d}t$

$= -2.13464...$

$= -2.13$ (m) ***A1***

| **Note**: Award ***(M1)A0*** if $-2.13$ is followed by $2.13$. |
|---|

***[2 marks]***

***Total [6 marks]***

**2.** (a) attempts to find an intersection point ***(M1)***

$a = -0.916562...$ or $b = 0$

$a = -0.917$, $b = 0$ ***A1A1***

***[3 marks]***

(b) let $A$ be the area of the region

**EITHER**

attempts to form the required integral involving subtraction (in any order). Accept absence of limits or incorrect limits. Accept absence of dx. ***(M1)***

**OR**

shows a graph with the required area shaded ***(M1)***

**THEN**

$A = \left( \int_{a}^{b} \left( f(x) - g(x) \right) \mathrm{d}x \right) = \int_{-0.916562...}^{0} \left( 1 - x^2 - \mathrm{e}^{2x} \right) \mathrm{d}x$ (or equivalent) ***(A1)***

$A = 0.239855...$

$A = 0.240$ ***A1***

***[3 marks]***

***Total [6 marks]***

**5.** (a) $v = -0.996114...$

$v = -0.996$ (ms$^{-1}$) ***A1***

***[1 mark]***

(b)

considers $v'(t) = 0$ ***(M1)***

$t = 0.405833...$

$v_{\max} = 1.18230...$

$v_{\max} = 1.18$ (ms$^{-1}$) ***A1***

***[2 marks]***

(c) recognizes that the particle changes direction when $v = 0$ ***(M1)***

**Note:** Award ***(M1)*** for $t = 1.65840...$ seen.

finds acceleration for their value of *t* for which $v(t) = 0$ ***(M1)***

$v'(1.65840...)$

$a = -2.53487...$

$a = -2.53$ (ms$^{-2}$) ***A1***

***[3 marks]***

***Total [6 marks]***

**8.** (a) let $A_R$ denote the area of the rectangle and $A_S$ denote the area of the semicircle

one correct area $A_R = 2xh$ OR $A_S = \frac{1}{2}\pi x^2$ ***(A1)***

$A = 2xh + \frac{1}{2}\pi x^2 \left( = x\left( 2h + \frac{1}{2}\pi x \right) \right)$ ***A1***

***[2 marks]***

(b) attempts to find a correct expression for the total perimeter ***(M1)***

$(P =) 2x + 2h + \pi x$, base + 2×height + half circunference

$2x + 2h + \pi x = 10$ OR $2h = 10 - 2x - \pi x$ ***A1***

$h = \frac{1}{2}(10 - 2x - \pi x)$ ***AG***

***[2 marks]***

*continued…*

*Question 8 continued.*

(c) $L = 3(2xh) + 1\left(\frac{1}{2}\pi x^2\right)$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for multiplying $2xh$ by $3$ and award ***(A1)*** for a fully correct expression.

substitutes $h = \frac{1}{2}(10 - 2x - \pi x)$ into their expression for $L$ ***M1***

$$L = 6x\left(\frac{1}{2}(10 - 2x - \pi x)\right) + \frac{1}{2}\pi x^2$$

$$= 30x - 6x^2 - 3\pi x^2 + \frac{1}{2}\pi x^2 \left(= 30x - \left(6 + \frac{5\pi}{2}\right)x^2\right)$$ ***A1***

$$L = 30x - 6x^2 - \frac{5}{2}\pi x^2$$ ***AG***

***[4 marks]***

*continued…*

*Question 8 continued.*

(d) (i) $\frac{\mathrm{d}L}{\mathrm{d}x} = 30 - 12x - 5\pi x \ \left(= 30 - (12 + 5\pi)x\right)$ (accept $\frac{\mathrm{d}L}{\mathrm{d}x} = 30 - 27.7x$ ) ***(M1)A1***

(ii) recognition that $\frac{\mathrm{d}L}{\mathrm{d}x} = 0$ (may be represented graphically) ***(M1)***

$x = 1.08272...$

$x = 1.08 \left(= \frac{30}{12 + 5\pi}\right)$ (m) ***A1***

correct reasoning to justify a maximum ***R1***

$L$ is a quadratic (function of $x$) with a negative coefficient of $x^2$ (may be represented as a sketch indicating maximum point) OR

a clearly labelled sign diagram showing the change in gradient OR

$\frac{\mathrm{d}^2L}{\mathrm{d}x^2} = -12 - 5\pi \ (= -27.7079...) \ (< 0)$

(iii) attempts to substitute their value of $x$ into $h$ ***(M1)***

$h = 2.21654...$

$h = 2.22 \left(= \frac{30 + 10\pi}{12 + 5\pi}\right)$ (m) ***A1***

***[7 marks]***

***Total [15 marks]***

**7.** (a) $f(x) = -2x + 12$ (accept $y = -2x + 12$, accept $m = -2$ and $c = 12$) ***A1A1***

**Note:** Award ***A1*** for correct gradient, ***A1*** for correct y-intercept

***[2 marks]***

(b) **METHOD 1** Axis of Symmetry

axis of symmetry is $\frac{-2+6}{2}(=2)$ **(*A1*)**

equating $\frac{-b}{2a}$ to their axis of symmetry **(*M1*)**

$\frac{-b}{-2} = 2$

$b = 4$ ***A1***

**METHOD 2** substitution

attempt to substitute $(-2,0)$ or $(6,0)$ into $g$ **(*M1*)**

$-(-2)^2 - 2b + 12 = 0$ or $-(6)^2 + 6b + 12 = 0$ **(*A1*)**

$b = 4$ ***A1***

**METHOD 3** factored form

attempt to write $g$ in factored form ***(M1)***

$g(x) = (x+2)(6-x)$

$-x^2 + 4x + 12$ **(*A1*)**

$b = 4$ ***A1***

**METHOD 4** quadratic formula

attempt to substitute into quadratic formula and set equal to -2 or 6 ***(M1)***

$6 = \frac{-b \pm \sqrt{b^2 - 4(-1)(12)}}{2(-1)}$ OR $-2 = \frac{-b \pm \sqrt{b^2 - 4(-1)(12)}}{2(-1)}$

$b^2 - 24b + 144 = b^2 + 48$ OR $b^2 + 8b + 16 = b^2 + 48$ (or equivalent, must not contain radical) **(*A1*)**

$b = 4$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued*

(c) recognizing to subtract $g - f$ (in correct order) ***(M1)***

$\int_0^6 \left(-x^2 + 4x + 12\right) - \left(-2x + 12\right) \mathrm{d}x$ ***A1***

$\int_0^6 \left(-x^2 + 6x\right) \mathrm{d}x$ ***AG***

***[2 marks]***

(d) attempt to integrate (***M1***)

$-\frac{1}{3}x^3 + 3x^2 \left(+C\right)$ ***A1***

attempt to substitute limits into their integrated function and find difference (***M1***)

$\left(-\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2\right) - \left(-\frac{1}{3} \cdot 0^3 + 3 \cdot 0^2\right)$

36 ***A1***

***[4 marks]***

(e) $g'(x) = -2x + 4$ ***A1***

attempt to equate their derivative of $g$ to their gradient of $f$ ***(M1)***

$-2x + 4 = -2$ ***A1***

$x = 3$ ***A1***

$y = 15$ ***A1***

***[5 marks]***

***Total [16 marks]***

**9.** (a) outer curved surface area is $2\pi(4r)h$ AND inner curved surface area is $2\pi rh$ ***(A1)***

area of each base (top and bottom) is $\pi(4r)^2 - \pi r^2$ ***(A1)***

$S = 2\left[\pi(4r)^2 - \pi r^2\right] + 2\pi(4r)h + 2\pi rh$ ***A1***

$= 30\pi r^2 + 10\pi rh$ ***AG***

***[3 marks]***

(b) $30\pi r^2 + 10\pi rh = 240\pi$

attempt to solve their equation for $h$ or $rh$ in terms of $r$ (must isolate $h$ or $rh$) ***(M1)***

$h = \frac{240 - 30r^2}{10r}\left(= \frac{24 - 3r^2}{r}\right)$ OR $rh = \frac{240 - 30r^2}{10}\left(= 24 - 3r^2\right)$ (or equivalent) ***A1***

uses volume = large cylinder – small cylinder ***(M1)***

$V = \pi(4r)^2 h - \pi r^2 h \quad \left(= 16\pi r^2 h - \pi r^2 h = 15\pi r^2 h\right)$ ***A1***

attempt to substitute in for $h$ or $rh$ ***(M1)***

$V = 15\pi r^2\left(\frac{24 - 3r^2}{r}\right)$ OR $V = 15\pi r\left(\frac{240 - 30r^2}{10}\right)\left(= 15\pi r\left(24 - 3r^2\right)\right)$ OR

$384\pi r - 48\pi r^3 - 24\pi r + 3\pi r^3$ ***A1***

$= 360\pi r - 45\pi r^3$ ***AG***

***[6 marks]***

(c) $\frac{\mathrm{d}V}{\mathrm{d}r} = 360\pi - 135\pi r^2$ ***A1A1***

***[2 marks]***

*continued…*

*Question 9 continued*

(d) **METHOD 1** (working with r)

recognition that (for a maximum) $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ ***M1***

$360\pi-135\pi r^2=0$

$r^2=\frac{360}{135}\left(=\frac{8}{3}\right)$

$r=\sqrt{\frac{360}{135}}\left(=\sqrt{\frac{8}{3}}\right)$ ***A1***

$p=2$ OR $r=2\sqrt{\frac{2}{3}}$ ***A1***

**METHOD 2** (working with $p\sqrt{\frac{2}{3}}$)

recognition that (for a maximum) $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ ***M1***

$360\pi-135\pi\left(p\sqrt{\frac{2}{3}}\right)^2=0$

$360-90p^2=0$

$p^2=4$ ***A1***

$p=2$ OR $r=2\sqrt{\frac{2}{3}}$ ***A1***

***[3 marks]***

*continued…*

*Question 9 continued*

(e) attempt to substitute their value of $r$ into $V = 360\pi r - 45\pi r^3$ ***M1***

$V = 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times \left(2\sqrt{\frac{2}{3}}\right)^3$

$360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times 8 \times \frac{2}{3} \times \sqrt{\frac{2}{3}} \quad \left(= 720\pi\sqrt{\frac{2}{3}} - 240\pi\sqrt{\frac{2}{3}}\right)$ ***(A1)***

$= 480\pi\sqrt{\frac{2}{3}}$ ***A1***

$(q = 480)$

***[3 marks]***

***Total [17 marks]***

**7.** (a) $f(x) = -2x + 8$ (accept $y = -2x + 8$, accept $m = -2$ and $c = 8$) ***A1A1***

**Note:** Award ***A1*** for correct gradient, ***A1*** for correct y-intercept

***[2 marks]***

(b) **METHOD 1** Axis of Symmetry

axis of symmetry is $\frac{-2+4}{2}(=1)$ ***(A1)***

equating $\frac{-b}{2a}$ to their axis of symmetry ***(M1)***

$\frac{-b}{-2} = 1$

$b = 2$ ***A1***

**METHOD 2** substitution

attempt to substitute $(-2,0)$ or $(4,0)$ into $g$ ***(M1)***

$-(-2)^2 - 2b + 8 = 0$ or $-(4)^2 + 4b + 8 = 0$ ***(A1)***

$b = 2$ ***A1***

**METHOD 3** factored form

attempt to write $g$ in factored form ***(M1)***

$g(x) = (x+2)(4-x)$

$-x^2 + 2x + 8$ ***(A1)***

$b = 2$ ***A1***

**METHOD 4** quadratic formula

attempt to substitute into quadratic formula and set equal to -2 or 4 ***(M1)***

$4 = \frac{-b \pm \sqrt{b^2 - 4(-1)(8)}}{2(-1)}$ OR $-2 = \frac{-b \pm \sqrt{b^2 - 4(-1)(8)}}{2(-1)}$

$b^2 - 16b + 64 = b^2 + 32$ OR $b^2 + 8b + 16 = b^2 + 32$ (or equivalent, must not contain radical) ***(A1)***

$b = 2$ ***A1***

***[3 marks]***

*continued…*

*Question 7 continued.*

(c) recognizing to subtract $g - f$ (in correct order) ***(M1)***

$\int_0^4 \left(-x^2 + 2x + 8\right) - \left(-2x + 8\right) \mathrm{d}x$ ***A1***

$\int_0^4 \left(-x^2 + 4x\right) \mathrm{d}x$ ***AG***

***[2 marks]***

(d) attempt to integrate (***M1***)

$-\frac{1}{3}x^3 + 2x^2 \left(+C\right)$ ***A1***

attempt to substitute limits into their integrated function and find difference ***(M1)***

$\left(-\frac{1}{3} \cdot 4^3 + 2 \cdot 4^2\right) - \left(-\frac{1}{3} \cdot 0^3 + 2 \cdot 0^2\right)$

$\frac{32}{3}$ ***A1***

***[4 marks]***

(e) $g'(x) = -2x + 2$ ***A1***

attempt to equate their derivative of $g$ to their gradient of $f$ ***(M1)***

$-2x + 2 = -2$ ***A1***

$x = 2$ ***A1***

$y = 8$ ***A1***

***[5 marks]***

***Total [16 marks]***

**9.** (a) outer curved surface area is $2\pi(4r)h$ AND inner curved surface area is $2\pi rh$ ***(A1)***

area of each base (top and bottom) is $\pi(4r)^2 - \pi r^2$ ***(A1)***

$S = 2\left[\pi(4r)^2 - \pi r^2\right] + 2\pi(4r)h + 2\pi rh$ ***A1***

$= 30\pi r^2 + 10\pi rh$ ***AG***

***[3 marks]***

(b) $30\pi r^2 + 10\pi rh = 240\pi$

attempt to solve their equation for $h$ or $rh$ in terms of $r$ (must isolate $h$ or $rh$) ***(M1)***

$h = \dfrac{240 - 30r^2}{10r}\left(= \dfrac{24 - 3r^2}{r}\right)$ OR $rh = \dfrac{240 - 30r^2}{10}\left(= 24 - 3r^2\right)$ (or equivalent) ***A1***

uses volume = large cylinder – small cylinder ***(M1)***

$V = \pi(4r)^2 h - \pi r^2 h \quad \left(= 16\pi r^2 h - \pi r^2 h = 15\pi r^2 h\right)$ ***A1***

attempt to substitute in for $h$ or $rh$ ***(M1)***

$V = 15\pi r^2\left(\dfrac{24 - 3r^2}{r}\right)$ OR $V = 15\pi r\left(\dfrac{240 - 30r^2}{10}\right)\left(= 15\pi r\left(24 - 3r^2\right)\right)$ OR $384\pi r - 48\pi r^3 - 24\pi r + 3\pi r^3$ ***A1***

$= 360\pi r - 45\pi r^3$ ***AG***

***[6 marks]***

(c) $\dfrac{\mathrm{d}V}{\mathrm{d}r} = 360\pi - 135\pi r^2$ ***A1A1***

***[2 marks]***

*Question 9 continued.*

(d) **METHOD 1** (working with $r$)

recognition that (for a maximum) $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ ***M1***

$360\pi-135\pi r^2=0$

$r^2=\frac{360}{135}\left(=\frac{8}{3}\right)$

$r=\sqrt{\frac{360}{135}}\left(=\sqrt{\frac{8}{3}}\right)$ ***A1***

$p=2$ OR $r=2\sqrt{\frac{2}{3}}$ ***A1***

**METHOD 2** (working with $p\sqrt{\frac{2}{3}}$)

recognition that (for a maximum) $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ ***M1***

$360\pi-135\pi\left(p\sqrt{\frac{2}{3}}\right)^2=0$

$360-90p^2=0$

$p^2=4$ ***A1***

$p=2$ OR $r=2\sqrt{\frac{2}{3}}$ ***A1***

***[3 marks]***

*continued…*

*Question 9 continued.*

(e) attempt to substitute their value of $r$ into $V = 360\pi r - 45\pi r^3$ ***M1***

$$V = 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times \left(2\sqrt{\frac{2}{3}}\right)^3$$

$$= 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times 8 \times \frac{2}{3} \times \sqrt{\frac{2}{3}} \quad \left(= 720\pi\sqrt{\frac{2}{3}} - 240\pi\sqrt{\frac{2}{3}}\right) \qquad \textbf{(A1)}$$

$$= 480\pi\sqrt{\frac{2}{3}} \quad (q = 480) \qquad \textbf{A1}$$

***[3 marks]***

***Total [17 marks]***

**3.** (a) $\frac{1}{3}e^{3x} + 5x + c$ ***A1A1A1***

**Note:** Award ***A1*** for each term.

***[3 marks]***

(b) recognition that $h(x) = \int h'(x)$ ***(M1)***

$h(x) = \frac{1}{3}e^{3x} + 5x + c$

$\left(h(2.1) = \right)\frac{1}{3}e^{3(2.1)} + 5(2.1) + c = 185$ ***(A1)***

$c = -7.02397...$

$h(x) = \frac{1}{3}e^{3x} + 5x - 7.02\left( = \frac{1}{3}e^{3x} + 5x + 174.5 - \frac{1}{3}e^{6.3} \right)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**8.** (a) (i) $a = 0.362$ (exact); $b = 30.5$ (exact) ***A1A1***

**Note**: Award ***A1A0*** if the values of $a$ and $b$ are interchanged or not labelled.

(ii) $a$ represents the (average) rate of increase (change) in population ($0.362$ millions of people per year). (or equivalent) ***R1***

***[3 marks]***

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

***[1 mark]***

(c) (i) attempt to find $B(100)$ ***(M1)***

$61.4707\ldots$

$61.5$ million OR $61{,}500{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.7%. ***A1***

**Note**: Description must include some reference to annual rate.

***[3 marks]***

(d) $58.1070\ldots$

$58.1$ million OR $58{,}100{,}000$ ***A1***

***[1 mark]***

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 45.9583$

$2045$ (accept $2046$) ***A1***

***[3 marks]***

*continued…*

*Question 8 continued*

(f) (i) $0.282151\ldots$

$B'(40) = 0.282$ ***A1***

(ii) $0.325546...$

$C'(40) = 0.326$ ***A1***

***[2 marks]***

(g) $B'(40) < C'(40)$ (or equivalent in words) ***A1***

the population in Cecilia's model is increasing at a faster rate than in Benoit's model (in $2040$) (or equivalent) ***R1***

| **Note**: Do not award ***A0R1***. |
|---|

***[2 marks]***

***Total [15 marks]***

**3.** (a) $\frac{1}{4}e^{4x} + 6x + c$ ***A1A1A1***

**Note:** Award ***A1*** for each term.

***[3 marks]***

(b) recognition that $h(x) = \int h'(x)$ ***(M1)***

$h(x) = \frac{1}{4}e^{4x} + 6x + c$

$\left(h(1.5) =\right)\frac{1}{4}e^{4(1.5)} + 6(1.5) + c = 105$ ***(A1)***

$c = -4.85719...$

$h(x) = \frac{1}{4}e^{4x} + 6x - 4.86\left(= \frac{1}{4}e^{4x} + 6x + 96 - \frac{1}{4}e^{6}\right)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**8.** (a) (i) $a = 0.358$ (exact); $b = 30.5$ (exact answer is $30.52$) ***A1A1***

**Note**: Award ***A1A0*** if the values of $a$ and $b$ are interchanged or not labeled.

(ii) $a$ represents the (average) rate of increase (change) in population ($0.358$ millions of people per year). (or equivalent) ***R1***

***[3 marks]***

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

***[1 mark]***

(c) (i) attempt to find $B(100)$ ***(M1)***

$55.1633\ldots$

$55.2$ million OR $55{,}200{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.5%. ***A1***

**Note**: Description must include some reference to annual rate.

***[3 marks]***

(d) $54.6094\ldots$

$54.6$ million OR $54{,}600{,}000$ ***A1***

***[1 mark]***

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 58.6283\ldots$

$2058$ (accept $2059$) ***A1***

***[3 marks]***

*continued…*

*Question 8 continued.*

(f) (i) $0.242876\ldots$

$B'(75) = 0.243$ ***A1***

(ii) $0.184941...$

$C'(75) = 0.185$ ***A1***

***[2 marks]***

(g) $B'(75) > C'(75)$ (or equivalent in words) ***A1***

the population in Benoit's model is increasing at a faster rate than in Cecilia's model (in 2075) (or equivalent) ***R1***

**Note**: Do not award ***A0R1***.

***[2 marks]***

***Total [15 marks]***

**2.** $f'(x) = 4x^2 - 16$ ***A1***

sets their derivative equal to zero ***(M1)***

$4x^2 - 16 = 0$ , $(x = \pm 2)$

$p = 2$ (accept $x = 2$) ***A1***

substitutes their **positive** *p* into $f(x)$ ***(M1)***

$y = \frac{4\left(2^3\right)}{3} - 16(2)\left( = \frac{32}{3} - 32 = -\frac{64}{3} \right)$

$q = -\frac{64}{3}$ (accept $y = \frac{-64}{3}$) ***A1***

***Total [5 marks]***



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**4. METHOD 1**

attempt to set up integral $e^x - \left(-e^x\right) = 2e^x$ or $e^x$ and then double ***(M1)***

$\int \left(e^x - \left(-e^x\right)\right)dx$ OR $2\int e^x\,dx$

$= 2\int_{-1}^{1} e^x\,dx$

$= 2\left[e^x\right]_{-1}^{1}$ ***(A1)***

attempt to substitute correct limits into their integrated function and subtract ***(M1)***

$= 2\left(e - \frac{1}{e}\right)$, $2e - \frac{2}{e}$, $2e - 2e^{-1}$ ***A1***

**METHOD 2**

$\int_{-1}^{1} e^x\,dx = \left[e^x\right]_{-1}^{1}$ and $\int_{-1}^{1} -e^x\,dx = \left[-e^x\right]_{-1}^{1}$ ***(A1)***

attempt to substitute correct limits into both their integrated functions and subtract ***(M1)***

$e^1 - e^{-1}$ and $-e^1 - (-e^{-1})$

subtracts their two integrals in correct order ***(M1)***

$e^1 - e^{-1} - \left(-e^1 + e^{-1}\right)$

$= 2\left(e - \frac{1}{e}\right)$, $2e - \frac{2}{e}$, $2e - 2e^{-1}$ ***A1***

***Total [4 marks]***

**7.** (a) (i) attempt to find $v(1)$ ***(M1)***

$v(1) = 30 + 20 - 10$

max velocity $40$ ($ms^{-1}$) ***A1***

(ii) attempt to find $v(5)$ ***(M1)***

$v(5) = 30 + 20(5) - 10(5)^2 = -120$

max speed 120 ($ms^{-1}$) ***A1***

***[4 marks]***

(b) (i) $T = 3$ ***A1***

(ii) attempt to set up an integral of $v(t)$ to find area under velocity graph ***(M1)***

$\int_0^3 \left(30 + 20t - 10t^2\right) \mathrm{d}t$

$\left[30t + 10t^2 - \frac{10t^3}{3}\right]_0^3$ ***A1***

attempt to substitute their limits into their integrated function and subtract ***(M1)***

$30(3) + 10(3)^2 - \frac{10(3)^3}{3} \left(= 90 + 90 - 90\right)$

distance $90$ (metres) ***A1***

***[5 marks]***

*continued…*

*Question 7 continued.*

(c) **METHOD 1**

attempt to find total displacement by setting up an integral of $v(t)$ between $t = 0$ and $t = 5$ ***(M1)***

$$\int_0^5 \left(30 + 20t - 10t^2\right) \mathrm{d}t$$

$$= \left[30t + 10t^2 - \frac{10t^3}{3}\right]_0^5 \quad \textbf{(A1)}$$

$$= 30(5) + 10(5)^2 - \frac{10(5)^3}{3}$$

$$= 400 - \frac{1250}{3} \left(= -\frac{50}{3}\right) \quad \textbf{A1}$$

total displacement is negative OR total displacement is not zero, (so the object does return to its initial position) ***R1***

**METHOD 2**

attempt to find displacement after change in direction by setting up an integral of $v(t)$ between $t = 3$ and $t = 5$ ***(M1)***

$$\int_3^5 \left(30 + 20t - 10t^2\right) \mathrm{d}t$$

$$= \left[30t + 10t^2 - \frac{10t^3}{3}\right]_3^5 \quad \textbf{(A1)}$$

$$= \left(30(5) + 10(5)^2 - \frac{10(5)^3}{3}\right) - \left(30(3) + 10(3)^2 - \frac{10(3)^3}{3}\right) = \left(400 - \frac{1250}{3}\right) - 90$$

$$= -\frac{50}{3} - 90 \left(= -\frac{320}{3}\right) \quad \textbf{A1}$$

$\frac{320}{3} > 90$ compares to distance before change of direction, (so the object does return to its initial position) ***R1***

***[4 marks]***

***Total [13 marks]***

**1.** (a) attempt to differentiate (at least one correct term seen) ***(M1)***

$f'(x) = 3x^2 + 10x$

$3 + 10$

$f'(1) = 13$ ***A1***

***[2 marks]***

(b) attempt to find $f(1)$ ***(M1)***

$1 + 5 - 8 \quad (= -2)$

$y + 2 = 13(x - 1)$ OR $y = 13x - 15$ ***A1***

***[2 marks]***

***Total [4 marks]***

**2.** recognises to integrate $g'(x)$ ***(M1)***

$\int g'(x)\,\mathrm{d}x$ OR $\int\left(\cos x+\mathrm{e}^{2x}\right)\mathrm{d}x$

$g(x)=\sin x+\frac{1}{2}\mathrm{e}^{2x}(+C)$ ***A1A1***

substitutes $(0,7)$ into their integrated expression (must involve $+C$) ***(M1)***

$g(0)=0+\frac{1}{2}+C=7\ \Rightarrow C=6.5$

$g(x)=\sin x+\frac{1}{2}\mathrm{e}^{2x}+6.5$ ***A1***

***[5 marks]***



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**6.**

(a) correct substitution into distance formula ***A1***

$l^2 = x^2 \ln x + 4 - x^2 + x^2$ OR $\sqrt{(x-0)^2 + \left(\sqrt{x^2 \ln x + 4 - x^2} - 0\right)^2}$ OR

$\sqrt{x^2 + x^2 \ln x + 4 - x^2}$

$l = \sqrt{x^2 \ln x + 4}$ ***AG***

***[1 mark]***

(b) recognising $\frac{\mathrm{d}l}{\mathrm{d}x} = 0$ (seen anywhere) ***(M1)***

**EITHER**

attempt to use chain rule with $l$ ***(M1)***

$\frac{1}{2}\left(x^2 \ln x + 4\right)^{-\frac{1}{2}} \times \frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 \ln x + 4\right)$

attempt to use product rule with $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 \ln x + 4\right)$ ***(M1)***

$\frac{1}{2}\left(x^2 \ln x + 4\right)^{-\frac{1}{2}} \times \left[x^2 \times \frac{1}{x} + \ln x \times 2x\right]$ ***A1***

**OR**

recognising to minimise $x^2 \ln x + 4$ ***(M1)***

attempt to use product rule ***(M1)***

$x^2 \times \frac{1}{x} + \ln x \times 2x$ ***A1***

**THEN**

$x + 2x \ln x = 0$ (or equivalent) ***A1***

$\Rightarrow \ln x = -\frac{1}{2}$

$x = \mathrm{e}^{-\frac{1}{2}} \left(= \frac{1}{\sqrt{\mathrm{e}}}\right)$ ***A1***

**Note:** Award ***A0*** for including $x = 0$ in the final answer.

***[6 marks]***

***Total [7 marks]***

## Section B

**4.** (a)

**Note:** Award a maximum of ***M1A0A0*** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

**METHOD 1**

recognizing difference of two squares ***(M1)***

$\left(\cos^4 x - \sin^4 x =\right)\ \left(\cos^2 x\right)^2 - \left(\sin^2 x\right)^2$

$\left(\cos^2 x + \sin^2 x\right)\left(\cos^2 x - \sin^2 x\right)$ ***A1***

$1 \times \cos 2x$ ***A1***

$\cos^4 x - \sin^4 x = \cos 2x$ ***AG***

**Note:** Award a maximum of ***M1A1A0*** if $\cos^2 x + \sin^2 x = 1$ not seen.

**METHOD 2**

expressing $\cos^4 x$ as $\left(\cos^2 x\right)^2$ ***(M1)***

$\left(\cos^4 x - \sin^4 x =\right)\ \left(\cos^2 x\right)^2 - \sin^4 x$

$\left(1 - \sin^2 x\right)^2 - \sin^4 x$ ***A1***

$1 - 2\sin^2 x$ ***A1***

$\cos^4 x - \sin^4 x = \cos 2x$ ***AG***

**METHOD 3**

expressing $\sin^4 x$ as $\left(\sin^2 x\right)^2$ ***(M1)***

$\left(\cos^4 x - \sin^4 x =\right)\ \cos^4 x - \left(\sin^2 x\right)^2$

$\cos^4 x - \left(1 - \cos^2 x\right)^2$ ***A1***

$-1 + 2\cos^2 x$ ***A1***

$\cos^4 x - \sin^4 x = \cos 2x$ ***AG***

***[3 marks]***

*continued…*

*Question 4 continued*

(b) $\int \left(\cos^4 x - \sin^4 x\right) \mathrm{d}x = \int \cos 2x \, \mathrm{d}x$

**EITHER**

attempt to integrate $\cos 2x$ by inspection ***(M1)***

**Note:** A valid attempt must include $\frac{1}{2}$ and/or $\sin 2x$.

**OR**

$u = 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ ***(M1)***

$\int \cos 2x \, \mathrm{d}x = \frac{1}{2}\int \cos u \, \mathrm{d}u$

**THEN**

$= \frac{1}{2}\sin 2x + c$ (must include $+c$) ***A2***

**Note:** Award ***(M1)A1*** for $\frac{1}{2}\sin 2x$ as a final answer.

***[3 marks]***

***Total [6 marks]***

**7.** (a) $4x \times 2y = 60$ **OR** $xy = \frac{60}{8} \left(= 7.5\right)$ (or equivalent) ***(A1)***

$y = \frac{15}{2x}$ ***A1***

***[2 marks]***

(b) $(T =)\ 10y + 12x$ ***(A1)***

$10\left(\frac{15}{2x}\right) + 12x$ ***A1***

**Note:** Award a maximum of ***(A1)A0FT*** if their answer to part (a) is incorrect.

$T = 12x + \frac{75}{x}$ ***AG***

***[2 marks]***

(c) $\left(\frac{\mathrm{d}T}{\mathrm{d}x} =\right) 12 - \frac{75}{x^2} \quad \left(= 12 - 75x^{-2}\right)$ ***A1A1***

**Note:** Award ***A1*** for each correct term.

***[2 marks]***

*continued…*

*Question 7 continued*

(d) (i) $12 - \frac{75}{k^2} = 0$ ***A1***

$k^2 = \frac{25}{4}$ **OR** $k = \pm\frac{5}{2}$ (or equivalent) ***(A1)***

$k = \frac{5}{2}$ ***A1***

(ii) substituting their value of $k$ into $T$ ***(M1)***

$(T =)\ 60\ (\text{m})$ ***A1***

(iii) substituting their value of $k$ into their $y$ or into their length equation ***(M1)***

$y = \frac{15}{2\left(\frac{5}{2}\right)}$ **OR** $12\left(\frac{5}{2}\right) + 10y = 60$ (or equivalent)

$(y =)\ 3\ (\text{m})$ ***A1***

***[7 marks]***

*continued…*

*Question 7 continued*

(e) (i) $\left(\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = \right)\frac{150}{x^3} \quad \left(=150x^{-3}\right)$ ***A1***

(ii) **METHOD 1 (using second derivative)**

(local) minimum since $\frac{150}{\left(\frac{5}{2}\right)^3} > 0 \quad \left(\frac{48}{5} > 0\right)$ (curve is concave up) ***R1***

**Note:** Award ***R0*** if candidate does not substitute their numerical values.

Award ***R1FT*** on their $\frac{\mathrm{d}^2T}{\mathrm{d}x^2}$ and their positive value of $k$.

**METHOD 2 (using first derivative from part (c))**

(local) minimum since the derivative is negative for a value of $x < \frac{5}{2}$, and positive for a value of $x > \frac{5}{2}$. ***R1***

**Note:** Award ***R0*** if candidate does not substitute their numerical values.

Award ***R1FT*** on their $\frac{\mathrm{d}T}{\mathrm{d}x}$ and their positive value of $k$.

***[2 marks]***

***Total [15 marks]***

**1.** (a) **METHOD 1**

attempt to expand brackets on the numerator ***M1***

$(x-1)^2 = x^2 - 2x + 1$

$\frac{(x-1)^2}{x} = \frac{x^2 - 2x + 1}{x} \left( = \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x} \right)$ ***A1***

$= x - 2 + \frac{1}{x}$ ***AG***

**METHOD 2**

attempt to express terms with a common denominator ***M1***

$x - 2 + \frac{1}{x} = \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x}$

$= \frac{x^2 - 2x + 1}{x}$ ***A1***

$= \frac{(x-1)^2}{x}$ ***AG***

***[2 marks]***

(b) $\int f(x)\mathrm{d}x = \frac{x^2}{2} - 2x + \ln|x|(+c)$ ***A1A1A1***

**Note:** Award ***A1*** for each correct term.

***[3 marks]***

***Total[5 marks]***



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**4. METHOD 1**

attempt to find change in population using a definite integral **(*M1*)**

$t = 4$ at the start of 2026 (seen anywhere) **(*A1*)**

$\int_0^4 -104000\mathrm{e}^{-0.0145t}\mathrm{d}t$ **(*A1*)**

$= -404165.8...$ **(*A1*)**

attempt to add initial population to their change in population from a definite integral **(*M1*)**

population at the start of 2026 $= 6.78\times10^6 - 404165.8...$

$= 6375834.1...$

$= 6380000\left(= 6.38\times10^6\right)$ ***A1***

**METHOD 2**

attempt to find population using an indefinite integral **(*M1*)**

$P = \int -104000\mathrm{e}^{-0.0145t}\mathrm{d}t$

$\frac{-104000\mathrm{e}^{-0.0145t}}{-0.0145} + c\left(= 7172413.7...\mathrm{e}^{-0.0145t} + c\right)$ **(*A1*)**

attempt to substitute $t = 0, P = 6.78\times10^6$ into equation with $c$. **(*M1*)**

$6.78\times10^6 = 7172413.7... + c \Rightarrow c = -392413.7...$

$P = 7172413.7...\mathrm{e}^{-0.0145t} - 392413.7...$ **(*A1*)**

$t = 4$ at the start of 2026 (seen anywhere) **(*A1*)**

population at the start of 2026 $7172413.7...\mathrm{e}^{-0.0145(4)} - 392413.7...$

$= 6375834.1...$

$= 6380000\left(= 6.38\times10^6\right)$ ***A1***

***Total [6 marks]***

**6.** (a) recognition of the need to differentiate (***M1***)

$h'(x) = \frac{\pi}{50}\left(-15\sin\left(\frac{\pi x}{50}\right)\right)\left(= -\frac{15\pi}{50}\sin\left(\frac{\pi x}{50}\right) = -\frac{3\pi}{10}\sin\left(\frac{\pi x}{50}\right)\right)$ ***A1A1***

$h'(k) = -\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right)\left(= -\frac{3\pi}{10}\sin\left(\frac{\pi k}{50}\right)\right)$

**Note:** Award ***A1*** for $-15\sin\left(\frac{\pi k}{50}\right)$ and ***A1*** for factor of $\frac{\pi}{50}$.

Award ***A1A0*** for a correct expression with additional terms or additional factors.

***[3 marks]***

(b) recognition that gradient of tangent = $-\tan\left(\frac{\pi}{8}\right)$ OR $\tan\left(\frac{7\pi}{8}\right)$ (***M1***)

**Note:** Accept $\tan\left(\frac{\pi}{8}\right)$ OR $-\tan\left(\frac{7\pi}{8}\right)$ for the (***M1***)

setting their $h'(k)$ equal to $-\tan\left(\frac{\pi}{8}\right)$ OR $\tan\left(\frac{7\pi}{8}\right)\left(= -0.414213...\right)$ (***A1***)

$-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = -\tan\left(\frac{\pi}{8}\right)$ OR $-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = \tan\left(\frac{7\pi}{8}\right)$

$k = 7.24211...$ , $k = 42.7578...$

$k = 7.24$ , $k = 42.8$ ***A1***

***[3 marks]***
***Total [6 marks]***

## Section B

**3.**

(a) one critical value (seen anywhere) ***(A1)***

recognizing that for $f$ to be decreasing $f' < 0$ ***(M1)***

$x = -1.73554...$ or $x = 0.517999...$

$x \leq -1.74$ and $x \geq 0.518$ OR $x < -1.74$ and $x > 0.518$ ***A1***

***[3 marks]***

(b) correct critical value $x = -0.405465... \left( = \ln \frac{2}{3} \right)$ ( seen anywhere) ***(A1)***

attempt to find maximum of $f'$ OR recognising that $f''(x) > 0$ OR sketching graph of $f''$ OR $f''(x) = 2 - 3e^x$ ***(M1)***

concave-up for $x < \ln \frac{2}{3}$ OR $x < -0.405$ ***A1***

***[3 marks]***

***Total [6 marks]***

**5.**

(a) recognition that maximum speed occurs when $|v|$ is greatest ***(M1)***

one correct coordinate for minimum $(4.71238..., -2.71828...)$

$2.72\ (=e)\ \ (\text{ms}^{-1})$ ***A1***

**Note:** Award ***(M1)A0*** for the answer $1$, from working with degrees.

***[2 marks]***

(b) substitution of limits into correct formula ***(M1)***

$\int_0^5 |v|\,dt$

$3.84591...$

$3.85\ (\text{metres})$ ***A1***

***[2 marks]***

(c) $v = 0$ (seen anywhere) ***(M1)***

$t = 2.35619...\left(=\frac{3\pi}{4}\right)$ ***(A1)***

$a = 0.986137...$

$a = 0.986\left(=2e^{-\frac{1}{\sqrt{2}}}\right)\ (\text{ms}^{-2})$ ***A1***

***[3 marks]***

***Total [7 marks]***

**6.** (a) use of quotient rule or product rule ***(M1)***

**EITHER**

$$f'(x) = \frac{3\times 2\times (2x+a)^2(x+5)^2 - 2(2x+a)^3(x+5)}{\left((x+5)^2\right)^2}$$ ***(A1)***

$$f'(x) = \frac{6(2x+a)^2(x+5)^2 - 2(2x+a)^3(x+5)}{(x+5)^4} \left(= \frac{2(x-a+15)(2x+a)^2}{(x+5)^3}\right)$$ ***A1***

**OR**

$$f'(x) = 3\times 2\times (2x+a)^2(x+5)^{-2} - 2(2x+a)^3(x+5)^{-3}$$ ***(A1)***

$$f'(x) = 6(2x+a)^2(x+5)^{-2} - 2(2x+a)^3(x+5)^{-3} \left(= 2(x-a+15)(2x+a)^2(x+5)^{-3}\right)$$ ***A1***

***[3 marks]***

(b) recognizing $f'(1)$ is equal to $\tan 70^\circ$ (=2.74747) ***(M1)***

$\frac{6(2+a)^2 6^2 - 2(2+a)^3\times 6}{6^4} = \tan 70^\circ$ **OR** $6(2+a)^2 6^{-2} - 2(2+a)^3\times 6^{-3} = \tan 70^\circ$ ***(A1)***

$2.72844\ldots$ and $14.96968\ldots$

$a = 2.73, 15.0$ ***A1A1***

***[4 marks]***

***Total [7 marks]***

## Section B

**7.** (a) (i) $0.636619\ldots$

(amplitude = ) $0.637\ \left(=\frac{2}{\pi}\right)$ ***A1***

(ii) $\frac{2\pi}{3\pi}$ **OR** $3$ complete waves every $2$ radians ***(A1)***

(period = ) $0.667\ \left(=\frac{2}{3}\right)$ ***A1***

***[3 marks]***

(b) substituting to find $f(1.63)$ ***(M1)***

$\left(f(1.63)=\right)\ 2.21564...$ ***A1***

$2.21564... > 2.16 \Rightarrow$ lies below (the graph of *f*) ***R1***

***[3 marks]***

(c) $(m_{L_1}=)\ 0.167\ \left(=\frac{1}{6}\right)$ ***A1***

***[1 mark]***

(d) (i) substituting their $m_{L_1}$ into $-\frac{1}{m_{L_1}}$ ***(M1)***

$(m_{L_2}=)-6$ ***A1***

(ii) $y-2=-6(x-1)$ **OR** $y=-6x+8$ **OR** $6x+y-8=0$ ***A1***

***[3 marks]***

(e) equating $L_1$ and *f* ***(M1)***

$\mathrm{B}(1.64856..., 2.10809...)$

$\mathrm{B}(1.65, 2.11)$ ***A1***

***[2 marks]***

*continued…*

*Question 7 continued*

(f) **EITHER**

Attempt to form the required integral involving subtraction in either order. ***(M1)***

$\int_{t_1}^{t_2} |L_1 - f(x)| \, dx$

correct expression with limits 1 and *their* $x$ value at B

$\int_{1}^{1.64856...} \left| (\frac{1}{6}x + \frac{11}{6}) - \left( \frac{2}{\pi} \sin 3\pi x + 2 \right) \right| dx$ (or equivalent) ***(A1)***

**OR**

intersection point between A and B is $(1.34287..., 2.05714...)$

subtracting $L_1$ and $f$, in either order, and substituting into area of region formula for one area with correct limits. ***(M1)***

Two correct expressions OR correct values ***(A1)***

$\int_{1}^{1.34287...} \left( (\frac{1}{6}x + \frac{11}{6}) - \left( \frac{2}{\pi} \sin 3\pi x + 2 \right) \right) dx = (0.144618...)$ **AND**

$\int_{1.34287...}^{1.64856...} \left( \left( \frac{2}{\pi} \sin 3\pi x + 2 \right) - \left( \frac{1}{6}x + \frac{11}{6} \right) \right) dx = (0.108585...)$

**THEN**

$0.253204\ldots$

(area =) $0.253$ ***A1***

***[3 marks]***

***Total [15 marks]***

**9.** (a) (i) $7.43076\ldots$

$v(1) = 7.43 \ (\text{m s}^{-1})$ ***A1***

(ii) equating $v(t)$ and $5$ ***(M1)***

$0.348114\ldots$

$0.348$ (seconds) ***A1***

***[3 marks]***

(b) recognizing $a = v'(t)$ ***(M1)***

$0.590930\ldots$

$0.591$ (seconds) ***A1***

***[2 marks]***

(c) (i) considering large values of $t$ ***(M1)***

$\left(\lim_{t \to \infty} (v(t)) = \right) 8.14 \ (\text{ms}^{-1})$ ***A1***

(ii) **EITHER**

the race lasts a finite time (e.g. it ends after (Fiona) crosses the line) ***R1***

**OR**

the graph approaches the limiting value but Fiona will never attain that speed ***R1***

**OR**

Fiona cannot maintain that speed for a long period of time ***R1***

**Note:** Award ***R1*** for a valid reason that does not contradict their answer to part (c)(i).

***[3 marks]***

*continued…*

*Question 9 continued*

(d) Fiona takes $t_f$ seconds to travel $200\ \text{m}$

**EITHER**

recognizing distance travelled by either athlete in the first $t$ seconds is

$\int_0^t v(t)\,\mathrm{d}t$ **OR** $\int_0^t w(t)\,\mathrm{d}t$ (seen anywhere) ***(M1)***

equating distance travelled by Fiona to $200\ (\text{m})$ ***(M1)***

$$\int_0^{t_f} v(t)\,\mathrm{d}t = 200$$

**OR**

attempt to integrate $v(t)$ ***(M1)***

$$s(t) = 8.14\sqrt{t^2 + 0.2} - 3.64031...$$

equating 200 to *their* $s(t)$ (must include a constant of integration) ***(M1)***

$$8.14\sqrt{t^2 + 0.2} - 3.64031... = 200$$

**THEN**

$t_f = 25.0132...$ (accept $25$) ***(A1)***

recognition that a definite integral of $w(t)$ with 0 and *their* $t_f$ is required ***(M1)***

(Lucy's distance =) $\int_0^{25.0132...} w(t)\,\mathrm{d}t$ $(= 195.772...)$

distance from finishing line = 200 – *their* Lucy's distance ***(M1)***

$4.22788\ldots$

$4.23\ (\text{m})$ ***A1***

***[6 marks]***

***Total [14 marks]***

**4.** (a) (i) $c = 9$ ***A1***

(ii) attempt to substitute $(3, 0)$ into $y = ax^2 + c$ with *their* $c$ ***(M1)***

$9a + 9 = 0$

$a = -1$ ***A1***

***[3 marks]***

(b) **METHOD 1** (single integral)

attempt to subtract areas (seen anywhere) ***(M1)***

$\int_0^3 \left(9 - x^2\right) - \left(9 - 3x\right)\, \mathrm{d}x \quad \left(= \int_0^3 3x - x^2\, \mathrm{d}x\right)$ OR $\int_0^3 \left(9 - x^2\right) \mathrm{d}x - \text{triangle area}$

correct integration $\left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$ **OR** $\left[9x - \frac{1}{3}x^3\right]_0^3$ ***(A1)***

attempt to substitute their limits into their integrated expression (condone absence of the $0$ substitution) ***(M1)***

$\frac{3}{2}(3)^2 - \frac{1}{3}(3)^3 - \left(\frac{3}{2}(0)^2 - \frac{1}{3}(0)^3\right)$ OR $9(3) - \frac{1}{3}(3)^3$

$\frac{27}{2} - 9$ OR $18 - \frac{27}{2}$

Area $= \frac{9}{2}$ (square units) ***A1***

*continued…*

*Question 4 continued.*

**METHOD 2** (evaluating two integrals separately)

$\int_0^3 \left(9-x^2\right) \mathrm{d}x = \left[9x-\frac{1}{3}x^3\right]_0^3$ **OR** $\int_0^3 (9-3x)\, \mathrm{d}x = \left[9x-\frac{3}{2}x^2\right]_0^3$ ***(A1)***

**Note**: Award ***A1*** for correct integration. Ignore absence of limits.

attempt to substitute their limits into both of their integrated expressions (condone absence of the $0$ substitution) ***(M1)***

$9(3)-\frac{1}{3}(3)^3-\left(9(0)-\frac{1}{3}(0)^3\right)$ and $9(3)-\frac{3}{2}(3)^2$

attempt to subtract the two integrals in any order (seen anywhere) ***(M1)***

$9(3)-\frac{1}{3}(3)^3-\left(9(3)-\frac{3}{2}(3)^2\right)$ OR $\int\left(9-x^2\right)\mathrm{d}x-\int(9-3x)\mathrm{d}x$

$18-\frac{27}{2}$

Area $=\frac{9}{2}$ (square units) ***A1***

***[4 marks]***

***Total [7 marks]***

**8.** (a) recognition that $f'(x) = 0$ (seen anywhere) ***(M1)***

attempt to solve quadratic equation (condone not seeing $=0$) ***(M1)***

$a = -5,\ b = 1$ ***A1***

***[3 marks]***

(b) (local) maximum ***A1***

($f'(x)$/gradient) changes from positive to negative (at $x = a$)

OR $f(x)$ changes from increasing to decreasing (at $x = a$)

OR $f''(a) = -18$ (accept $f''(a) < 0$) ***R1***

**Note:** Do not award ***A0R1***.

***[2 marks]***

(c) $f''(x) = 6x + 12$ ***(A1)***

$6c + 12 = 0$ ***(A1)***

$c = -2$ ***A1***

***[3 marks]***

(d) yes (there is a point of inflexion at $x = c$) ***A1***

($f''(x)$/second derivative) changes sign at $x = c$

OR concavity (of $f$) changes at $x = c$

OR $f'(x)$/gradient decreasing to the left of $c$, and increasing to the right of $c$ ***R1***

***[2 marks]***

**Note:** Award ***R0*** if their only reason is that $f''(c) = 0$. Do not award ***A0R1***.

*continued…*

*Question 8 continued.*

(e) recognize to find $\int f'(x)\mathrm{d}x$ ***(M1)***

$x^3 + 6x^2 - 15x(+C)$ ***(A1)***

substitution of $(-2, 36)$ into their integrated expression (must contain $+C$) ***(M1)***

$36 = (-2)^3 + 6(-2)^2 - 15(-2) + C$

$C = -10$ ***A1***

$f(x) = x^3 + 6x^2 - 15x - 10$

***[4 marks]***

***Total [14 marks]***

**2.** recognition to integrate $4\cos x - \sin x$ ***(M1)***

$\int 4\cos x - \sin x \, \mathrm{d}x$ (accept absence of $\mathrm{d}x$)

$= 4\sin x + \cos x \, (+c)$ ***A1A1***

substitute $x = \pi$ and $y = 3$ into their integrated expression containing '$+c$' : ***(M1)***

$3 = 0 - 1 + c$ or $c = 4$

$(y =) 4\sin x + \cos x + 4$ ***A1***

**Note:** Award no marks for candidates who differentiate rather than integrate, (obtaining $-4\sin x - \cos x$ ).

***[5 marks]***

**8.** (a)

**Note:** In this question, the marks are independent and may be awarded in either order.

finding correct discriminant $k^2 - 26$ ***A1***

recognition that discriminant $\left(b^2 - 4ac\right) < 0$ ***M1***

$b^2 - 4ac < 0$ or $k^2 - 26 < 0$ or $k^2 < 26$

(since $k \in \mathbb{Z}^+$) $k = 5$ ***AG***

***[2 marks]***

(b) (i) $x = -5$ (must be an equation) ***A1***

(ii) attempt to substitute their value of $x$ (from axis of symmetry) into $\frac{1}{2}x^2 + 5x + 13$

OR attempt to use completing the square ***(M1)***

$(y =) \frac{1}{2}(-5)^2 + 5(-5) + 13 \left(= \frac{1}{2}\right)$ OR $(y =) \frac{1}{2}(x+5)^2 + \frac{1}{2}$

$\left(-5, \frac{1}{2}\right)$ ***A1***

***[3 marks]***

(c) at $x = -3,\ y = \frac{5}{2}$ (seen anywhere) ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = x + 5$ ***A1***

at $x = -3$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ ***A1***

normal gradient is negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = -3$ ***(M1)***

gradient of normal $= -\frac{1}{2}$

$y - \frac{5}{2} = -\frac{1}{2}(x+3)$ OR $\frac{5}{2} = -\frac{1}{2}(-3) + c \Rightarrow c = 1$ ***A1***

$y = -\frac{1}{2}x + 1$ ***AG***

***[5 marks]***

*continued…*

*Question 8 continued*

(d) attempt to describe shaded area as the difference of two integrals or one integral and the area of a trapezium ***(M1)***

$(A=)\ \int_{-3}^{0}\left(\frac{1}{2}x^2+5x+13\right)\mathrm{d}x-\int_{-3}^{0}\left(-\frac{1}{2}x+1\right)\mathrm{d}x$ OR $\int_{-3}^{0}\left(\frac{1}{2}x^2+\frac{11}{2}x+12\right)\mathrm{d}x$

OR $\int_{-3}^{0}\left(\frac{1}{2}x^2+5x+13\right)\mathrm{d}x-\frac{1}{2}\left(\frac{5}{2}+1\right)3$

correct integration ***A2***

$\left[\frac{x^3}{6}+\frac{5x^2}{2}+13x\right]_{-3}^{0}-\left[-\frac{x^2}{4}+x\right]_{-3}^{0}$ OR $\left[\frac{x^3}{6}+\frac{11x^2}{4}+12x\right]_{-3}^{0}$ OR $\left[\frac{x^3}{6}+\frac{5x^2}{2}+13x\right]_{-3}^{0}-\frac{21}{4}$

**Note:** If not completely correct, award ***A1*** for at least two terms correctly integrated.

substituting $(0$ and$)\ -3$ into their integrated expression and subtracting ***(M1)***

$0-\left(\frac{(-3)^3}{6}+\frac{11(-3)^2}{4}+12(-3)\right)$ OR $0-\left(\frac{(-3)^3}{6}+\frac{5(-3)^2}{2}+13(-3)\right)-\frac{21}{4}$ (or equivalent)

$=\frac{63}{4}$ ***A1***

***[5 marks]***
***Total [15 marks]***

**9.** (a)

(i) substituting $x = 0$ into $f(x) = \dfrac{5e^x - e^{2x}}{e^x + 1}$ ***(M1)***

$(0, 2)$ ***A1***

**Note**: Award ***A1*** for $x = 0,\ y = 2$.
Award ***A0*** if only $2$ or $y = 2$ is seen.

(ii) attempt to find root ***(M1)***

$\dfrac{5e^x - e^{2x}}{e^x + 1} = 0$ OR $5e^x - e^{2x} = 0$ OR $5e^x = e^{2x}$

**EITHER**

attempt to factorise $5e^x - e^{2x} = 0$ or divide it through by $e^x$ ***(M1)***

$e^x(5 - e^x) = 0$ OR $5 - e^x = 0$ (since $e^x \neq 0$)

$e^x = 5$ OR $x = \ln 5$ ***A1***

**OR**

take ln of both sides of $5e^x = e^{2x}$ ***(M1)***

$\ln(5e^x) = \ln e^{2x}$

$\ln 5 + x = 2x$ OR $x = \ln 5$ ***A1***

**THEN**

$(\ln 5, 0)$ ***A1***

**Note:** Award ***A1*** for $x = \ln 5, y = 0$. Award ***A0*** if only $\ln 5$ or $x = \ln 5$ is seen.

***[6 marks]***

(b) $f(x) \leq 7 - 2\sqrt{6}$ OR $y \leq 7 - 2\sqrt{6}$ OR $\left(-\infty, 7 - 2\sqrt{6}\right]$ (or equivalent) ***A1***

***[1 mark]***

*continued…*

*Question 9 continued*

(c) **EITHER**

attempt to use quotient rule with $u = 5e^x - e^{2x}$ and $v = e^x + 1$ ***(M1)***

$$f'(x) = \frac{\left(e^x + 1\right)\left(5e^x - 2e^{2x}\right) - e^x\left(5e^x - e^{2x}\right)}{\left(e^x + 1\right)^2}$$ ***A1***

$$f'(x) = \frac{5e^{2x} - 2e^{3x} + 5e^x - 2e^{2x} - 5e^{2x} + e^{3x}}{\left(e^x + 1\right)^2}$$ ***A1***

**OR**

attempt to use product rule with $u = 5e^x - e^{2x}$ and $v = \left(e^x + 1\right)^{-1}$ ***(M1)***

$$f'(x) = \left(5e^x - e^{2x}\right) \times -e^x \times \left(e^x + 1\right)^{-2} + \left(e^x + 1\right)^{-1}\left(5e^x - 2e^{2x}\right)$$ ***A1***

$$f'(x) = \frac{-5e^{2x} + e^{3x} + 5e^{2x} - 2e^{3x} + 5e^x - 2e^{2x}}{\left(e^x + 1\right)^2}$$ ***A1***

**THEN**

$$f'(x) = \frac{-e^{3x} - 2e^{2x} + 5e^x}{\left(e^x + 1\right)^2}$$ ***AG***

***[3 marks]***

*continued…*

*Question 9 continued*

(d) setting $f'(x) = 0$ (since Q is a maximum point) ***(M1)***

$-e^{3x} - 2e^{2x} + 5e^{x} = 0$

attempt to factorise by $e^{x}$ or divide through by $e^{x}$ ***(M1)***

$e^{2x} + 2e^{x} - 5 = 0$ ***A1***

attempt to solve their quadratic ***(M1)***

$e^{x} = \frac{-2 \pm \sqrt{24}}{2}$ OR $e^{x} = -1 \pm \sqrt{6}$ (or equivalent) ***A1***

(reject $e^{x} = -1 - \sqrt{6}$ since $e^{x} > 0$ for all $x$) $\Rightarrow e^{x} = -1 + \sqrt{6}$ ***A1***

$(q =) \ln\left(\sqrt{6} - 1\right)$ ***AG***

***[6 marks]***

***Total [16 marks]***

***7.*** (a) (i) $x = 2$ ***A1***

(ii) $y = \frac{3}{2}$ ***A1***

**Note:** Award ***A1A0*** for answers $x \neq 2$ and $y \neq \frac{3}{2}$.

***[2 marks]***

(b) **EITHER**

attempt to use the quotient rule ***(M1)***

$f'(x) = \frac{(2x-4)\times 3 - (3x+1)\times 2}{(2x-4)^2}$ ***A1A1***

**Note:** Award ***A1*** for each term in the numerator with correct signs, provided correct denominator is seen.

**OR**

attempt to use the product rule ***(M1)***

$f'(x) = 3(2x-4)^{-1} + (-1)(3x+1)(2x-4)^{-2}\times 2$ ***A1A1***

**Note:** Award ***A1*** for each term.

**THEN**

$f'(x) = \frac{-14}{(2x-4)^2}$ ***A1***

***[4 marks]***

*continued…*

*Question 7 continued.*

(c) recognising $f'(x) < 0$ (when $f$ is decreasing), **OR** $(2x-4)^2 > 0$

**OR** the gradient (function) is negative (may be seen on a labelled graph of $f'$) ***A1***

for all values of $x$ (with $x \neq 2$) **OR** always (or equivalent) ***A1***

$f$ is never increasing ***AG***

**Note:** Do not accept a sketch and argument based on the graph of $f$.

***[2 marks]***

(d) attempt to use the distance formula ***(M1)***

$d = \sqrt{(x-3)^2 + \left(\frac{3x+1}{2x-4} - 5\right)^2}$ ***A1***

**Note:** Accept $d^2 = (x-3)^2 + (f(x)-5)^2$.

***[2 marks]***

(e) attempt to find value of $x$ that gives minimum value of $d$ ***(M1)***

graph of $d$, $d' = 0$,

$(x =) -0.305218...$, $(f(-0.305218...) =) -0.0182941...$

P has coordinates $(-0.305, -0.0183)$ ***A1A1***

**Note:** Award ***A1*** for each value.

***[3 marks]***

***Total [13 marks]***

**8.** (a) correct substitution of $t = 12$ into $F(t)$ ***(A1)***

$6130.82...$

$6130$ (accept $6131$) answer must be integer ***A1***

***[2 marks]***

(b) $1850 \times 1.105^x \geq 35000$ (accept strict inequality or equality) ***(A1)***

attempt to solve for $x$ using table, graph, or logarithms ***(M1)***

$29.4471\ldots$ OR $(x = 29 \Rightarrow)\ 33471.7...$ OR $(x = 30 \Rightarrow)\ 36986.2...$

$x = 30$ (months) ***A1***

**Note:** Condone use of $t$.

***[3 marks]***

(c) $\left(1.105^t = \left(e^k\right)^t \Rightarrow\right) e^k = 1.105$ **OR** $\left(1.105^t = e^{kt} \Rightarrow\right) kt = \ln 1.105^t$ (or equivalent) ***(A1)***

$k = \ln 1.105$ (do not accept $k = 0.0998$) ***A1***

**Note:** Do not award any marks if the candidate substitutes an approximate value obtained from using $F(t) = 1850 \times 1.105^t$ into $F(t) = 1850 \times e^{kt}$, for example $F(12) = 1850 \times e^{12k} = 6130.82...$, as this will not lead to an exact value for $k$.

***[2 marks]***

(d) recognizing the need to find $F'$ or $F'(15)$ ***(M1)***

$825.911\ldots$

$F'(15) = 826$ (frogs per month) ***A1***

***[2 marks]***

(e) setting $G(15)$ equal to $6995$ ***(M1)***

$17.8890...$

$A = 17.9$ ***A1***

***[2 marks]***

*continued…*

*Question 8 continued.*

(f) recognize need for $G'' = 0$ or maximum of $G'$ ***(M1)***

$28.8996...$

$t = 28.9$ ***A1***

***[2 marks]***

(g) any one of the following reasons ***(A1)***

- $G(t)$ accounts for frog deaths in the population.
- $G(t)$ accounts for limited space/resources.
- $G(t)$ has carrying capacity.
- $F(t)$ has the population growing forever, without bound.

***[1 mark]***

***Total [14 marks]***

**9.** (a) (i) speed of airplane $= 60\ (\text{ms}^{-1})$ ***A1***

(ii) speed of car $= 0\ (\text{ms}^{-1})$ ***A1***

***[2 marks]***

(b) (i) recognizing $v_{\text{air}} = v_{\text{car}}$ when speeds are the same ***(M1)***

$6.35564...$

$t = 6.36\ (\text{s})$ ***A1***

(ii) $31.7782\ldots$

speed $= 31.8\ (\text{ms}^{-1})$ ***A1***

***[3 marks]***

(c) let $s_{\text{air}}(t)$ be the displacement of the airplane and $s_{\text{car}}(t)$ be the displacement of the car at time $t$

recognizing to integrate either velocity function ***(M1)***

$\int 60\text{e}^{-0.1t}\ \text{d}t,\ \int 5t\ \text{d}t$

correct integration ***(A1)(A1)***

$-600\text{e}^{-0.1t},\ \frac{5}{2}t^2\ (+C)$ (award ***A1*** for each term)

recognize to subtract displacements (seen anywhere) ***(M1)***

**EITHER**

substituting initial condition into their $d(t)$ ***(M1)***

$100 = -600\text{e}^{-1(0)} - \frac{5}{2}\times 0^2 + C$ OR $-100 = 600\text{e}^{-1(0)} + \frac{5}{2}\times 0^2 + C$ (or equivalent)

**OR**

substituting initial condition into their $s_{\text{air}}(t)$ ***(M1)***

$\left(s_{\text{car}}(0) = 0 \Rightarrow\right) 100 = -600\text{e}^{-1(0)} + C$

*continued…*

*Question 9 continued.*

**THEN**

$C = 700$ (or $-700$) ***(A1)***

$d(t) = -600e^{-0.1t} - \frac{5}{2}t^2 + 700$ (accept $d(t) = \left|600e^{-0.1t} + \frac{5}{2}t^2 - 700\right|$) ***A1***

***[7 marks]***

(d) recognizing to solve their $d(t) = 0$ ***(M1)***

$15.0586...$

$t = 15.1$ (s) ***A1***

***[2 marks]***

(e) attempt to find the distance travelled by the car ***(M1)***

$\int_0^{15.0586} 5t \, dt$, $\frac{5}{2}(15.0586)^2$ ***(A1)***

$566.904\ldots$

$567$ (m) (accept $570$ (m) from 3 sf answer in (d)) ***A1***

**Note:** Award ***(M1)(A1FT)A0***, where appropriate, if the candidate has an incorrect $d(t)$ and/or incorrect value of $t$ which leads to their distance travelled by the car to be less than $100$ (m).

For example, where the candidate finds $d(t) = 600e^{-0.1t} - \frac{5}{2}t^2 - 500$ in part (c), award ***(M1)(A1FT)A0*** to $7.08$, or $7.06$ from using 3 sf value.

***[3 marks]***

***Total [17 marks]***

**9.** (a) (i) attempt to find a maximum point ***(M1)***

$(x =) 0.718442...$ or $(y =) 1.80645\ldots$ or $(x =) 3.71844...$ or $(y =) 1.19181\ldots$ etc

$1.80645\ldots$

(maximum displacement) $= 1.81$ (cm) ***A1***

(ii) recognition that $v = \frac{\mathrm{d}s}{\mathrm{d}t}$ ***(M1)***

recognition that the gradient at $t = 0$ represents the maximum velocity ***(M1)***

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$ at $t = 0$

$v = 4.18879025...\left(\frac{4\pi}{3}\right)$

$v = 4.19\left(\frac{4\pi}{3}\right)$ ($\mathrm{cms}^{-1}$) ***A1***

***[5 marks]***

(b) (i) attempt to find a minimum point ***(M1)***

$(x =) 2.21844...$ or $(y =) -1.46729...$ OR $(x =) 5.21844...$ OR $(y =) -0.968053\ldots$

$-1.46729...$

$-1.47$ ***A1***

(ii) $(t = 3.5)\ s = 1.06620...$

$s = 1.07$ ***A1***

***[3 marks]***

*continued…*

*Question 9 continued*

(c) **METHOD 1**

attempt to sum **distances** from O involving maximum, minimum and distance at $t = 3.5$ ***(M1)***

$1.80645\ldots \times 2 + 1.46729... \times 2 + 1.06620...$ ***(A1)***

$= 7.61369...$

$7.61\,(\text{cm})$ ***A1***

**METHOD 2**

attempt to integrate the absolute value of $\frac{\mathrm{d}s}{\mathrm{d}t}$ ***(M1)***

$\int_0^{3.5} \left| \frac{d}{dt}\left( 2^{1-\frac{t}{5}} \sin\left(\frac{2\pi t}{3}\right)\right)\right| \mathrm{d}t$ ***(A1)***

$= 7.61369...$

$7.61\,(\text{cm})$ ***A1***

***[3 marks]***

(d) $(T =) 1.5$ ***A1***

***[1 mark]***

*continued…*

*Question 9 continued*

(e)

(i) attempt to divide two of $u_1, u_2, u_3 \ldots$ ***(M1)***

$\frac{u_2}{u_1}\left(=\frac{1.46729...}{1.80645...}\right)$ or $\frac{u_3}{u_2}\left(=\frac{1.19181...}{1.46729...}\right)$ or $r^2=\frac{u_3}{u_1}\left(=\frac{1.19181...}{1.80645...}\right)$

$0.812252...$

$(r=)0.812$ ***A1***

(ii) attempt to use the sum to infinity of the GP (***M1)***

$S_\infty=\frac{1.80645...}{1-0.812252...}(=9.62167...)$

recognition that the total distance is double their $S_\infty$ (***M1)***

$2\times S_\infty=2\times\frac{1.80645...}{1-0.812252...}$ OR $2\times 9.62167..$

$19.2433...$

$19.2(\text{cm})$ ***A1***

**Note**: Award (***M1)***(***M1)A0*** for values of $r$ that lead to an answer that is smaller than 2 x first maximum displacement $(3.6129)$.

***[5 marks]***

***Total [17 marks]***